This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

EducT
118
88.785

HARVARD COLLEGE
LIBRARY

TRANSFERRED FROM THE
GRADUATE SCHOOL
OF
BUSINESS ADMINISTRATION

3 2044 097 001 978

Smithdeal Business College,
1113 Main Street,
Richmond, Va.

John F. May

JAN 9 1889

THE

COMMERCIAL ARITHMETIC

A PRACTICAL TEXT-BOOK

FOR

BUSINESS COLLEGES, HIGH SCHOOLS
AND ACADEMIES

[Sadler, Warren Hunt]

FIRST EDITION.

BALTIMORE, MD.
W. H. SADLER, PUBLISHER
6 AND 8 N. CHARLES STREET
1888

Edu T 118,88,785

HARVARD COLLEGE LIBRARY

Apr 24, 1928

ALL RIGHTS RESERVED.

Infringements will be prosecuted to the full extent of the law.

Entered, according to Act of Congress, in the year 1886, by

WARREN H. SADLER,

in the Office of the Librarian of Congress, at Washington.

ELECTROTYPED BY
SMITH & McDOUGAL, N. Y.

PRINTED BY
FERGUSON BROS. & CO.

[illegible]
4 [illegible]
[illegible]
[illegible]

PREFACE.

IN the preparation of this volume, the authors have sought to furnish an arithmetic of convenient size, and embracing the whole field of modern business practice to the exclusion of every useless theory or obsolete subject.

At the very beginning, they have demonstrated the law of increase and decrease of numbers, exhibited the application of this general law to all classes of problems as they are successively reached, and thus released pupils from their former bondage to arbitrary rules, adapted only to a specific class of problems.

Brief and clear in its definitions and explanations, simple and labor-saving in its methods of solution, strictly utilitarian in its large collection of problems, it will be found a reliable exponent of the best modern thought upon the subjects which it treats.

By its exclusion of non-essentials, it is enabled to be unusually complete in every essential of written arithmetic, containing an ample supply of just the class of problems which our boys and girls will be required to solve, and of the simple, business methods of solution which they will find it convenient to practice, when they become men and women.

This arithmetic will be published in two editions, differing materially from each other in contents: one intended for Business Colleges, and the other specially adapted for use in other schools. Both editions are confidently submitted to teachers and school officers as meeting every reasonable requirement of the classes of schools for which they are intended.

THE AUTHORS.

TO TEACHERS.

To secure an intelligent solution of arithmetical problems, pupils should be thoroughly conversant with the generalizations of **29, 58, 59, 60, 61,** and **62,** relating to integers; of **128, 129, 130,** and **132,** relating to common and decimal fractions; and of **288,** relating to percentage and its various applications.

Pupils should be required to show the application of the general law of increase and decrease of numbers to particular problems as they are successively solved, until they have mastered its principles.

Rules and arbitrary processes of solution are soon forgotten; but the constant iteration and reiteration of a few general principles, simple in character, are likely to leave a permanent impression upon the most backward pupils. It is by thus generalizing that Business Colleges have attained their wide popularity and success; and the same methods are as practicable in our common schools.

Every new topic should be introduced with suitable oral exercises relating thereto; but the only *suitable* oral exercises are inductive, which require no previous study by the pupil. For this reason, oral exercises have been omitted, and the space thus saved has been utilized with additional written problems. If the teacher cannot originate these exercises, he should provide himself with a desk copy of Sadler's Inductive Arithmetic, or one of the many other arithmetics which excel in this respect.

CONTENTS.

INTEGERS.

PAGE
Numeration and Notation . . 7
Addition 12
Subtraction 16
Multiplication 20
Division 25
United States Money 32
PAGE
Relation of Integers 36
Review 38
Factoring 41
Common Divisors 43
Common Multiples. 46
Cancellation 47

COMMON FRACTIONS.

Preliminary Definitions . . . 49
Reduction 51
Addition 56
Subtraction. 58
Multiplication 60
Division 63
Relation of Common Fractions . 67
Review. 72

DECIMAL FRACTIONS.

Preliminary Definitions . . . 76
Numeration and Notation . . 78
Reduction 80
Addition. 82
Subtraction 84
Multiplication. 85
Division 86
Review. 88

COUNTING-HOUSE PRACTICE.

Commercial Calculations . . . 91
Bills and Accounts 95

DENOMINATE NUMBERS.

Preliminary Definitions. . . 103
Measures of Value. 104
" " Weight 107
" " Extension . . . 108
" " Surface 109
" " Volume 110
" " Capacity 111
Circular Measure 112
Measure of Time 113
Miscellaneous Measures . . 113
Reduction Descending. . . . 114
" Ascending. . . . 116
" of Denom. Fractions 118
Addition of Comp. Numbers 121
Subtraction " " " 122
Multiplication" " " 125
Division " " " 126
Review " " " 128
Latitude and Longitude . . . 131
Metric System 135

PRACTICAL MEASUREMENTS.

PAGE
Rectangular Surfaces 142
Carpeting, Paving, etc. . . . 144
Rectangular Volumes 146
Board Measure 149
Stone work 151
Brick work 153
Bins, Granaries, etc. 155
Cisterns 156

PERCENTAGE.

Simple Percentage 157
Profit and Loss 168
Trade Discount 176
Commission 183
Ordinary Interest 193
Accurate " 206
Interest Formulas 208
Review 212
Annual Interest 214
Compound Interest 217
True Discount 223
Commercial Paper 226
Bank Discount 230
Savings-Banks 238
Partial Payments:
  United States Rule 244
  Mercantile Rule. 247
  Other Rules 250
Stocks and Bonds 251
Exchange 262
  Domestic 264
  Foreign 268
Custom-House Business . . . 274
Taxes 278
Insurance 281
  Fire and Marine 282
  Life 286

RATIO AND PROPORTION.

Ratio 290
Proportion 291
Single Rule of Three 293
Double Rule of Three . . . 297
Alligation 300
Bankruptcy 307
General Average 310
Storage 315
Equation of Payments . . . 321
Averaging Accounts 333
Averaging Accounts Sales . . 343
Cash Balance 346
Partnership 350

APPENDIX.

The Stock Exchange 367
Rapid Addition 377
Rapid Subtraction 379
Rapid Multiplication 380
Marking Goods 390
Answers 393

ARITHMETIC.

1. **Arithmetic** treats of numbers, and teaches how they may be variously applied to produce required results.

2. A **Number** is an expression of a quantity by means of characters or figures.

3. The **Unit** of a number is *one* of the things which it expresses. Thus, *one* gallon is the unit of nine gallons, *one* acre of seventy acres.

4. A **Concrete Number** names the kind of units which it represents; as five *gallons,* three *pounds,* six *books.*

5. An **Abstract Number** is one whose unit is not named.

REMARK.—In business practice, there are no abstract numbers; but concrete numbers are frequently considered abstractly as denoting *times* (number of repetitions of) some other number.

6. **Like Numbers** are those which express the same kind of units; as two *acres* and ten *acres,* five *books* and nine *books.*

7. **Unlike Numbers** are those which express different kinds of units; as three *feet* and five *hours,* two *miles* and four *gallons.*

8. An **Integer** or **Integral Number** is one which is composed exclusively of whole units.

9. An **Example** is a proposition containing certain given conditions, previously explained, from which can be derived one or more conclusions or answers, which are not given.

10. A **Solution** is a process for obtaining a required answer.

11. A **Rule** is a description of the successive steps of a solution.

NUMERATION AND NOTATION.

12. **Numeration** is the art of *reading* numbers when expressed by figures.

13. **Notation** is the art of *writing* numbers.

Rem.—The method of expressing numbers by *figures* is called *Arabic Notation;* and by letters, *Roman Notation.*

14. The Arabic method embraces the following ten figures:

0	1	2	3	4	5	6	7	8	9
Naught,	*One,*	*Two,*	*Three,*	*Four,*	*Five,*	*Six,*	*Seven,*	*Eight,*	*Nine.*

Each of the above figures, when standing alone, represents the number written under it. Consequently, *nine* is the greatest number that can be expressed by one figure, and numbers greater than nine are expressed by the artifice of increasing the local value of each figure *tenfold* by placing *one* figure to the right of it; ten times ten or a *hundredfold* by placing *two* figures to the right of it; ten times ten times ten, or a *thousandfold,* by placing *three* figures to the right of it, etc.

Rem.—The distance of a figure from the right of a number is called its order; thus, the *first* figure at the right is the *first* or *units'* order; the second figure, the *second* or *tens'* order; the *third* figure, the *third,* or *hundreds'* order; the *fourth* figure, the *fourth,* or *thousands'* order, etc.; *ten units of any order equaling one unit of the next order on the left.* Hence, the representative value of a figure increases tenfold for each order that it is moved to the left, and decreases tenfold for each order that it is moved to the right. For the names and position of the several orders, see Decimal Notation Table, **138.**

ILLUSTRATIVE EXERCISE.

15. Read 9342.

Explanation.—9342 is read *nine thousand three hundred and forty-two;* the local value of 9 being *thousand* because it has *three* figures at its right, the local value of 3 being *hundred* because it has *two* figures at its right, the local value of 4 being *tens* because it has *one* figure at its right, and the local value of 2 being its simple value (two) because it has *no* figure at its right.

Read, or express by written words, the following numbers:

1. 6.
2. 15.
3. 19.
4. 36.
5. 48.
6. 52.
7. 67.
8. 71.
9. 85.
10. 98.
11. 109.
12. 173.
13. 248.
14. 472.
15. 805.
16. 960.
17. 2137.
18. 8250.
19. 7635.
20. 5096.
21. 6008.
22. 3209.
23. 9500.
24. 1350.
25. 4006.

16. With numbers greater than hundreds, the further artifice is employed of considering them in *periods* or sets of *three figures* each. The first set of three figures at the right is named the *units' period;*

the second set, the *thousands'* period; the third, *millions*; the fourth, *billions*; the fifth, *trillions*; the sixth, *quadrillions*; etc.

ILLUSTRATIVE EXERCISE.

17. Read 95342783678.

Explanation.—Begin at the right and separate the number into periods of three figures each; as follows, 95, 342, 783, 678. As the right-hand period (678) is pointed off, think of its name (units); as the second period (783) is pointed off, think of its name (thousands); and so continue with the remaining periods until the left-hand period (95) is reached, which will thus prove to be billions.

Commence with this left-hand period and read it as if it stood alone, adding its name (95 billion); then read the succeeding period as if it stood alone, adding its name (342 million); then the next period (783 thousand); and finally the last period (678), omitting its name (units). Hence,

Rule.—I. *Commence at the right and separate the number into periods of three figures each.*

II. *Then begin at the left and in regular order read each period as if it stood alone, adding the name of each period except that of units.*

Read, or express by written words, the following numbers:

1. 37548.
2. 1218326.
3. 716532.
4. 56319265.
5. 76285.
6. 9135264378.
7. 813461217.
8. 5196272.
9. 9293718249.
10. 58376.
11. 92162715208508.
12. 9000273008.
13. 620005107.
14. 5000000018.
15. 6200108300.
16. 500096.
17. 7200905.
18. 2800009500060.
19. 63178290005.
20. 1218000.
21. 57000328.

ILLUSTRATIVE EXERCISE.

18. Express, by means of figures, three trillion, seventy-five billion, one hundred and forty-eight million, five hundred and ninety-two.

Ans. 3,075,148,000,592.

Explanation.—Commence with the highest period of the given number (trillions), setting it down and placing a comma after it (thus, 3,); then write the next lower period (billions), but as 75 contains only two of the *three* figures required for a period (**16**), write a cipher in the place of the omitted figure (thus, 3,075,); then write the next lower period (millions), placing a comma after it (thus, 3,075,148,); then write the next lower period (thousands), and, as there are no thousands in the given number, indicate the omission of the three figures belonging to that period by writing three ciphers (thus, 3,075,148,000,); then, as it is the lowest period of the scale, conclude by writing the next lower period (units), obtaining 3,075,148,000,592.

Rule.—I. *Commence with the highest period of the given number, writing it and each succeeding period in regular order, and filling vacant periods with three ciphers.*

II. *If any period except the highest should contain only one or two figures, prefix ciphers thereto until it contains three figures.*

Express, by means of figures, the following numbers:

1. Five thousand two hundred and forty-seven.
2. Sixty-five thousand eight hundred and seventy-one.
3. Eight thousand and fifty-three.
4. Twelve thousand and nine.
5. Ninety thousand and thirty-six.
6. Two hundred and eight thousand and ninety-four.
7. Six million, two hundred and three thousand, and twenty-five.
8. Nine million, thirty-seven thousand, five hundred and forty.
9. Two million, five thousand, seven hundred and six.
10. Five million, two hundred and eight thousand, and thirty-two.
11. Six billion, ninety-five thousand, one hundred and forty-eight.
12. Nine billion, forty-three million, and seven thousand.
13. Ninety-six trillion, thirty-five thousand and nine.
14. Fifteen trillion, seven hundred and thirty-nine billion, fifty million, one hundred and ninety thousand, and seventy-six.
15. Twelve trillion, six hundred and twenty-nine million, two hundred and twelve thousand, five hundred and sixty-one.
16. Ninety-two quadrillion, eight trillion, two hundred and four thousand, one hundred and forty.

ROMAN NOTATION AND NUMERATION.

19. The **Roman Notation** employs the following seven letters:

I	V	X	L	C	D	M
One,	*Five,*	*Ten,*	*Fifty,*	*One hundred,*	*Five hundred,*	*One thousand.*

20. The method of combining these seven characters to express numbers is shown in the following

ROMAN NOTATION TABLE.

I	One.	XIV	Fourteen.	XC	Ninety.
II	Two.	XV	Fifteen.	C	One hundred.
III	Three.	XVI	Sixteen.	CC	Two hundred.
IV	Four.	XVII	Seventeen.	CCC	Three hundred.
V	Five.	XVIII	Eighteen.	CCCC	Four hundred.
VI	Six.	XIX	Nineteen.	D	Five hundred.
VII	Seven.	XX	Twenty.	DC	Six hundred.
VIII	Eight.	XXX	Thirty.	DCC	Seven hundred.
IX	Nine.	XL	Forty.	DCCC	Eight hundred.
X	Ten.	L	Fifty.	DCCCC	Nine hundred.
XI	Eleven.	LX	Sixty.	M	One thousand.
XII	Twelve.	LXX	Seventy.	MM	Two thousand.
XIII	Thirteen.	LXXX	Eighty.	MDCCC	Eighteen hundred.

ILLUSTRATIVE EXERCISE.

Write 2784 by the Roman method.

Explanation.—First write its highest order (thousands), thus, MM; then its next highest order (hundreds), thus, MMDCC; then its next highest order (tens), thus, MMDCCLXXX; and finally its lowest order (units), thus, MMDCCLXXXIV. Therefore,

Rule.—*Write the highest order, then the next highest order, and so continue until the lowest order is written.*

Express the following numbers by the Roman notation:

1. 18.
2. 35.
3. 19.
4. 28.
5. 42.
6. 57.
7. 75.
8. 81.
9. 63.
10. 99.
11. 117.
12. 125.
13. 178.
14. 216.
15. 524.
16. 839.
17. 421.
18. 676.
19. 949.
20. 354.
21. 1346.
22. 1886.
23. 1027.
24. 2948.
25. 2729.

Express the following numbers by the Arabic notation:

26. XXIV.
27. XLVI.
28. XIX.
29. LXXIX.
30. XCVIII.
31. LXXXIV.
32. CXVII.
33. CCCCXXVIII.
34. CCLXIV.
35. CCXCVII.
36. DCCCXXII.
37. DCXLIX.
38. MCCXVIII.
39. MMDCXXXIX.
40. MMMCCLXII.
41. MDXXXIV.
42. DCCCCXVI.
43. MDCCXLIV.

ADDITION.

21. **Addition** is the process of finding *one number* which will express as many units as two or more other numbers.

Rem. 1.—The several numbers to be added are called **Addends**; the number which expresses as many units as its addends is called the **Sum** or **Amount.**

Rem. 2.—The **Sign of Addition** is an erect cross (+). It is read *plus*, and is placed between numbers to denote that they are to be added.

Rem. 3.—The **Sign of Equality** is two short, horizontal, and parallel lines (=). It is read *equals*, or *equal to*, and denotes that the quantity or quantities on its left are equal to the quantity or quantities on its right.

22. The **Principles of Addition** are:

1. *Only like numbers* (6) *and like orders can be added.*
2. *The sum expresses units of the same name as its addends.*
3. *The sum of two or more numbers equals the sum of their respective similar orders, added separately.*

Note.—Orders are said to be **like** or **similar** when they are at the same distance from the right of their respective numbers.

ILLUSTRATIVE EXAMPLE.

23. What is the sum of $532, $758, $427, and $263?

Write the addends so that their right-hand figures shall fall in the same column. Add the figures of the right-hand column, obtaining 3+7+8+2 = 20, or 2 tens and 0 units. Place 0 units under the units' column, and carry 2 tens to the tens' column. Next add the tens' column, obtaining 2 (the carrying figure) +6+2+5+3 = 18 tens, or 1 hundred and 8 tens. Place 8 tens under the tens' column, and carry 1 hundred to the hundreds' column. Next add the hundreds' column, obtaining 1 (the carrying figure) +2+4+7+5 = 19 hundreds. Hence,

Solution.

$$\begin{array}{r} \$532 \\ 758 \\ 427 \\ \underline{263} \\ \$1980 \\ {\scriptstyle 1\,2\;} \end{array}$$

Rule.—I. *Write the numbers to be added so that units of the same order shall fall in the same column, and draw a horizontal line beneath.*

II. *Beginning at the right, add each column separately; and write the sum underneath, if it does not exceed nine.*

III. *If the sum of any column exceeds nine, set down its right-hand figure only, and add the number expressed by its remaining figures to the next column on the left.*

IV. *Set down the entire sum of the left-hand column.*

24. Proof of Addition.—*Add each column in the contrary direction. If the same results are obtained, the sum is supposed to be correct.*

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
$3	$6	$8	$5	$32	$82	$327	$982
2	5	7	9	76	15	582	758
5	6	9	4	85	36	264	329
4	3	5	8	47	75	429	678
3	1	7	3	93	28	843	825
1	4	6	7	25	34	736	597

(9.)	(10.)	(11.)	(12.)	(13.)
$4175	$9286	$3582	$48275	$2681753
5698	5735	6896	27368	5176382
8527	8267	5275	59487	9628625
3489	3489	9182	61735	8763897
7678	9261	7364	98275	6374859
2537	7082	5385	63842	6283926

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)	(21.)
$3	$7	$5	$41	$31	$85	$253	$516
8	3	2	58	27	54	138	273
6	4	1	34	62	48	847	832
5	8	8	72	16	32	413	127
2	5	2	67	75	76	272	983
7	3	7	25	83	27	765	275
1	2	1	13	28	63	994	764
5	7	6	92	54	84	283	353
3	2	2	84	12	12	812	626
8	1	8	38	47	41	647	189
6	4	1	12	83	38	733	841
2	8	4	79	35	29	528	237
1	3	5	23	21	75	264	723
9	2	3	62	43	11	652	812
6	5	4	56	54	44	546	254

24

Find the sum of

22. $32 + $183 + $54 + $3486 + $569 + $9375 + $85 + $4103.

23. $875 + $284 + $37 + $5200 + $398 + $62 + $74 + $2168 + $720.

24. 763 ft. + 827 ft. + 49 ft. + 5283 ft. + 768 ft. + 2175 ft.

25. 1536 lb. + 8973 lb. + 5178 lb. + 926 lb. + 8259 lb. + 36 lb. + 867 lb.

26. 9365 bu. + 8375 bu. + 1473 bu. + 826 bu. + 4123 bu. + 15378 bu.

27. 986 yd. + 325 yd. + 7261 yd. + 5820 yd. + 569 yd. + 8371 yd.

28. 6275 gal. + 5183 gal. + 985 gal. + 3267 gal. + 75 gal. + 1528 gal.

29. 1738 lb. + 9168 lb. + 8273 lb. + 5298 lb. + 9 lb. + 6832 lb. + 65 lb.

30. $78352 + $4127 + $83745 + $968173 + $4826 + $91278 + $9175 + $68312 + $4183 + $59187 + $29183 + $75832 + $5867.

31. 46235 miles + 8976 miles + 5807 miles + 98397 miles + 68325 miles + 892 miles + 5140 miles + 6839 miles + 326 miles + 2125 miles.

32. A grocer's sales were as follows: Monday, $675; Tuesday, $709; Wednesday, $530; Thursday, $695; Friday, $918; and Saturday, $1230. What were his total sales for the week?

33. A man bought a boat for $15835, paid $2307 for repairs, $375 for painting, $525 for upholstering, and then sold it at a profit of $1200. What did he receive for the boat?

34. Eight hogsheads of tobacco weigh respectively 825, 750, 890, 735, 773, 815, 745, and 836 pounds. What is the total weight?

35. In a certain city there are 85290 Americans, 19275 Irish, 6241 Germans, 528 French, and 5238 of other nationalities. What is the population of that city?

36. I sold a farm for $6785 and lost $2340. How much would I have received if I had sold the farm at a gain of $659?

37. A man was born in 1823 and died when he was 58 years old. In what year did he die?

38. What is the sum of five hundred and ninety-one dollars, two thousand three hundred and eight dollars, sixty-nine dollars, and nineteen thousand one hundred dollars?

39. A man bought four building lots. For the first he paid $1275; for the second, $1520; for the third, $256 more than the first; and for the fourth, $325 more than the second. How much did he pay for the four lots?

40. At a certain saw-mill, 14134 feet of lumber were sawed in January; 18275 feet in February; 19213 feet in March; 18372 feet in April; 15204 feet in May, and 19213 feet in June. How many feet were sawed during the six months?

41. A man bought a lot for $1375, paid $408 to have it graded, $75 to have it enclosed, and afterward sold it at a profit of $238. How much did he receive for the lot?

Rem.—To arrange addends in vertical columns involves unnecessary labor. Pupils should therefore be trained to add items horizontally, in which manner they are frequently written. The process is as follows: *First add all the right-hand orders of the several addends; then all the second orders from the right; then all the third orders, etc., setting down and carrying as usual.*

Find the sum by horizontal addition:

42. $382 + $487 + $245 + $413.
43. $575 + $732 + $419 + $826.
44. $912 + $826 + $375 + $250.
45. $78 + $2158 + $97 + $23 + $6.
46. $2732 + $4158 + $8976.
47. $8276 + $497 + $565 + $4173.
48. $5183 + $9876 + $96 + $185.
49. $8250 + $376 + $65 + $2341.
50. $41 + $35 + $183 + $2573.
51. $815 + $9137 + $28 + $72348.

In the following examples, add vertically, then horizontally; and prove both by comparing the sums of their respective totals, which must agree.

52. POPULATION OF A CITY.

	FIRST WARD.	SECOND WARD.	THIRD WARD.	FOURTH WARD.	FIFTH WARD.	SIXTH WARD.	TOTALS.
Americans........	2175	3251	2876	4531	3250	1760	*****
Germans..........	837	735	2531	1260	924	853	****
Irish............	1235	1526	627	837	1537	526	****
French...........	125	271	182	206	193	175	****
Italians.........	376	582	163	293	185	116	****
Others...........	975	1236	927	2301	627	823	****
Totals..........	****	****	****	****	****	****	*****

53. GRAIN EXPORT OF A CITY FOR ONE WEEK (in bushels).

	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTALS.
Corn......	28325	15236	35715	29128	75183	46217	******
Wheat....	35719	41719	50108	32546	59275	81126	******
Oats.......	12136	9237	18265	7268	6950	17230	*****
Barley.....	18230	15738	21375	15928	19263	13637	******
Rye.......	5275	6829	7201	11325	7825	13261	*****
Totals...	*****	*****	*****	*****	******	******	******

SUBTRACTION.

25. Subtraction is the process of taking one number from another number of the same kind.

REM. 1.—The number to be taken is called the **Subtrahend**; the number from which the subtrahend is taken is called the **Minuend**; and the number which is left after taking the subtrahend from the minuend is called the **Remainder** or **Difference.**

REM. 2.—The **Sign of Subtraction** is a short, horizontal line (—). It is read *minus*, and is placed between two numbers to indicate that the one on the right is to be subtracted from the one on the left.

26. The **Principles of Subtraction** are:

1. *Only like numbers and like orders can be subtracted, one from the other.*
2. *The remainder or difference expresses units of the same name as those expressed by the minuend and subtrahend.*
3. *The increase of any order in the subtrahend is equivalent to a corresponding decrease of the same order of the minuend.*
4. *The total difference of two numbers equals the differences of the several orders of the less number separately subtracted from similar orders of the greater number.*

ILLUSTRATIVE EXAMPLE.

27. Subtract $681 from $923.

Solution.

$923
681
$242

Explanation.—Subtract the right-hand order of the subtrahend (1) from the right-hand order of the minuend (3), obtaining 2 as the right-hand order of the remainder. Then subtract the second order of the subtrahend (8) from the second order of the minuend (2); but since this is impossible, borrow 1 from the next higher order of the minuend (9) and add this borrowed unit of the third order (equal to 10 units of the second order.—See Rem., **14**) to the 2 units in the second order, obtaining 12 units of the second order, from which subtract 8 units of the second order, obtaining 4 units of the second order in the remainder.

Next subtract the 6 units of the third order of the subtrahend from 9—1 borrowed unit, or the 8 remaining units of the third order of the minuend, obtaining 2 units of the third order in the remainder. The two preceding subtractions from 9, first of the borrowed unit and then of the 6 in the subtrahend, could have been consolidated in one subtraction of 6 plus the borrowed unit, or 7, from 9, obtaining the same result, 2. Hence,

Rule.—I. *Place the subtrahend under the minuend so that units of the same order shall fall in the same column.*

II. *Begin at the right and subtract each order of the subtrahend from the like order of the minuend, writing the result under the order subtracted.*

III. *If any order of the subtrahend be greater than the like order of the minuend, add 10 to that order of the minuend and then subtract.*

IV. *If 10 are added to any order of the minuend, 1 should be mentally added to the next higher order of the subtrahend.*

28. Proof of Subtraction.—*Add the remainder to the subtrahend; the result should equal the minuend. Or, subtract the remainder from the minuend; the result should equal the subtrahend.*

EXAMPLES FOR PRACTICE.

Find the difference between

1. 7243 bu. and 3926 bu.
2. $82651 and $51762.
3. $583769 and $278293.
4. 927183 ft. and 691827 ft.
5. $5173289 and $2891726.
6. $82153716 and $52681725.
7. $6912835 and $1098276.
8. 827135 lb. and 751682 lb.
9. $2618351 and $2583179.
10. $65173828 and $91783182.
11. $90817308 and $21705019.
12. $81075050 and $60208384.
13. $26007005 and $50003098.
14. $700810002 and $309000280.
15. $937000000 and $205890786.
16. $528300007 and $200600280.
17. $281007009 and $100602085.
18. $500800706 and $308070608.
19. $608070706 and $402090785.
20. $700050002 and $200400601.

21. The production of wheat in the United States was 308102700 bushels in 1874, and 448756630 bushels in 1879. How many more bushels were raised in 1879 than in 1874?

22. The salary of a clerk is $850 per annum, of which he spends $498. How much does he save in a year?

23. If A lives 325 miles due north of a certain city and B 419 miles due north of the same city, how far apart do they live?

24. A man deposited $19075 in bank and afterwards withdrew $5107. How much had he remaining in bank?

25. A man is worth $307200 in real estate and personal property.

If the value of his personal property is $9075, what is the value of his real estate ?

26. The Revolutionary War commenced in 1775, at which time Washington was 43 years old. In what year was Washington born ?

27. Gen. U. S. Grant was born in 1822 and died in 1885. How old was he when he died ?

28. A speculator bought a house and lot for $19250, and sold them for $26075. What was his gain ?

29. I sold a farm for $9350, which was $1200 more than it cost. What did I pay for the farm ?

30. The minuend is 58210 bushels and the remainder 15216 bushels. What is the subtrahend ?

31. The minuend is 4860 miles and the subtrahend 1970 miles. What is the remainder ?

REVIEW OF ADDITION AND SUBTRACTION.

29. Addition and Subtraction Compared.—(*a*) Addition is employed to find the *total* when all the addends are given, and (*b*) subtraction to find *one* addend when the total and the remaining addends are given. (*c*) The minuend in subtraction corresponds with the total in addition ; (*d*) the subtrahend in subtraction corresponds with the given addend [if one] or the sum of the given addends [if more than one] ; and (*e*) the remainder in subtraction corresponds with the *required* addend.

General Direction: *If the total of all the addends is required, add ; if the total of all the addends is given, subtract.*

EXAMPLES.

1. What is my total indebtedness, if I owe A $532, B $650, C $375, and D $176 ?

REM.—To find the total, *all* the addends being given, involves addition (*a*, **29**).

2. I owe $1733 to A, B, C, and D. How much do I owe A, if I owe B $650, C $375, and D $176 ?

REM.—To find one addend (what I owe A) when the total ($1733) and the remaining addends ($650, $375, $176) are given, involves subtraction (*b*, **29**), the given total ($1733) being the minuend (*c*, **29**), the sum of the given addends ($650 + $375 + $176 = $1201) being the subtrahend (*d*, **29**).

3. If the minuend is 15082 bushels, and the remainder 2156 bushels, what is the subtrahend ?

Rem.—The minuend corresponds with total in addition (*c*, **29**) and is *given*, hence subtract (Gen. Direction, **29**) the given addend, 2156 bu. to find the required addend.

4. The remainder is 2175 miles and the subtrahend 1216 miles. What is the minuend ?

Rem.—Since the minuend or total is *required*, add (Gen. Dir., **29**).

5. A dealer bought 6850 bushels of wheat, of which he sold 975 bushels to A, 394 bushels to B, and 1625 bushels to C. How many bushels remained unsold ?

6. A merchant deposited $3250 in bank, against which he drew checks for $275, $195, $826, and $93. How much had he still on deposit ?

7. The sum of two quantities is 19506 gallons. If one of the quantities is 8953 gallons, what is the other ?

8. A merchant's receipts for the first half of a year were $8275. His receipts in January were $825, in February $946, in March $1050, in April $925, and in May $1536. What were his receipts in June ?

9. A gentlemen bought 5 houses for $28350, paying $3800 for the first, $4500 for the second, $5300 for the third, and $6975 for the fourth. What did he pay for the fifth house ?

10. An army went into battle with 75830 men, and came out of the battle with 59275 men. How many men were lost ?

11. Grover Cleveland was born in the year 1837 and inaugurated President of the United States in 1885. How old was he when inaugurated ?

12. A gentleman willed $75835 to his wife and son. He gave $30250 to his son. How much did he give his wife ?

13. A man bought a lot for $1925, paid $425 to have it graded, $75 to have it enclosed, and then sold it at a loss of $275. How much did he receive for the lot ?

14. I bought a house for $4850, spent $756 in repairs, and then sold it for $5320. How much did I lose ?

15. A man had $3500 in bank. He afterward drew out $685, then deposited $284, and then drew out $850. How much had he remaining in bank ?

16. The salary of a bookkeeper is $1250 per annum ; of which he spends $300 per annum for board and $375 for other expenses. How much per annum does he save ?

29

17. Two men engaged in partnership with a joint capital of $85917. If one partner invested $32715, what was the investment of the other?

18. A mechanic's income in 1885 was as follows: wages $600, rent of houses $325, and interest on funds in a savings-bank $42. His expenditures were: butcher's bill $130, baker's $75, grocer's $264, insurance and taxes $58, clothing $130, and other expenses $70. How much did he save during that year?

19. The arrivals of fruit at a certain port were as follows: On Monday, 1731 boxes of peaches, 750 boxes of apples, 1230 boxes of pears, and 5600 watermelons; on Tuesday, 375 boxes of apples, 748 boxes of pears, 1245 boxes of peaches, and 3500 watermelons; on Wednesday, 2400 boxes of peaches, 960 boxes of pears, 530 boxes of apples, and 1830 watermelons; on Thursday, 850 boxes of apples, 1890 boxes of peaches, 875 boxes of pears, and 3208 watermelons; on Friday, 735 boxes of apples, 1050 boxes of pears, 2350 boxes of peaches, and 2500 watermelons; and on Saturday, 2125 boxes of peaches, 1340 boxes of pears, 1220 boxes of apples, and 8200 watermelons. What was the total arrival of each during the week?

20. A cyclopedia is divided into three volumes, the first containing 450 pages, the second 425 pages, and the third 509 pages. How many pages does the cyclopedia contain?

MULTIPLICATION.

30. **Multiplication** is a short process of finding the sum, *when the several addends are equal.*

Rem. 1.—The **Multiplicand** is the addend to be repeated; the **Multiplier** denotes how many repetitions of the addend are required; and the **Product** is the result obtained by repeating the multiplicand as many times as there are units in the multiplier.

Rem. 2.—The multiplicand and multiplier are called **factors** of the product.

Rem. 3.—The **Sign of Multiplication** is an oblique cross (×). It is read *multiplied by,* or *times;* and is placed between numbers to indicate that they are to be multiplied.

31. The **Principles of Multiplication** are:

1. *The multiplicand in business practice is a concrete number.*

2. *The multiplier, though usually a concrete number, is always regarded abstractly as denoting* **times.**

3. *The product expresses units of the same name as the multiplicand.*

4. *The total product equals the sum of the products of the several orders of the multiplicand separately multiplied by each order of the multiplier.*

32. MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**

NOTE.—It will not be a waste of time to memorize the whole of the above table, especially if the learner is preparing himself for a mercantile life.

ILLUSTRATIVE EXAMPLE.

33. Multiply $627 by 9.

Explanation.—To find 9 times $627, separately take 9 times each of the orders of $627, commencing with the lowest. Thus, 9 times 7 units equal 63 units, or 6 tens and 3 units (Rem., **14**). Write 3, and carry 6 tens to 9 times 2 tens, or 18 tens, obtaining 24 tens, or 2 hundreds and 4 tens. Write 4 tens, and carry 2 hundreds to 9 times 6 hundreds, or 54 hundreds, obtaining 56 hundreds.

Solution.

$$\begin{array}{r} \$627 \\ 9 \\ \hline \$5643 \end{array}$$

EXAMPLES FOR PRACTICE.

Multiply

1. 38754 bu. by 5.
2. 9182634 lb. by 8.
3. 5179138 gal. by 6.
4. 2153827 yr. by 4.
5. 590100897 bu. by 7.
6. 602508007 yd. by 3.

Multiply

7. 850703005 lb. by 9.
8. 5207100806 ft. by 2.
9. $2710350278 by 12.
10. $82635051206 by 8.
11. $71625803005 by 11.
12. $52007008034 by 10.

ILLUSTRATIVE EXAMPLE.

34. Multiply $9283 by 64.

Solution.

$9283
64
37132
55698
$594112

Explanation.—First multiply $9283 by the right-hand order of the multiplier (4 units) as shown in **33**, obtaining $37132 as a partial product.

Then similarly multiply $9283 by the second order of the multiplier (6 tens), obtaining 55698 tens of dollars. As it is necessary to add similar orders of both partial products (Prin. 1, **22**) to obtain the complete product, arrange the second partial product (55698 tens) so that its right-hand figure (8 tens) shall fall under the same order (3 tens) of the first partial product.

Add the partial products, obtaining $594112 as the complete product.

When one of the factors consists of only one figure.

Rule.—I. *Place the factor having one figure under the right-hand order of the remaining factor.*

II. *Commence with the lowest order of the upper factor, and multiply it and each of its successive higher orders by the lower factor, setting down and carrying as in addition.*

When both factors consist of more than one figure.

Rule.—I. *Under the factor which has the greater number of figures, write the remaining factor so that units shall fall under units, tens under tens, etc.*

II. *Applying the preceding Rule, successively multiply the upper factor by each order of the lower factor, as if that order stood alone.*

III. *Place the first figure of each partial product under that order of the lower factor which was used to produce it.*

IV. *Add the several partial products to find the complete product.*

34

35. Proof of Multiplication.—*Reverse the position of the factors, and multiply. If the same product is obtained, the result may be presumed to be correct.*

EXAMPLES FOR PRACTICE.

Multiply

1. 7234178 lb. by 62.
2. 5918372 bu. by 75.
3. 1683425 yd. by 81.
4. 9287365 yr. by 93.
5. 517836252 gal. by 527.
6. 678329183 ft. by 648.
7. 2651378 yd. by 913.
8. 4268317 oz. by 765.

Multiply

9. $875034082 by 2185.
10. $507083005 by 5091.
11. $687034806 by 9807.
12. $205008324 by 70091.
13. $622900508 by 53716.
14. $415382691 by 900106.
15. $68250003 by 52018.
16. $97003628 by 82713.

17. A gentleman bought a square of ground, containing 82 building lots, at $895 per lot. What was the total cost?

18. The produce of a field of 236 acres averaged 28 bushels of wheat per acre. How many bushels of wheat did the field produce?

19. What is the total weight of 385 bales of cotton which average 419 pounds per bale?

20. A bookkeeper's salary is $85 per month, and his expenses $54 per month. How much does he save in a year?

21. Two men are 500 miles apart, and approach each other, one traveling at the rate of 24 miles per day and the other at 35 miles per day. How far apart will they be after each has traveled 8 days?

22. A has 375 bushels of wheat, B 7 times as much as A, and C 5 times as much as B. How many bushels have they together?

23. How many pounds of flour in 837 barrels, allowing 196 pounds to each barrel?

24. What is the cost of building 238 miles of railroad at $23478 per mile?

25. A man bought 425 acres of land at $56 per acre, sold 125 acres at $50 per acre, 230 acres at $65 per acre, and the remainder at $75 per acre. What was his total gain?

26. If a man buy 75 horses at $130 per head, spend $8 per head in taking them to market, and then sell them at $150 per head, what will be his total gain?

27. In a certain orchard are 25 rows, each containing 42 fruit trees. How many fruit trees does the orchard contain?

28. If I buy a farm of 370 acres for $20720, expend $532 in improvements, and then sell it at $64 per acre, what will be my total gain?

36. A **Continued Product** is one which is produced by the multiplication of *more than two factors*.

ILLUSTRATIVE EXAMPLE.

37. What is the cost of 72 organs at $156 each?

Explanation.—One of the factors (72) is capable of being separated into two other factors (8 × 9). Multiply $156 by one of these factors (8), obtaining $1248 as the cost of 8 organs, and multiply this result ($1248) by the remaining factor (9), obtaining $11232 as the cost of 9 times 8, or 72 organs.

Solution.

$$\begin{array}{r} 72 = 8 \times 9 \\ \$156 \\ \underline{8} \\ \$1248 \\ \underline{9} \\ \$11232 \end{array}$$

EXAMPLES FOR PRACTICE.

In like manner, multiply

1. 5768 lb. by 63.
2. 92815 yr. by 32.
3. 76234 bu. by 49.
4. 39826 gal. by 96.
5. 16275 men by 42.
6. $91837526 by 132.
7. $40827063 by 108.
8. $57003807 by 84.
9. $62783251 by 144.
10. $3278265 by 378 (7 × 6 × 9).

ILLUSTRATIVE EXAMPLES.

38. Multiply $928 by 100.

Explanation.—Simply place at the right of 928 as many ciphers as are found in 100.

Solution.

$928 × 100 = $92800

39. Multiply $32500 by 41000.

Explanation.—$32500 = $325 × 100 (**38**); and 41000 = 41 × 1000. Applying **37**, first multiply $325 by 41, obtaining $13325; then multiply this result by 100 × 1000, by annexing *two* plus *three*, or *five* ciphers.

Solution.

$$\begin{array}{r} \$32500 \\ \underline{41000} \\ 325 \\ \underline{1300} \\ \$1332500000 \end{array}$$

When the multiplier is 10, 100, 1000, 10000, etc.

Rule.—*At the right of the multiplicand place as many ciphers as are found in the multiplier.*

When either or both factors terminate in ciphers.

Rule.—I. *Arrange the factors so that the right-hand significant figure of the lower factor shall fall under the right-hand significant figure of the upper factor.*

II. *Multiply as if there were no ciphers at the right of either factor, and to the product thus obtained annex as many ciphers as are found at the right of both factors.*

EXAMPLES FOR PRACTICE.

Multiply

1. 536 yr. by 100.
2. 98756 bu. by 10000.
3. 6297 gal. by 1000.
4. 625 acres by 600.
5. 4275 men by 5000.
6. 3978 ft. by 700.

Multiply

7. $395000 by 1800.
8. $62800 by 95000.
9. $5308200 by 700.
10. $6281000 by 32500.
11. $910000 by 8000.
12. $53200 by 710.

DIVISION.

40. **Division** is the process of finding one factor (Rem. 2, **30**) when the product and the remaining factor are given.

Rem. 1.—The given product is called the **dividend**; the given factor, the **divisor**; and the required factor, the **quotient**.

Rem. 2.—After the integral part of the required factor has been found, whatever is left of the dividend is called the **remainder**.

Rem. 3.—Division is indicated by the sign (÷) or (:), usually the former. Either sign is read *divided by;* and is written after the dividend and before the divisor.

41. The **Principles of Division** are:

1. *The dividend is a concrete number* (Prin. 1 and 3, **31**).

2. *The divisor is a concrete number if the given factor is the multiplicand* (Prin. 1, **31**), *or an abstract number if the given factor is the multiplier* (Prin. 2, **31**).

3. *The quotient is concrete or abstract according to whether the required factor is the multiplicand or the multiplier.*

4. *The remainder is concrete, and expresses units of the same name as the dividend.*

Summary.—*That term of division is abstract which corresponds with the given or required multiplier; the remaining terms are concrete and express units of the same name.*

5. *The total quotient equals the sum of the quotients of the several orders of the dividend, separately divided by the divisor.*

42. To divide when the divisor is 12 or less.

ILLUSTRATIVE EXAMPLE.

Divide $942 by 6.

Solution.

Divisor. Dividend.
6) $942
$157 Quotient.

Explanation.—Since the entire dividend cannot conveniently be divided by the divisor in one operation, apply Prin. 5, **41**, and separately divide each order of the dividend, commencing with the highest. 9 (hundreds) ÷ 6 = 1 (hundred) and 3 (hundred) over. Write this partial quotient (1) under the order divided, carry the remainder (3 hundreds or 30 tens) to the next order of the dividend (4 tens), and divide the result (34 tens) by 6, obtaining 5 (tens) as the next quotient figure, and 4 (tens) as the next remainder. Carry the last remainder (4 tens or 40 units) to the next order of the dividend (2 units), and divide the result (42 units) by 6, obtaining 7 units as the last quotient figure. Hence,

Rule.—I. *From the left of the dividend, select the least number of orders that will contain the divisor. Find how many times the divisor is contained in these orders and write the result as the first quotient figure.*

II. *If the division is not exact, conceive the remainder to be placed before the next right-hand order of the dividend, find how many times the divisor is contained in the number thus formed, and write the result as the second quotient figure. Proceed in a similar manner until the entire dividend has been divided.*

43. Proof of Division.—*Multiply the quotient by the divisor, and to the product add the remainder, if any. The result should equal the dividend.*

EXAMPLES FOR PRACTICE.

Divide

1. 2875632 lb. by 3.
2. 6783936 yr. by 8 yr.
3. 9273840 gal. by 5 gal.
4. $397821616 by 4.
5. 482178 bbl. by 6 bbl.
6. 31827690 ft. by 9.
7. 51382192 bu. by 8.
8. $321802000 gal. by 5.
9. 518322 trees by 7 trees.
10. $513981765 by $11.

Divide

11. $8500170800 by 10.
12. 310500300 pens by 12 pens.
13. 4187309008 lb. by 8 lb.
14. $14009080071 by 7.
15. 4193008002 bu. by 6 bu.
16. 51837204 ft. by 12 ft.
17. $28100907009 by 9.
18. $8200600030 by $5.
19. $413078060514 by 7.
20. $6628173924 by $6.

44. Short Division is the method of obtaining the successive quotient figures by *mental processes,* as in the Ill. Ex., **42.** It should always be preferred when the divisor is not greater than 12.

45. Long Division is the method of obtaining the successive quotient figures by written processes. It is usually employed when the divisor exceeds 12.

46. To divide when the divisor exceeds 12.

ILLUSTRATIVE EXAMPLE.

Divide $39679 by 48.

Solution.

$$\begin{array}{rrl} \text{Divisor} & \text{Dividend} & \text{Quotient} \\ 48\,) & 39679\,(& \$826\tfrac{31}{48} \\ & \underline{384} & \\ & 127 & \\ & \underline{96} & \\ & 319 & \\ & \underline{288} & \\ & 31 & \text{Remainder.} \end{array}$$

Explanation.—396 (hundreds) divided by 48 equals 8 (hundreds). Write 8 as the first quotient figure; multiply the divisor (48) by this quotient figure; place the product (384 hundreds) under the orders divided (396 hundreds); and subtract, obtaining 12 (hundreds) as a remainder.

To this remainder annex the succeeding order of the dividend (7 tens), obtaining 127 (tens) as the next partial dividend which, divided by the divisor (48), will produce 2 (tens) as the second quotient figure. Multiply the divisor (48) by the second quotient figure (2 tens); write the result (96 tens) under 127 (tens); and subtract, obtaining 31 (tens) as a remainder, to the right of which bring down the next order of the dividend (9 units).

Divide the second partial dividend by 48, obtaining 6 as the next quotient figure. Multiply and subtract as before, obtaining 31 as a final remainder; under which write the divisor, and annex the result ($\frac{41}{48}$) to the quotient, obtaining \$826$\frac{41}{48}$ as the complete quotient. Hence,

Rule.—I. *Find the first quotient figure as directed in Rule for short division,* **42.**

II. *Multiply the divisor by the first quotient figure; write the product under the first partial dividend; subtract; and to the right of the remainder place the next undivided order of the dividend for a second partial dividend.*

III. *Divide the second partial dividend by the divisor, and write the result as the second quotient figure. Multiply, subtract, and bring down, as with the first partial dividend; and continue in this manner until all the orders of the dividend have been divided.*

IV. *If any partial dividend is less than the divisor, place a cipher in the quotient, and annex the next undivided order of the dividend for a new partial dividend.*

NOTE 1.—If any product be greater than the partial dividend, the quotient figure is too large.

NOTE 2.—If any remainder be equal to or greater than the divisor, the quotient figure is too small.

NOTE 3.—A convenient method of finding the quotient figure is to point off from the right of any partial dividend as many figures lacking 1 as there are figures in the divisor, and divide the remaining figures of the partial dividend by the left-hand figure of the divisor (if the following figure of the divisor is less than 5) or by the left-hand figure of the divisor increased by 1 (if the following figure is 5 or more).

EXAMPLES FOR PRACTICE.

Divide

1. 537164983 bu. by 47 bu.
2. 42178374 gal. by 54.
3. 7198342 yd. by 42 yd.
4. 917080878 yr. by 34.
5. 4871639 bbl. by 87 bbl.
6. 918215763 ft. by 38 ft.
7. 21748329 lb. by 89 lb.
8. 14736281 tons by 364 tons.
9. 482798 men by 243 men.
10. 734189261 bu. by 832 bu.

Divide

11. 879386453 lb. by 5218 lb.
12. 1378256 yr. by 7831.
13. 731924676 yd. by 9324.
14. 3182798 bu. by 144 bu.
15. \$31400819 by \$4832.
16. 804726 miles by 3258.
17. \$37919114144 by 4328.
18. \$123984567 by \$6298.
19. \$73829156 by \$57382.
20. \$5177306171268 by 63756.

46

21. A farm of 428 acres was sold for $36380. At what price per acre was it sold ?

22. At $175 each, how many horses can be bought for $14700 ?

23. How many boxes will contain 5292 pieces of muslin, if the capacity of each box is 54 pieces ?

24. The product of two factors is 186750 gallons and one of the factors is 498 gallons. What is the other factor ?

25. An orchard contains 31188 trees planted in 113 equal rows. How many trees are in each row ?

26. A jeweler invested $3151 in watches bought at $137 each. How many watches did he buy ?

27. A farmer harvested 3504 bushels of corn from a field containing 48 acres. What was the average yield per acre ?

28. A steamer completed a voyage of 7975 miles in 29 days. What was its average speed per day ?

47. In division and multiplication, much unnecessary labor may frequently be avoided by understanding the relation to each other of the three terms of division, *divisor, quotient, dividend,* and their respective equivalent terms of multiplication, *multiplicand, multiplier, product,* as shown in the following

General Principles governing Division and Multiplication.

1. *To multiply the dividend* (product) *or divide the divisor* (multiplicand) *by any number is equal to multiplying the quotient* (multiplier) *by the same number.*

2. *To divide the dividend* (product) *or multiply the divisor* (multiplicand) *by any number is equal to dividing the quotient* (multiplier) *by the same number.*

3. *To multiply or divide both dividend* (product) *and divisor* (multiplicand) *by the same number does not change the value of the quotient* (multiplier).

4. *To multiply the divisor* (multiplicand) *and divide the quotient* (multiplier) *by the same number, or vice versâ, does not change the value of the dividend* (product).

48. To divide by the factors of a divisor.

ILLUSTRATIVE EXAMPLE.

Divide $9128 by 126.

Explanation.—Since the divisor is equal to 3 × 7 × 6, the division of $9128 by 126 may be accomplished by dividing successively by these factors.

Solution.

$$126 = 3 \times 7 \times 6.$$

$$\begin{array}{rlr} 3\,)\,\underline{\$9128} & & \\ 7\,)\,\underline{3042} & +2= & 2 \\ 6\,)\,\underline{434} & +4 \times 3= & 12 \\ \$72 & +2 \times 3 \times 7= & \underline{42} \\ & & \$56 \end{array}$$

Quotient, $\$72\frac{56}{126}$.

Dividing $9128 by 3 (or one forty-second of the true divisor, 126) produces $3042 (or 42 times the true quotient, Prin. 1, **47**) and a remainder of $2. Since this remainder is left from the true dividend, it must be a part of the true remainder.

Dividing $3042 (one forty-second of the true quotient) by the second factor, 7, produces $434 (which must be one-seventh of 42, or 6 times the true quotient), and a remainder of $4. Since $4 is left from dividing one-third of the true dividend, this remainder must be one-third of the true remainder ($4 × 3=$12, second part of true remainder).

Dividing $434 (one-sixth of the true quotient) by the remaining factor, 6, produces $72 (the true quotient) and a remainder of $2. Since $2 is the remainder from dividing $434 (one-third of one-seventh of the true dividend) this remainder must be one-third of one-seventh of the true remainder ($2 × 3 × 7=$42, third part of the true remainder).

Add the several parts of the true remainder, obtaining $56 as the total true remainder.

Rule.—I. *Separate the divisor into convenient factors.*

II. *Divide the dividend by one of these factors; divide the resulting quotient by one of the remaining factors; and so continue until each factor has been used as a divisor. The final quotient will be the required quotient.*

III. *Multiply each remainder by all the divisors that precede its own divisor; and add these products to the remainder, if any, of the first division, to find the true remainder.*

EXAMPLES FOR PRACTICE.

Find the quotient and remainder by the preceding rule.

1. 182756 lb. ÷ 63.
2. 917836 gal. ÷ 36.
3. 517918 yd. ÷ 72 yd.
4. 8251983 ft. ÷ 48 ft.
5. 5173916 bu. ÷ 121 bu.
6. 3748265 bbl. ÷ 81.
7. 5138921 yr. ÷ 35.
8. 7917362 in. ÷ 56 in.
9. $928735 ÷ $125 ($5 × 5 × 5).
10. $713456 ÷ 648 (9 × 9 × 8).
11. $3271826 ÷ 147 (7 × 7 × 3).
12. $682754 ÷ $336 ($8 × 6 × 7).
13. $917583 ÷ 224 (8 × 7 × 4).
14. $153927 ÷ 168 (3 × 8 × 7).
15. $517831 ÷ $360 ($8 × 5 × 9).
16. $175193 ÷ 385 (11 × 7 × 5).

49. To Divide by 10, 100, 1000, 10000, etc.

ILLUSTRATIVE EXAMPLE.

Divide 37589 lb. by 100.

Explanation.—Cut off the two right-hand figures (89) of the dividend, which is equivalent to moving the remaining figures of the dividend two places to the right, thus decreasing the dividend one hundredfold (Rem., **14**), and producing the required quotient 375 lb.

The figures cut off (89) being the undivided part of the true dividend, must be the true remainder.

Solution.

1|00) 375|89

375 quo., 89 rem.

$374\frac{89}{100}$ lb., *Ans.*

Rule.—*Cut off from the right of the dividend as many figures as there are ciphers in the divisor. The remaining figures will express the quotient, and the figures cut off will express the remainder.*

EXAMPLES FOR PRACTICE.

Divide

1. 89734 gal. by 100.
2. $689756 by $1000.
3. 487536 bu. by 10 bu.
4. 192753 yd. by 10000.

Divide

5. $68973591 by 100000.
6. 537869 yr. by 1000 yr.
7. 8273625 ft. by 100 ft.
8. 5917826 bbl. by 10000.

50. When any divisor has ciphers on the right.

ILLUSTRATIVE EXAMPLE.

Divide $51987 by 1900.

Explanation. The divisor (1900) equals 100 × 19. Applying **48,** divide by one of these factors (100) as shown in **49,** obtaining $519 as one-nineteenth of the true quotient, and $87 dollars as part of the true remainder.

Divide $519 by 19, obtaining the true quotient ($27) and a second remainder of $6, which is one-hundredth of the true remainder. Multiply $6 by 100, and add $87 to the product, obtaining $687 as the total true remainder.

Solution.

19|00)$519|87($27 Quo.

38

139

133

$687 Rem.

Rule.—I. *Cut off the ciphers at the right of the divisor and an equal number of figures at the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor to find the true quotient.*

III. *Prefix the remainder, if any, to the figures cut off from the dividend, to obtain the true remainder.*

Examples for Practice.

Divide

1. 9183456 lb. by 700.
2. 1528376 bbl. by 5000.
3. 5827913 gal. by 900.
4. 268349 yd. by 8900.
5. 2738291 bu. by 123000.

Divide

6. $4137861 by $67000.
7. $172835918 by $26300.
8. $68739251 by $31200.
9. $58263719 by $41000.
10. $25713826 by $927000.

UNITED STATES MONEY.

51. In **United States Money,** 10 cents equal one dime; and 10 dimes or 100 cents equal one dollar.

Rem. 1.—The *Sign* of U. S. money is $.

Rem. 2.—A period or point (.) is used to distinguish the several denominations of U. S. money; all the orders on the left of the point expressing *dollars;* the first order at the right of the point, *tens of cents* or dimes; and the second order at the right of the point, *units* of cents (less than ten cents).

Rem. 3.—U. S. money is usually expressed in two denominations, *dollars* and *cents.* If the cents are less than 10, a cipher is written before the cents and after the point, to denote the absence of tens of cents or dimes.

Write

1. 32 dollars and 16 cents.
2. 145 dollars and 95 cents.
3. 708 dollars and 60 cents.
4. 65 dollars and 8 cents.
5. 346 dollars and 5 cents.
6. 1275 dollars and 9 cents.

Write

7. 5362 dollars and 85 cents.
8. 7900 dollars and 6 cents.
9. 472 dollars and 34 cents.
10. 2075 dollars and 4 cents.
11. 8123 dollars and 75 cents.
12. 6274 dollars and 3 cents.

Read

13. $ .04.
14. $ .35.
15. $ .07.
16. $ .03.

17. $725.45.
18. $1250.09.
19. $7283.75.
20. $1275.50.

Read

21. $625.06.
22. $4285.35.
23. $9276.18.
24. $7253.04.

25. $573.08.
26. $129.65.
27. $325.05.
28. $4283.12.

52. Since there are 100 cents in 1 dollar, dollars may be changed to cents by annexing *two* ciphers (**38**); and cents may be changed to dollars by cutting off two figures from the right (**49**).

Change to cents.

1. $287.
2. $2965.
3. $4537.
4. $9269.
5. $6230.
6. $4175.
7. $918.50
8. $625.45

Change to dollars.

9. 27158 cents.
10. 295 cents.
11. 5873 cents.
12. 7500 cents.
13. 9000 cents.
14. 8275 cents.
15. 412580 cents.
16. 6275 cents.

53. Addition of United States money.

ILLUSTRATIVE EXAMPLE.

Add $536, $275.08, $625.45, $572, and $6298.86.

Explanation.—Only similar units can be added (Prin. 1, **22**). Therefore, arrange the several addends so that cents shall fall under cents, dimes under dimes, units of dollars under units of dollars, tens of dollars under tens of dollars, etc.

Add as in ordinary addition. The sum of the dimes' orders being 18 dimes (Prin. 2 and 3, **22**) or 1 dollar and 8 dimes, place a point to the left of the 8 dimes to denote the denomination of its units (Rem. 2, **51**).

Solution.

```
 $536.
  275.08
  625.45
  572.
 6298.86
 --------
$8307.39
```

Rule.—I. *Arrange the addends so that their respective points shall fall in a vertical column.*

II. *Add and carry as in ordinary addition.*

III. *Place a point in the sum directly under those of the addends.*

EXAMPLES FOR PRACTICE

1. Add $430.50, $8215.09, $725, $62.75, $250.45, and $91.08.
2. Add $2580.70, $269.04, $290, $2140.38, and $13248.60.
3. Add $712.41, $25.38, $5, $19.60, $275.45, and $1239.50.
4. Add $915.80, $520.75, $68.30, $25.95, $1.25, and $536.78.
5. What is the sum of 9 dollars and 5 cents, 25 dollars and 40 cents, 7 dollars and 38 cents, 25 cents, 9 cents, and 2 dollars and 75 cents?
6. I bought of a grocer as follows: sugar, 75 cents; coffee, 30 cents; flour, $6.75; molasses, 60 cents; soap, 6 cents; and matches, 2 cents. What was the amount of my bill?

54. Subtraction of United States money.

ILLUSTRATIVE EXAMPLE.

Subtract 19 dollars and 8 cents from 45 dollars.

Explanation.—Only similar units can be subtracted (Prin. 1, **26**). Hence, arrange the terms so that the point in the subtrahend shall fall under the point of the minuend, causing similar orders to fall in the same column. Since the minuend contains no cents, either write, or imagine to be written, a cipher in each of the missing orders of the minuend. Subtract, borrow, and carry as in ordinary subtraction.

Solution.

```
 $45.
  19.08
 ------
 $25.92
```

Rule.—I. *Arrange the subtrahend so that its point shall fall under the point of the minuend.*

II. *Subtract, borrow, and carry as in ordinary subtraction.*

III. *Place a point in the remainder directly under those of the minuend and subtrahend.*

EXAMPLES FOR PRACTICE.

Subtract

1. $275.15 from $425.90.
2. $2183.50 from $5275.85.
3. $3425 from $8250.60.
4. $2951.85 from $3126.
5. $627.35 from $4285.
6. $2178 from $3240.95.

Subtract

7. $29135.38 from $29275.06.
8. $518.30 from $725.
9. $25 and 9 cents from $31.
10. 6 cents from $2 and 5 cents.
11. 3 cents from 5 dimes.
12. $37 from $225 and 8 cents.

55. Multiplication of United States money.

ILLUSTRATIVE EXAMPLE.

Multiply 735 dollars and 5 cents by 53.

Explanation.—Place the multiplier under the multiplicand and proceed as in ordinary multiplication. Since the multiplicand expresses 73505 cents, 53 times the multiplicand or 3895765 must also express cents (Prin. 3, **31**). Hence, cut off two figures from the right of the product to make it express dollars (**52**).

Solution.

```
   $735.05
        53
 ---------
   2205 15
  36752 5
 ---------
 $38957.65
```

Rule.—I. *Arrange the factors and multiply as in ordinary multiplication.*

II. *If the multiplicand contains cents, point off two figures, for cents, from the right of the product.*

EXAMPLES FOR PRACTICE.

Multiply

1. $2349 by 75.
2. $8729.15 by 28.
3. $629.70 by 146.
4. $5128.38 by 327.
5. $628.62 by 43.
6. $4275.50 by 671.

Multiply

7. $325 and 5 cents by 27.
8. $8150 and 9 cents by 325.
9. $576 and 24 cents by 92.
10. $6125 and 7 cents by 168.
11. 9 cents by 3251.
12. 15 cents by 467.

56. Division of U. S. money by an abstract divisor.

ILLUSTRATIVE EXAMPLES.

1. Divide $101.84 by 76.

Explanation.—Divide $101.84 by 76 as in ordinary division.

Since the dividend expresses 10184 cents, one seventy-sixth of the dividend or the quotient (134) must also express cents (Summary, **41**). Hence, point off two figures from the right of the quotient to make it express dollars.

Solution.

76) $101.84 ($1.34
76
258
228
304
304

2. Divide $105 by 84.

Explanation.—If, as in the present example, the dividend ($105) is an exact number of dollars; and it is found that the division of the dividend as it stands will produce a remainder, add two ciphers to the dividend (changing it to cents) and continue the division until these two ciphers have been brought down. Then point off two figures in the quotient, as explained in the preceding example.

Solution.

84) $105.00 ($1.25
84
210
168
420
420

Rule.—*Divide as in ordinary division. If the dividend contains cents, point off two figures, for cents, from the right of the quotient.*

NOTE 1.—If the dividend does not contain cents, and is not exactly divisible by the divisor, annex two ciphers to the dividend to make it express cents, and continue the division until these ciphers have been brought down.

NOTE 2.—It is not customary to carry the quotient beyond the cents' order. If the final remainder is less than half the divisor, omit it; if more, add 1 to the cents' order of the quotient.

EXAMPLES FOR PRACTICE.

Divide

1. $592.50 by 6.
2. $712.92 by 12.
3. $22480.57 by 47.
4. $477.09 by 93.
5. $642.20 by 845.

Divide

6. $178169.60 by 328.
7. $105320 by 32.
8. $292023 by 426.
9. $167889 by 764.
10. $5691.75 by 7589.

57. Division of U. S. money by U. S. money.

ILLUSTRATIVE EXAMPLE.

Divide $497 by $3.50.

Explanation.—Since the divisor expresses cents (350 cents) annex two ciphers to the dividend that it may also express cents (Summary, **41**). Divide as in ordinary division, finding that 350 cents are contained in 49700 cents, 142 times.

Solution.

$3.50) $497.00 (142
350
1470
1400
700
700

Rule.—*If either the divisor or dividend expresses cents, make both express cents. Divide as in ordinary division.*

EXAMPLES FOR PRACTICE.

Divide

1. $2358.23 by $4.13.
2. $1635.90 by $.05.
3. $72275 by $1.75.
4. $52808 by $5.74.
5. $39570.50 by $682.25.

Divide

6. $108 by 25 cents.
7. $406602 by 9 cents
8. $9875 by 5 cents.
9. $111 by $1.48.
10. $18991.15 by $283.45.

RELATION OF INTEGERS.

58. All arithmetical operations involve either an increase or decrease of some given number. The law of increase and decrease of integral numbers is simple in character and uniform in application. A given integer is increased by considering it an addend and augmenting it by one or more other addends; and is decreased by considering it the sum of two or more addends and taking away one or more of those addends. The former process is called addition and the latter subtraction. A full comparison of these operations is shown in **29.** Multiplication, however, is a more convenient process when the increase is by *equal* addends (**30**); and division when the decrease is by *equal* addends. Hence,

To find the TOTAL *of two or more addends,* { *if unequal, requires addition;*
if equal, requires multiplication.

To find ONE *of two or more addends,* { *if unequal, requires subtraction;*
if equal, requires division.

59. It is thus seen that addition and subtraction produce opposite results involving unequal addends; that division and multiplication produce opposite results involving equal addends; that multiplication and addition produce similar results, and the employment of each is dependent respectively upon the equality or inequality of the considered addends; so with division and subtraction. The processes of addition and subtraction on the one hand, and of multiplication and division on the other, are illustrated, comparatively, by the following simple problems:

60. Addition and Subtraction compared.

1. A has $75 and B $50. How much have both?
2. A and B have $125. If A has $75, how much has B?

Solution.

Components of given minuend. { subtrahend, if given = $75=unequal addend } Components of required sum.
remainder, if required= 50=unequal addend
Minuend, if given=$125=total of unequal addends=sum, if required.

Deduction.—The total of two or more unequal addends is the ***sum if required***, or the ***minuend if given;*** and the components of a required sum or given minuend are the addends of which it is composed. Hence, add the components to find a required sum; and subtract one component from a given sum to find the remaining component.

61. Multiplication and Division compared.

1. Find the total weight of 35 bu. of wheat, allowing 60 lb. to the bu.
2. If 35 bu. of wheat weigh 2100 lb., what is the weight of 1 bu.?

3. How many bushels in 2100 lb. of wheat, allowing 60 lb. to the bu.?

Solution.

Factors of given dividend. { concrete divisor = 60 lb. = weight of 1 equal addend = multiplicand } Factors of required product.
{ abstract quotient = 35 = number of equal addends = multiplier }
Dividend, if given = 2100 lb. = total of equal addends = product, if required.

Deduction.—The total of two or more equal addends is the *product if required*, or the *dividend if given*. *One factor* of a required product or given dividend is the weight, measure, value, etc., of *one equal addend;* and the *other factor* is the *number* of such equal addends. Hence, if a product is required, multiply its two given factors; and if a product is given, divide it by its given factor, and the quotient will be its remaining factor.

62. Summary.—To determine which of the fundamental rules should be employed in the solution of a given example, ascertain whether its considered addends are equal or unequal.

I. *If the considered addends are unequal*, (a) *employ* ADDITION *if the total weight, measure, value, etc., is* REQUIRED; *or* (b) SUBTRACTION *if the total weight, measure, value, etc., is* GIVEN.

II. *If the considered addends are equal*, (c) *employ* MULTIPLICATON *if the total weight, measure, value, etc., is* REQUIRED; *or* (d) DIVISION *if the total weight, measure, value, etc., is* GIVEN.

NOTE. 1.—An example may involve *two or more* given or required totals of equal addends, in which case there will be as many separate divisions as there are given totals, and as many separate multiplications as there are required totals.

63. REVIEW EXAMPLES.

1. A man bought 450 acres of land at $17.25 per acre, expended $1735 in improvements, and then sold it at $25 per acre. What was his gain?

2. What is the cost of 13 car loads of wheat, each car containing 735 bushels, at $1.08 per bushel?

3. How much must be paid for 18200 pounds of corn bought at 75 cents per bushel, allowing 56 pounds to the bushel?

4. A lumber dealer sold 35286 feet of lumber on Monday, 47239 feet on Tuesday, 19275 feet on Wednesday, 50260 feet on Thursday, 9525 feet on Friday, and 109275 feet on Saturday. What were his total sales for that week?

5. I deposited in bank at various times the following sums: $825.40, $325.75, $90.50, $1350.48, and $75.15. After making the

last deposit, I found there was a balance to my credit of $1960.85. How much had I withdrawn?

6. A merchant had $91.20 remaining after buying 2180 bushels of rye at 75 cents per bushel and 5237 bushels of wheat at $1.05 per bushel. How many bushels could he have purchased if he had bought no rye and invested all his money in wheat?

7. A man first owed $5384.25, but after making four payments thereon, he owed only $930.50. If his first payment was $750, his second $1275.30, and his third $960.75, what was his fourth payment?

8. A dealer bought cattle for $16694.50 and sold them for $19047.30, gaining $6.80 per head. How many cattle did he buy?

9. A farmer sold 125 acres of land at $45 per acre, received $3150 in cash, and took a note for the remainder. What was the face of the note?

10. I owe A $525.75, B $240.30, C $875.50, and D $340. If I pay A $175.28, B $100, C $420, and D $150.75, what will be my total remaining indebtedness?

11. I bought 15 yards of muslin at 7 cents per yard, 18 yards of alpaca at 45 cents per yard, 9 yards of sheeting at 30 cents per yard, 8 yards of cloth at $1.75 per yard, and in payment gave the clerk 3 ten-dollar notes. How much change should he return?

12. If a man buys 230 acres of land for $6210, sells 50 acres at $29 per acre, 76 acres at $28.50 per acre, and 41 acres at $31.25 per acre, at what price per acre must he sell the remainder to make a total profit of $1349?

13. A farmer sold to a merchant 15 pounds of butter at 28 cents per pound, 25 dozen eggs at 14 cents per dozen, 5 dozen chickens at $2.20 per dozen, and in payment received 4 gallons of molasses at 55 cents per gallon, 9 pounds of coffee at 18 cents per pound, 1 bbl. of flour worth $6.40, and the remainder in sugar at 8 cents per pound. How many pounds of sugar did the farmer receive?

14. A man bought 89 acres of land for $3337.50, sold 32 acres at $40 per acre and the remainder at $45 per acre. What was his gain?

15. If $350 be paid for 56 barrels of flour, how many barrels can be bought for $256.25, at the same price per barrel?

16. I bought a farm for $6737.50 and sold it for $7218.75, gaining $2.75 per acre. How many acres did the farm contain?

17. A dealer having 750 bushels of corn bought 830 bushels additional, then sold 235 bushels, then bought 340 bushels, then sold 465

bushels, then bought 190 bushels, and then sold enough to reduce his stock of corn to 620 bushels. What was the amount of his last sale at 68 cents per bushel?

18. A farmer has $180.50 in cash, and by selling his crop of apples at 75 cents per bushel, 3 calves at $8 each, 7 lambs at $4.25 each, and borrowing $72 from his neighbor, he is able to pay off a mortgage upon his farm amounting to $550. How many bushels of apples did he sell?

19. A merchant's sales were as follows: Monday, $315.30; Tuesday, $405.75; Wednesday, $298.45; Thursday, $325.62; Friday, $318.25; and Saturday, $480.55. What were his average sales per day?

REM. — Averages are found by dividing the sum of two or more unequal addends by the number of addends, thus changing them to equal addends.

20. A, B, and C joined in buying a farm containing 420 acres for $7665. How much ought each to pay if A took 75 acres, B 130 acres, and C the remainder, the land being of uniform value?

21. A grocer mixed 350 pounds of Rio coffee costing 12 cents per pound with 175 pounds of Laguayra coffee costing 16 cents per pound. At what price per pound must he sell the mixture to gain $8.75?

22. It required 9 men to dig A's cellar in 5 days, and 30 men to dig B's cellar in 6 days. How many times the size of A's cellar is B's?

23. A man bought 12 pounds of sugar at 7 cents per pound, 3 pounds of coffee at 22 cents per pound, 12 pounds of buckwheat at 4 cents per pound, and 3 gallons of syrup at 65 cents per gallon, tendering a ten-dollar bill in payment. How much change should he receive?

24. If 30 men can complete a contract in 48 days, in how many days can 40 men complete it?

25. If 12 men can build a boat in 17 days, how long will it take to build it if 8 men leave at the close of the sixth day?

26. A man bought 3500 bushels of wheat and 2763 bushel of rye for $5778.55. If the cost of the wheat was 98 cents per bushel, what was paid per bushel for the rye?

27. A man has $2016.62, bought 370 bushels of wheat at $1.12 per bushel, 980 bushels of corn at 74 cents per bushel, 536 bushels of rye at 82 cents per bushel, and invested the remainder of his money in flour at $6.25 per barrel. How many barrels of flour did he buy?

28. An army consisted of 3 divisions, each division of 5 brigades, each brigade of 6 regiments, each regiment of 10 companies, and each company of 85 men. How many men did the army contain?

29. In 1880, the population of the city of New York was 1206577, and of Philadelphia 847170. How much does New York lack of having double the population of Philadelphia?

30. A and B start from the same point and travel in the same direction, one at the average speed of 28 miles per day, and the other at the average speed of 4 miles per hour, traveling 9 hours per day. How far apart will they be at the close of the ninth day?

31. If A and B start from the same point and travel in opposite directions, A at the rate of 3 miles per hour and 8 hours per day, and B at the rate of 4 miles per hour and 9 hours per day, how far apart will they be at the end of the sixth day?

32. I bought 24 yards of goods for $8.40, and a friend bought the remainder of the piece at the same price per yard, paying $6.30. How many yards did the piece contain?

33. The temperature of a certain city averaged 80° Fahrenheit for the month of June, 90° for July, and 91° for August. What was the average temperature for the three months?

34. A contractor expected to complete a contract in 38 days by employing 75 men; but after working 28 days, 25 of these men were discharged. In how many days can the remainder of the men complete the contract?

35. If there are 32 quarts in a bushel, how much will be received for 3 bushels of chestnuts by retailing them at 8 cents per quart?

FACTORING.

64. The **Factors** of a number are such integers as will produce it, if multiplied together.

Thus, 6 and 7 are factors of 42; 3, 2, and 5 are factors of 30.

65. A **Prime Number** is a number which has no integral factors except itself and 1.

66. A **Composite Number** is a number which has *other* integral factors than itself and 1.

Thus, 15 is a composite number because 3 × 5 = 15, but 19 is a prime number because its only factors are 1 × 19.

DIVISIBILITY OF NUMBERS.

67. An **Exact Divisor** of a number is one which will divide it and leave no remainder.

68. One number is said to be *divisible* by another when it contains that other an *exact* number of times. Any number is divisible by itself and 1; and it will be divisible—

1. By 2, if it is an *even* number.
2. By 3, if the sum of all its figures is divisible by 3.

Thus, 285 is divisible by 3 because 2+8+5, or 15 is so divisible.

3. By 4, if its two right-hand figures are either ciphers or express a number which is divisible by 4.

Thus, 4 is an exact divisor of 900 because its two right-hand figures are ciphers; and of 1756 because the number expressed by its two right-hand figures (56) is divisible by 4.

4. By 5, if its right-hand figure is either 5 or 0.
5. By 6, if it is an even number and the sum of its figures is divisible by 3.

Thus, 6 is an exact divisor of 348, because the latter is an even number, and the sum of its orders (3+4+8=15) is divisible by 3.

6. By 8, if its three right-hand figures are either ciphers or express a number which is divisible by 8.

Thus, 8 is an exact divisor of 5000 because its three right-hand figures are ciphers; and of 62176 because the number expressed by its three right-hand figures (176) is divisible by 8.

7. By 9, if the sum of its figures is divisible by 9.

Thus, 56178 is divisible by 9 because 5+6+1+7+8, or 27 is so divisible.

8. By 10, if its right-hand figure is a cipher.

PRIME FACTORS.

69. A **Prime Factor** of a number is any prime number taken as a factor.

Rem. 1.—3 × 5 are prime factors of 15, since each of these factors is a prime number. 4 × 8 are composite factors of 32, since each of these factors is a composite number.

Rem. 2.—All the prime or composite factors of a number are exact divisors of that number. Therefore, any prime number that is an exact divisor of a given number must also be a prime factor of that number.

70. **To find the prime factors of a composite number.**

ILLUSTRATIVE EXAMPLE.

What are the prime factors of 924 bushels?

Explanation.—Since 924 is an even number, divide it by 2, a prime number (1, **68**), obtaining 462. Since 462 is another even number, divide it by 2, obtaining 231. Since the sum of the figures of 231 (2+3+1=6) is divisible by 3, a prime number, take 3 as the next divisor (2, **68**), obtaining 77. Divide 77 by 7, a prime number, obtaining 11, another prime number. Hence, the several prime divisors (2, 2, 3, 7) and the prime quotient (11 bu.) are the prime factors of 924 bu.

Solution.

2) 924 bu.
2) 462 "
3) 231 "
7) 77 "
11 bu.

Rule.—*Divide the given number by its least prime factor; similarly divide the resulting quotient; and so continue until a prime quotient is obtained. The several divisors and the last quotient are the prime factors.*

Proof. — *The continued product of all the prime factors should equal the given number.*

EXAMPLES FOR PRACTICE.

Find the prime factors of

1. 40 lb.
2. 45 gal.
3. 126 ft.
4. 132 yd.
5. 210 bbl.
6. 274 days.
7. 392 yr.
8. 525 men.
9. 6435 tons.
10. 10472 bbl.

Find the prime factors of

11. 18216 yr.
12. 13356 bu.
13. 86832 oz.
14. 46494 yr.
15. 29184 hr.
16. 30240 yd.
17. $108900.
18. 56250 ft.
19. $366025.
20. $4266108.

COMMON DIVISORS.

71. A **Common Divisor** of two or more numbers is any number that will exactly divide each of them.

72. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

Rem.—When numbers have no common divisor except 1, they are said to be *prime to each other.*

73. Principles.—1. *One number is an exact divisor of only such other numbers as contain all its prime factors.*

2. *A common divisor of two or more numbers contains only such prime factors as are common to those numbers.*

3. *The greatest common divisor of two or more numbers contains all the prime factors which are common to those numbers, but no others.*

4. *The greatest common divisor of two numbers is a divisor of their sum, of their difference, and of the sum or difference of one number and any number of times the other.*

74. To find the greatest common divisor of two or more numbers by factoring.

ILLUSTRATIVE EXAMPLE.

Find the greatest common divisor of $42, $60, and $84.

Solution.

$3) $42—$60—$84

2) 14— 20— 28

7— 10— 14

$3 × 2=$6, *G. C. D.*

Explanation.—Since $3 will exactly divide each of the given numbers, it is one factor of their greatest common divisor (Prin. 2, **73**). Since 2 will exactly divide each of the resulting quotients, it is a second factor of the greatest common divisor. Since no number will exactly divide *all* the quotients in the lower line, they contain no other common factor, and therefore the product of the common factors already found ($3 × 2=$6) must be the greatest common divisor.

Rule.—I. *Arrange the numbers in a line and divide by any prime number that will exactly divide all of them.*

II. *Similarly divide the resulting quotients; and so continue until no number will exactly divide all of the last set of quotients.*

III. *The product of the several divisors will be the greatest common divisor.*

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of

1. 18 bu., 36 bu., 72 bu., 90 bu.
2. 16 yr., 24 yr., 48 yr., 80 yr.
3. 24 yd., 36 yd., 48 yd., 72 yd.
4. 30 ft., 42 ft., 54 ft., 78 ft.
5. 24 lb., 36 lb., 60 lb., 132 lb.
6. 28 oz., 42 oz., 84 oz., 112 oz.
7. $12, $18, $24, $36, $54.
8. 63 bbl., 105 bbl., 147 bbl.
9. 36 ft., 84 ft., 108 ft., 144 ft.
10. 105 bu., 160 bu., 400 bu.
11. $17, $51, $68, $102.
12. $27, $81, $135, $162.
13. 180 yd., 216 yd., 276 yd.
14. 140 oz., 315 oz., 385 oz.

75. **To find the greatest common divisor of two numbers which cannot be conveniently factored.**

ILLUSTRATIVE EXAMPLE.

Find the greatest common divisor of $779 and $1653.

Explanation.—The G. C. D. of $779 and $1653 must divide the difference between $1653 and two times $779 (Prin. 4, **73**), and that difference is $95.

The G. C. D. of $95 and $779 must divide the difference between $779 and eight times $95, and that difference is $19.

The G. C. D. of $19 and $95 must be $19, for $19 will divide itself and is contained exactly 5 times in $95. Hence, $19 is the G. C. D. of $779 and $1653.

Solution.

$779) $1653 (2
1558
$95) $779 (8
760
$19) $95 (5
95

$19 = G. C. D.

Rule.—I. *Divide the greater number by the less. If there is no remainder, the less number will be the greatest common divisor.*

II. *If there should be a remainder, divide the preceding divisor by this remainder; and continue to divide the last divisor by the last remainder until there is no remainder. The last divisor will be the greatest common divisor.*

NOTE 1.—*To find the greatest common divisor of more than two numbers:* First find the greatest common divisor of any two of the numbers; then the greatest common divisor of this result and one of the remaining numbers; and so continue until all the numbers have been used. The last common divisor will be the greatest common divisor of all the numbers.

NOTE 2.—If, at any time, 1 should be obtained as a remainder, the numbers considered have no common divisor.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of

1. 172 yr. and 258 yr.
2. $235 and $495.
3. 387 lb and 473 lb.
4. 462 yd. and 714 yd.
5. 1377 bu. and 1581 bu.
6. 1083 bbl. and 1387 bbl.
7. 13661 ft. and 32355 ft.
8. 16499 doz. and 25927 doz.
9. 1980 tons and 3120 tons.
10. 1179 gal. and 1703 gal.
11. $323, $513, and $589.
12. $1938, $2074, and $5151.

COMMON MULTIPLES.

76. A **Multiple** of a number is any number that is exactly divisible by it.

77. A **Common Multiple** of two or more numbers is *any* number that is exactly divisible by each of them.

78. The **Least Common Multiple** of two or more numbers is the *least* number that is exactly divisible by each of them.

79. **Principles.**—1. *A multiple of a number must contain all the prime factors of that number.*

2. *A common multiple of two or more numbers must contain all the prime factors of each of those numbers.*

3. *The least common multiple of two or more numbers is the least number that contains all the prime factors of each of those numbers.*

Rem.—Two or more numbers have an *unlimited* number of common multiples; but they can have only *one* least common multiple.

80. To find the least common multiple of two or more numbers.

ILLUSTRATIVE EXAMPLE.

Find the least common multiple of $18, $30, and $36.

Solution.

$$\begin{array}{r|rcrcr} \$2\,) & \$18 & - & \$30 & - & \$36 \\ \hline 3\,) & 9 & - & 15 & - & 18 \\ \hline 3\,) & 3 & - & 5 & - & 6 \\ \hline & 1 & - & 5 & - & 2 \end{array}$$

$$\$2 \times 3 \times 3 \times 5 \times 2 = \$180.$$

Explanation.—Each of the given numbers contains the prime factor $2; hence, their least common multiple must contain the factor $2 *at least once.*

Since 3 is a prime factor of the unfactored parts of $18 (9), of $30 (15), and of $36 (18), their least common multiple must also contain the factor 3 *at least once.*

Since 3 is a prime factor of the yet unfactored parts of $18 (3) and of $36 (6), their least common multiple must contain the factor 3 *at least one more time, or twice.*

In addition to the factors already obtained ($2 x 3 x 3), the required least common multiple must contain the factor 5 (the unfactored part of $30), else it will not be divisible by $30; and it must also contain the factor 2 (the unfactored part of $36), else it will not be divisible by $36. Hence, the least common multiple of $18, $30, and $36 must be $2 x 3 x 3 x 5 x 2, or $180.

Rule.—I. *Omit such of the smaller numbers as may be exact divisors of the larger numbers.*

II. *Write the remaining numbers in a horizontal line, and divide by any prime number that will exactly divide two or more of them, placing the quotients and the undivided numbers in a line beneath.*

III. *Divide the quotients and undivided numbers by any prime number that will exactly divide two or more of them; and so continue until all the numbers in the lower line are prime to each other.*

IV. *The product of the divisors and the numbers in the lower line will be the least common multiple.*

NOTE.—When no two of the given numbers have a common prime factor, the *product of all the given numbers* will be their least common multiple.

EXAMPLES FOR PRACTICE.

Find the least common multiple of

1. 24 bu., 36 bu., and 42 bu.
2. 5 yd., 12 yd., and 15 yd.
3. 12 gal., 27 gal., and 30 gal.
4. 18 lb., 24 lb., and 42 lb.
5. 15 hr., 35 hr., and 45 hr.
6. 8 doz., 15 doz., and 24 doz.
7. 27 yr., 36 yr., and 42 yr.
8. 17 ft., 34 ft., and 68 ft.
9. 96 oz., 168 oz., and 264 oz.
10. $315, $525, and $735.
11. 13 ft., 15 ft., 18 ft., 21 ft.
12. 13 lb., 54 lb., 78 lb., 84 lb.
13. $35, $56, $60, $75.
14. $27, $36, $54, $63, $72.
15. $18, $21, $24, $30, $42.
16. $5, $15, $25, $40, $45, $50.
17. 3 miles, 5 miles, and 7 miles.
18. 7 gal., 11 gal., and 13 gal.
19. $24, $28, $32, $36, $38, $45.
20. $5, $6, $7, $8, $9, $10, $12.
21. What is the shortest distance that may be exactly measured by each of three poles, whose lengths are respectively 3, 6, and 8 feet?

CANCELLATION.

81. **Cancellation** is the omission of the same factor from both divisor and dividend to shorten the labor of division.

82. **Principles.**—1. *To cancel any factor of a number is equivalent to dividing the number by the canceled factor.*

2. *To cancel the same factor, or equal factors, from both divisor and dividend does not affect the quotient* (*Prin.* 3, **47**).

83. To find a quotient by cancellation.

ILLUSTRATIVE EXAMPLE.

A farmer traded 160 pounds of butter, worth 21 cents per pound, for 2 barrels of sugar averaging 240 pounds per barrel. How much per pound did the farmer pay for the sugar?

Explanation.—\$.21 × 160 = the total value of the butter; and since the butter was exchanged for sugar, the total value of the butter is also the total cost of the sugar. Therefore, divide the cost (\$.21 × 160) of all the pounds of sugar by the number of pounds (240 lb. × 2), obtaining 7 cents as the required price of one pound.

Solution.

$$\frac{\overset{\$.07}{\cancel{\$.21}} \times \overset{80}{\cancel{160}}}{\underset{3}{\cancel{240}} \times \cancel{2}} = \$.07.$$

The solution may be simplified by canceling all factors which are common to both divisor (240 × 2) and dividend (\$.21 × 160), and placing over the canceled numbers their uncanceled factors, if any. Thus, cancel the common factor 2 from the divisor ($\cancel{2}$) and the dividend ($\overset{80}{\cancel{160}}$); then the common factor 80 from the divisor (240) and the dividend ($\cancel{80}$); then the common factor 3 from the divisor (3) and the dividend ($\overset{7}{\cancel{21}}$), leaving only the uncanceled factor \$.07 in the dividend, which is the required quotient.

Rule.—I. *Place the factors of the dividend above a horizontal line and the factors of the divisor below it.*

II. *Cancel all factors which are common to both divisor and dividend; and divide the product of the uncanceled factors of the dividend by the product of the uncanceled factors of the divisor.*

NOTE.—If, in canceling, the quotient 1 is obtained, it should be expressed in the dividend (above the horizontal line), but not necessarily in the divisor (below the line).

EXAMPLES FOR PRACTICE.

1. Divide \$350 × 9 by 25 × 6.
2. Divide \$480 × 160 by \$60 × 32.
3. Divide 75 bbl. × 504 by 15 × 8 × 9.
4. Divide 315 gal. × 64 × 56 by 360 gal. × 392.
5. Divide 630 yd. × 352 × 65 by 90 × 88 × 91.
6. How many horses at \$175 each are worth as much as 7 horses at \$125 each?
7. How many barrels of flour worth \$8 per barrel should a miller give for 40 bushels of wheat worth \$1.40 per bushel?

8. I exchanged 18 tons of hay worth $15 per ton for corn worth 90 cents per bushel. How many bushels of corn did I receive ?

9. How many chickens at $3 per dozen should be given for 5 pieces of muslin, averaging 47 yards per piece and worth 10 cents per yard ?

10. A merchant sold 12 barrels of pork, averaging 200 pounds per barrel, at 8 cents per pound, and took in payment 160 hams, averaging 10 pounds each. At what price per pound were the hams estimated ?

11. A seamstress received 8 pieces of bleached muslin, averaging 45 yards to the piece, out of which she made 10 bundles of shirts with 12 shirts in each bundle. What was the amount of muslin used in making a shirt ?

12. I bought 8 loads of corn, each load containing 15 bags, and each bag 2 bushels; and paid for the corn with 8 barrels of sugar, averaging 250 pounds per barrel, and worth 9 cents per pound. What price per bushel did I allow for the corn ?

13. A mechanic worked for a farmer at 25 cents per hour and averaged 8 hours per day for 32 days. He accepted his wages in wheat at $1.60 per bushel. How many bushels did he receive ?

14. A manufacturer exchanged stockings at 15 cents per pair for 5 bales of cotton, averaging 474 pounds per bale, at 12 cents per pound. How many dozen pairs of stockings were required to pay for the cotton ?

15. How many pieces of calico averaging 42 yards per piece, and worth 5 cents per yard, are equivalent to 35 pieces of bunting, averaging 52 yards to the piece, and worth 18 cents per yard ?

COMMON FRACTIONS.

84. A **Fraction** expresses one or more of the equal parts of a unit.

REM.—*One of the equal parts* into which a unit is divided is called a *fractional unit*, to distinguish it from whole or integral units.

85. The **Denominator** of a fraction is written below a horizontal line, and expresses the number of equal parts into which the integral unit is divided.

REM. 1.—In the fraction $$\frac{3}{4}$, the denominator 4 denotes that the integral unit ($1) is conceived to be divided into 4 equal parts, thus *denominating* (naming) the *value* or *size* of the fractional units.

Rem. 2.—The greater the denominator (the more the parts into which any unit is divided), the less the value of each part; and the less the denominator (the fewer the parts), the greater their value.

86. The **Numerator** of a fraction is written above a horizontal line, and expresses the *number* of equal parts of an integral unit which are taken to form the fraction.

Rem.—In the fraction $\$\frac{3}{4}$, the num(*b*)erator 3 expresses the *number* of fractional units. Hence, $\$\frac{3}{4}$ denotes 3 fractional units, each of which is the fourth of a dollar.

87. The **Terms** of a fraction are its numerator and denominator.

Rem.—In the fraction $\$\frac{3}{4}$, 3 is one term, and the denominator 4 is the other.

EXERCISE.

Express with figures the following fractions, and state what each term denotes:

1. One-half of an apple.
2. One-third of an orange.
3. Two-thirds of a melon.
4. One-fourth of a mile.
5. Three-fourths of an inch.
6. Two-fifths of a bushel.
7. Five-sixths of a gallon.
8. Three-eighths of a dollar.
9. Seven-twelfths of a yard.
10. Three-sixteenths of an acre.
11. Five-thirtieths of a foot.
12. Nine forty-fifths of a rod.

88. A **Common Fraction** is one which requires a written denominator.

89. Common Fractions may be **Proper or Improper.**

90. A **Proper Fraction** is one whose numerator is less than its denominator.

Rem.—The *value* of a proper fraction is less than a whole unit. $\$\frac{1}{2}$, $\$\frac{3}{5}$, $\$\frac{5}{8}$ are proper fractions.

91. An **Improper Fraction** is one whose numerator is either equal to, or greater than its denominator.

Rem.—The *value* of an improper fraction is either equal to, or greater than a whole unit. $\$\frac{5}{5}$, $\$\frac{9}{4}$, $\$\frac{9}{7}$, $\$\frac{11}{5}$ are improper fractions; the first because its numerator is equal to its denominator, and the others because their numerators are greater than their denominators.

92. A **Mixed Number** is a whole number and a fraction united.

Rem.—$\$3\frac{1}{4}$, $15\frac{5}{8}$ pounds, $48\frac{5}{8}$ gallons are mixed numbers.

93. The **Value of a Fraction** is the quotient arising from dividing its numerator by its denominator.

Rem.—A fraction expresses an *unexecuted* division, its numerator being the dividend, and its denominator the divisor.

94. Since a fraction expresses an unexecuted division, any change made in its numerator or denominator will produce the same change in the value of the fraction as a like change in a dividend or divisor will produce in the value of a quotient (Gen. Prin., **47**). Hence,

95. Principles.—1. *To multiply the numerator or divide the denominator by any number multiplies the fraction by the same number* (Prin. 1, **47**).

2. *To divide the numerator or multiply the denominator by any number divides the fraction by the same number* (Prin. 2, **47**).

3. *To multiply or divide both numerator and denominator by the same number does not change the value of the fraction* (Prin. 3, **47**).

REDUCTION OF FRACTIONS.

96. Reduction of Fractions means to change their *form* without changing their value.

Rem.—A fraction is said to be reduced to *higher terms* when Prin. 3, **95**, is employed to express its numerator and denominator in *larger* numbers; to *lower terms* when the same principle is employed to express its numerator and denominator in *smaller* numbers; and to *lowest terms* when its numerator and denominator are thus made *prime to each other* (Rem., **72**).

97. Reduction of integers to improper fractions.

ILLUSTRATIVE EXAMPLE.

Reduce \$17 to fifths of a dollar.

Explanation.—There are $\frac{5}{5}$ of a dollar in one whole dollar; therefore in 17 whole dollars, there must be 17 times \$$\frac{5}{5}$, or \$$\frac{85}{5}$ (Prin. 1, **95**).

Solution.

$\$\frac{5}{5} \times 17 = \$\frac{85}{5}$

Principles.—1. ONE *whole unit is expressed as a fraction by making both numerator and denominator the* SAME.

2. *Two or more whole units are expressed as a fraction by making the numerator as many times the denominator as the number of whole units is times 1.*

Rule.—*Multiply the integer by the required denominator, and write the product over the required denominator.*

Note.—If no particular denominator is required, an integer may be made to assume the form of a fraction by writing the denominator 1 under it. Thus, $\$2=\$\frac{2}{1}$; 5 lb. $=\frac{5}{1}$ lb.

EXAMPLES FOR PRACTICE.

Reduce

1. 9 lb. to fourths of a pound.
2. 21 bu. to ninths of a bushel.
3. $38 to thirds of a dollar.
4. 27 yd. to thirtieths of a yard.
5. 56 ft. to nineteenths of a foot.
6. 11 in. to fifteenths of an inch.

Reduce

7. 71 miles to eighths of a mile.
8. 15 yr. to sixths of a year.
9. 20 bu. to fifty-ninths of a bu.
10. 32 lb. to halves of a pound.
11. 153 yr. to 124ths of a year.
12. 9 ft. to 347ths of a foot.

98. Reduction of mixed numbers to improper fractions.

ILLUSTRATIVE EXAMPLE.

Reduce $\$8\frac{2}{3}$ to an improper fraction.

Explanation.—There are $\$\frac{3}{3}$ in $1 (Prin. 1, **97**), and 8 times $\$\frac{3}{3}$, or $\$\frac{24}{3}$ in $8 (Prin. 2, **97**); and in $8 and $\$\frac{2}{3}$ there must be $\$\frac{24}{3}+\$\frac{2}{3}$, or $\$\frac{26}{3}$.

Solution.

$\$\frac{3}{3}\times 8=\$\frac{24}{3}$

$\$\frac{24}{3}+\$\frac{2}{3}=\$\frac{26}{3}$

Principle.—*A mixed number contains as many fractional units as there are fractional units in the integer + the fractional units in the terminal fraction.*

Rule.—*Multiply the integer by the denominator, and to the product add the numerator for a new numerator. Keep the same denominator.*

EXAMPLES FOR PRACTICE.

Reduce to improper fractions:

1. $6\frac{3}{4}$ miles.
2. $8\frac{2}{3}$ rods.
3. $12\frac{4}{5}$ tons.
4. $18\frac{1}{6}$ acres.
5. $35\frac{5}{7}$ pecks.
6. $41\frac{7}{9}$ cords.
7. $79\frac{3}{11}$ inches.
8. $135\frac{5}{8}$ melons.
9. $763\frac{6}{17}$ ft.
10. $895\frac{13}{16}$ bbl.
11. $473\frac{3}{19}$ lb.
12. $8275\frac{37}{46}$ bu.
13. $1526\frac{143}{267}$ yr.
14. $6829\frac{5}{8}$ yd.
15. $196\frac{310}{738}$ gal.
16. $573\frac{31}{171}$ yd.

99. Reduction of improper fractions to integers or mixed numbers.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{75}{8}$ to a mixed number.

Explanation.—$\$\frac{8}{8}=1$ whole dollar, and there must be as many whole dollars in 75 eighths as 8 eighths are contained times in 75 eighths, or 9 whole dollars and 3 eighths remaining.

Solution.

$\$75\div 8=\$9\frac{3}{8}$

Rule.—*Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

Reduce the following to integers or mixed numbers:

1. $\frac{18}{3}$ ft.
2. $\frac{42}{7}$ yr.
3. $\frac{342}{9}$ bbl.
4. $\frac{182}{13}$ qt.
5. $\frac{253}{59}$ tons.
6. $\frac{712}{6}$ yd.
7. $\frac{412}{9}$ gal.
8. $\frac{3173}{7}$ days.
9. $\frac{2763}{113}$ yr.
10. $\frac{1925}{271}$ miles.
11. $\frac{4120}{329}$ bbl.
12. $\frac{1256}{23}$ gal.
13. $\frac{1623}{371}$ days.
14. $\frac{12126}{517}$ yd.
15. $\frac{37183}{19}$ rods.
16. $\frac{218271}{7}$ oz.

100. Reduction of fractions to higher terms of a required denominator.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{2}{3}$ to twelfths of a dollar.

Explanation.—The required denominator (12) is 4 times the given denominator (3); therefore, without changing the value of the fraction, the required numerator must be 4 times the given numerator (2), or 8. Hence, $\$\frac{2}{3}=\$\frac{8}{12}$.

Solution.

$12 \div 3 = 4$

$\$\frac{2}{3} \times \frac{4}{4} = \$\frac{8}{12}$

Principle.—*To multiply both numerator and denominator by the same number does not change the value of the fraction.*

Rule.—I. *Divide the required denominator by the denominator of the given fraction and multiply the quotient by its numerator.*

II. *Write the product, as the new numerator, over the required denominator.*

EXAMPLES FOR PRACTICE.

Reduce

1. $\frac{4}{5}$ lb. to fifteenths.
2. $\frac{5}{7}$ yd. to twenty-eighths.
3. $\frac{3}{8}$ gal. to fifty-sixths.
4. $\frac{1}{4}$ qt. to sixteenths.
5. $\frac{3}{7}$ doz. to thirty-fifths.
6. $\frac{5}{16}$ gal. to forty-eighths.
7. $\frac{7}{13}$ hr. to ninety-firsts.

Reduce

8. $\frac{5}{9}$ bbl. to sixty-thirds.
9. $\frac{3}{14}$ bu. to 126ths.
10. $\frac{5}{32}$ yr. to 128ths.
11. $\frac{7}{27}$ yd. to eighty-firsts.
12. $\frac{11}{15}$ ft. to 165ths.
13. $\frac{18}{19}$ yr. to 152ds.
14. $\frac{17}{196}$ gal. to 1176ths.

101. Reduction of fractions to lower terms of a required denominator.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{24}{72}$ to ninths of a dollar.

Explanation.—The required denominator (9) is one-eighth of the given denominator (72); therefore, the required numerator must be one-eighth of the given numerator (24); and 24+8=3. Hence, $\$\frac{24}{72}+\$\frac{3}{9}$.

Solution.

$72\div9=8$

$\$\frac{24\div8}{72\div8}=\$\frac{3}{9}$

Principle.—*To divide both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

Rule.—I. *Divide the denominator of the fraction by the required denominator, and divide the numerator of the fraction by the quotient thus obtained.*

II. *Write the result, as the new numerator, over the required denominator.*

EXAMPLES FOR PRACTICE.

Reduce

1. $\frac{6}{27}$ ft. to ninths.
2. $\frac{15}{36}$ hr. to twelfths.
3. $\frac{24}{48}$ gal. to sixths.
4. $\frac{15}{40}$ oz. to eighths.
5. $\frac{25}{125}$ bbl. to twenty-fifths.
6. $\frac{36}{66}$ yr. to elevenths.

Reduce

7. $\frac{18}{54}$ yd. to ninths.
8. $\frac{202}{606}$ yd. to thirds.
9. $\frac{366}{915}$ hr. to fifths.
10. $\frac{185}{296}$ bu. to eighths.
11. $\frac{28}{294}$ tons to 42ds.
12. $\frac{2972}{3715}$ gal. to fifths.

102. Reduction of fractions to lowest terms.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{36}{144}$ and $\$\frac{393}{1048}$ to their lowest terms.

Explanation.—Divide both terms of $\$\frac{36}{144}$ by 9, a common factor, obtaining $\$\frac{4}{16}$ as an equivalent fraction in lower terms. Divide both terms of the obtained fraction ($\$\frac{4}{16}$) by 4, a common factor, obtaining $\$\frac{1}{4}$. Since the terms of $\$\frac{1}{4}$ are prime to each other, it must be the required fraction in lowest terms (Rem. 1, **96**).

First Solution.

$\$\frac{36\div9}{144\div9}=\$\frac{4}{16}$

$\$\frac{4\div4}{16\div4}=\$\frac{1}{4}$, *Ans.*

Explanation.—If the common factor cannot be discovered by inspection, find the greatest common divisor of both terms as shown in **75**, obtaining 131.

Divide both terms by 131, obtaining 3 for the numerator and 8 for the denominator of the equivalent fraction in lowest terms.

Second Solution.

G. C. D. of 393 and 1048 is 131

$\$\frac{393\div131}{1048\div131}=\$\frac{3}{8}$, *Ans.*

First Rule.—*Divide both terms by any common factor; divide both terms of the resulting fraction by any common factor; and continue the operation until a fraction is obtained whose terms are prime to each other.*

Second Rule.—*Divide both terms of the fraction by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

Reduce to equivalent fractions in lowest terms:

1. $\frac{48}{72}$ ft.
2. $\frac{8}{16}$ yd.
3. $\frac{27}{81}$ bu.
4. $\frac{24}{40}$ yr.
5. $\frac{63}{81}$ tons.
6. $\frac{28}{35}$ gal.
7. $\frac{28}{98}$ bbl.
8. $\frac{90}{145}$ ft.
9. $\frac{56}{104}$ hr.
10. $\frac{36}{132}$ lb.
11. $\frac{64}{160}$ oz.
12. $\frac{80}{112}$ yd.
13. $\frac{39}{143}$ qt.
14. $\frac{114}{133}$ bbl.
15. $\frac{205}{328}$ yr.
16. $\frac{498}{830}$ yd.
17. $\frac{485}{679}$ bbl.
18. $\frac{549}{671}$ yd.
19. $\frac{604}{1359}$ bu.
20. $\frac{3068}{4602}$ lb.
21. $\frac{1683}{3927}$ tons.
22. $\frac{5608}{7711}$ bbl.
23. $\frac{4707}{6799}$ gal.
24. $\frac{14287}{17188}$ yd.

COMMON DENOMINATOR.

103. A **Common Denominator** of two or more fractions is *any* denominator to which all of them can be reduced.

104. The **Least Common Denominator** of two or more fractions is the *least* denominator to which all of them can be reduced.

REM.—A fraction can be reduced to only such higher denominators as its own denominator can exactly divide (**100**). Hence, a common denominator of two or more fractions must be a multiple of the several denominators of those fractions; and the least common denominator of two or more fractions must be the least common multiple of their several denominators.

105. Reduction of two or more fractions to their least common denominator.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{3}{4}$, $\$\frac{5}{6}$, and $\$\frac{7}{9}$ to equivalent fractions of their least common denominator.

Explanation.—First find the least common multiple of the denominators (4, 6, 9) as shown in **80**, and take the result (36) as the required least common denominator.

Then reduce the several fractions to this common denominator, as shown in **100**; finding that $\$\frac{3}{4}=\$\frac{27}{36}$; $\$\frac{5}{6}=\$\frac{30}{36}$; and $\$\frac{7}{9}=\$\frac{28}{36}$.

Solution.

L. C. M. of 4, 6, 9=36

$\$\frac{3}{4}=(36\div4)\times3=\$\frac{27}{36}$

$\$\frac{5}{6}=(36\div6)\times5=\$\frac{30}{36}$

$\$\frac{7}{9}=(36\div9)\times7=\$\frac{28}{36}$

Rule.—I. *The least common multiple* (**80**) *of the given denominators will be the required least common denominator.*

II. *Reduce each fraction to the required common denominator as shown in* **100.**

Note.—If necessary, reduce all fractions to their lowest terms (**102**) before applying the above rule.

Examples for Practice.

Reduce to fractions having the least common denominator:

1. $\frac{3}{4}$ lb., $\frac{1}{6}$ lb., $\frac{2}{3}$ lb.
2. $\frac{5}{6}$ oz., $\frac{7}{12}$ oz., $\frac{4}{9}$ oz.
3. $\frac{1}{3}$ bu., $\frac{7}{8}$ bu., $\frac{5}{12}$ bu.
4. $\frac{1}{4}$ ft., $\frac{5}{6}$ ft., $\frac{7}{12}$ ft.
5. $\frac{2}{5}$ yr., $\frac{7}{10}$ yr., $\frac{4}{15}$ yr.
6. $\frac{2}{3}$ yd., $\frac{9}{10}$ yd., $\frac{11}{12}$ yd.
7. $\frac{1}{2}$ gal., $\frac{2}{3}$ gal., $\frac{5}{7}$ gal.
8. $\frac{3}{4}$ oz., $\frac{5}{9}$ oz., $\frac{8}{11}$ oz.
9. $\frac{5}{6}$ yr., $\frac{4}{7}$ yr., $\frac{5}{14}$ yr., $\frac{13}{21}$ yr.
10. $\frac{4}{5}$ hr., $\frac{5}{6}$ hr., $\frac{7}{15}$ hr., $\frac{19}{25}$ hr.
11. $\frac{3}{4}$ bbl., $\frac{5}{6}$ bbl., $\frac{7}{8}$ bbl., $\frac{8}{9}$ bbl.
12. $\frac{2}{3}$ ft., $\frac{5}{7}$ ft., $\frac{7}{9}$ ft., $\frac{17}{28}$ ft.
13. $\frac{5}{16}$ bu., $\frac{19}{28}$ bu., $\frac{16}{21}$ bu., $\frac{7}{9}$ bu.
14. $\frac{13}{18}$ rd., $\frac{7}{24}$ rd., $\frac{25}{36}$ rd., $\frac{31}{63}$ rd.
15. $\frac{5}{36}$ yr., $\frac{25}{96}$ yr., $\frac{37}{112}$ yr., $\frac{19}{132}$ yr.
16. $\frac{5}{39}$ bbl., $\frac{16}{17}$ bbl., $\frac{9}{11}$ bbl.

ADDITION OF FRACTIONS.

106. Addition of Fractions is the process of finding the sum of two or more fractions.

107. Principles.—1. *To be added, fractions must express similar fractional units, and therefore have a common denominator.*

2. *The numbers of fractional units* (the numerators) *are the addends.*

3. *The sum of the several numerators expresses fractional units of the same name* (same denominator) *as the fractions added.*

108. To find the sum of two or more fractions.

Illustrative Example.

1. Find the sum of $\$\frac{1}{3}$, $\$\frac{3}{4}$, and $\$\frac{5}{6}$.

Solution.

12 = least com. denom.

$\$\frac{1}{3}$... 4 = new numerator.

$\$\frac{3}{4}$... 9 = " "

$\$\frac{5}{6}$... $\underline{10}$ = " "

$\$\frac{23}{12} = \$1\frac{11}{12}$

Explanation.—Fractions must have a common denominator before they can be added (Prin. 1, **107**). The least common denominator of the several fractions is 12 (**105**). Reduce each fraction to this common denominator, and write the corresponding numerators in a column. Add these numerators (Prin. 2, **107**), obtaining 23 as the numerator of the sum, under which write the common denominator 12 (Prin. 3, **107**), finding $\$\frac{23}{12} = \$1\frac{11}{12}$ to be the required sum.

2. Find the sum of \$$95\frac{2}{3}$, \$$28\frac{5}{8}$, \$$137\frac{1}{6}$, \$$45\frac{5}{9}$.

Explanation.—First add the fractions as in the preceding example, obtaining \$$\frac{145}{72}$=\$$2\frac{1}{72}$. Carry the integral part of this result (2) and add the integers, obtaining \$307, to the right of which annex tne fractional part $\frac{1}{72}$, obtaining \$$307\frac{1}{72}$ as the required sum.

Solution.

$$\begin{array}{rl} & \underline{72} \\ \$95\frac{2}{3} \ldots & 48 \\ 28\frac{5}{8} \ldots & 45 \\ 137\frac{1}{6} \ldots & 12 \\ 45\frac{5}{9} \ldots & \underline{40} \\ \underline{\quad 2} \ldots & \frac{145}{72} = 2\frac{1}{72} \\ \$307\frac{1}{72} & \end{array}$$

Rule.—*Reduce to equivalent fractions of a common denominator* **(105)**; *add the resulting numerators; and place the sum over the common denominator.*

NOTE 1.—To find the sum of two or more mixed numbers: *add the fractions and integers separately, and combine the two results.*

NOTE 2.—When necessary, all results if proper fractions should be reduced to lowest terms; and if improper fractions, to equivalent mixed numbers.

EXAMPLES FOR PRACTICE.

Find the sum of

1. $\frac{7}{8}$ bu., $\frac{2}{3}$ bu., $\frac{3}{4}$ bu.
2. $\frac{5}{6}$ yr., $\frac{1}{9}$ yr., $\frac{5}{12}$ yr.
3. $\frac{1}{3}$ hr., $\frac{6}{7}$ hr., $\frac{3}{8}$ hr.
4. $\frac{3}{5}$ ft., $\frac{5}{6}$ ft., $\frac{7}{15}$ ft.
5. $\frac{1}{2}$ gal., $\frac{3}{4}$ gal., $\frac{1}{3}$ gal.
6. $\frac{4}{9}$ yd., $\frac{7}{12}$ yd., $\frac{1}{8}$ yd.
7. $\frac{2}{3}$ bbl., $\frac{5}{8}$ bbl., $\frac{6}{7}$ bbl.
8. $\frac{3}{4}$ oz., $\frac{2}{3}$ oz., $\frac{6}{11}$ oz.

Find the sum of

9. $\frac{5}{14}$ yd., $\frac{8}{21}$ yd., $\frac{17}{27}$ yd., $\frac{11}{45}$ yd.
10. $\frac{5}{6}$ ft., $\frac{4}{5}$ ft., $\frac{7}{15}$ ft., $\frac{13}{20}$ ft.
11. $\frac{29}{36}$ bu., $\frac{23}{63}$ bu., $\frac{37}{42}$ bu., $\frac{3}{84}$ bu.
12. $\frac{5}{7}$ qt., $\frac{2}{3}$ qt., $\frac{1}{3}$ qt., $\frac{5}{12}$ qt., $\frac{19}{21}$ qt.
13. $37\frac{2}{3}$ lb., $48\frac{3}{4}$ lb., $59\frac{5}{6}$ lb.
14. $387\frac{5}{8}$ yd., $275\frac{1}{6}$ yd., $69\frac{5}{9}$ yd.
15. $132\frac{19}{35}$ yr., $216\frac{5}{7}$ yr., $249\frac{2}{25}$ yr.
16. $72\frac{13}{16}$ lb., $95\frac{5}{8}$ lb., $416\frac{19}{32}$ lb.

17. Add $293\frac{3}{4}$ rods, $75\frac{2}{3}$ rods, $145\frac{4}{15}$ rods, and $618\frac{5}{6}$ rods.

18. Add $9\frac{2}{35}$ bu., $5\frac{3}{7}$ bu., $8\frac{5}{21}$ bu., $7\frac{2}{3}$ bu., and $16\frac{5}{6}$ bu.

REM.—The small figures at the right of the following addends express fourths. This mode of expressing fractions of a yard is common among business men.

19. Add 15^1 yd., 32 yd., 18^2 yd., 27^1 yd., 25^3 yd., 31^1 yd., 19^2 yd.

20. Add 41^2 yd., 37^3 yd., 29 yd., 38^1 yd., 37^1 yd., 36 yd., 43^3 yd.

21. Add 9^2 yd., 8^3 yd., 5^1 yd., 7^1 yd., 11^3 yd., 6^2 yd., 8^1 yd., 5^3 yd.

22. A grocer packed in a box for shipment $15\frac{5}{8}$ lb. sugar, $19\frac{3}{4}$ lb. coffee, $2\frac{5}{16}$ lb. tea, $17\frac{3}{8}$ lb. ham, and $25\frac{1}{2}$ lb. bacon. What was the total weight of the contents of the box?

23. I bought 4 barrels of sugar, the net weight of each being

108

respectively $275\frac{5}{8}$ lb., $285\frac{3}{4}$ lb., $291\frac{1}{3}$ lb., and $269\frac{7}{16}$ lb. What was the total net weight?

24. A man inherited $\frac{5}{17}$ of an estate in his own right, and bought the shares of two fellow-legatees who had inherited respectively $\frac{11}{51}$ and $\frac{7}{34}$ of the same estate. What part of the estate did he then own?

25. A lady purchased the following goods: $5\frac{3}{4}$ yards of ribbon for \$$\frac{5}{8}$, $1\frac{3}{4}$ yards of lace for \$$\frac{31}{32}$, and $5\frac{1}{3}$ yards of silk for \$$15\frac{3}{16}$. How much money did she expend altogether?

SUBTRACTION OF FRACTIONS.

109. Subtraction of Fractions is the process of finding the difference between two fractions.

110. Principles.—1. *To be subtracted, the fraction of the subtrahend must express fractional units of the same name as the fraction of the minuend; and both fractions must therefore have a common denominator.*

2. *The numbers of fractional units* (the numerators) *are the terms of subtraction.*

3. *The difference of the numerators expresses fractional units of the same name* (same denominator) *as those of the subtrahend and minuend.*

111. To find the difference between two fractions.

ILLUSTRATIVE EXAMPLE.

Subtract \$$\frac{4}{9}$ from \$$\frac{5}{6}$.

Explanation.—First find the least common denominator (Prin. 1, **110**). Reduce each fraction to this common denominator; write the new numerator of the subtrahend under the new numerator of the minuend; and subtract (Prin. 2, **110**), obtaining 7 as the numerator of the difference, under which write the common denominator (Prin. 3, **110**).

Solution.

$$\begin{array}{ll} & 18 = \text{com. denom.} \\ \$\tfrac{5}{6} \ldots & 15 = \text{new numerator.} \\ \$\tfrac{4}{9} \ldots & \underline{\ \ 8} = \text{"} \qquad \text{"} \\ & \$\tfrac{7}{18} = \textit{Answer.} \end{array}$$

Rule.—*Reduce to equivalent fractions of a common denominator, and place the difference of the resulting numerators over the common denominator.*

NOTE.—If *small* mixed numbers are given, reduce to improper fractions before applying the rule.

EXAMPLES FOR PRACTICE.

Find the difference between

1. $\frac{5}{12}$ ft. and $\frac{3}{4}$ ft.
2. $\frac{3}{8}$ in. and $\frac{5}{7}$ in.
3. $\frac{1}{6}$ bbl. and $\frac{3}{4}$ bbl.
4. $\frac{7}{9}$ lb. and $\frac{4}{21}$ lb.
5. $\frac{4}{11}$ yr. and $\frac{5}{7}$ yr.
6. $\frac{5}{13}$ yd. and $\frac{3}{52}$ yd.
7. $\frac{11}{16}$ hr. and $\frac{5}{24}$ hr.
8. $\frac{7}{8}$ gal. and $\frac{5}{6}$ gal.
9. $\frac{1}{2}$ oz. and $\frac{1}{3}$ oz.
10. $\frac{3}{4}$ bbl. and $\frac{5}{9}$ bbl.
11. $1\frac{1}{2}$ yr. and $2\frac{5}{6}$ yr.
12. $3\frac{1}{4}$ bu. and $2\frac{2}{3}$ bu.
13. $5\frac{1}{7}$ lb. and $9\frac{2}{3}$ lb.
14. $3\frac{4}{5}$ qt. and $5\frac{7}{15}$ qt.
15. $1\frac{5}{8}$ yr. and $3\frac{1}{3}$ yr.

112. Subtraction of mixed numbers.

ILLUSTRATIVE EXAMPLES.

1. Subtract \$275 from \$$328\frac{3}{4}$.

Solution.

$$\begin{array}{r} \$328\frac{3}{4} \\ 275 \\ \hline \$53\frac{3}{4} \end{array}$$

Explanation.—Since *no* fourths of a dollar are to be subtracted from \$$\frac{3}{4}$, the fractional part of the remainder will be \$$\frac{3}{4}$, which should be annexed to the difference of the integers, obtaining \$$53\frac{3}{4}$.

2. Subtract \$$275\frac{3}{4}$ from \$328.

Solution.

$$\begin{array}{r} \$328 \\ 275\frac{3}{4} \\ \hline \$52\frac{1}{4} \end{array}$$

Explanation.—\$$\frac{3}{4}$ cannot be taken from *no* fourths of a dollar; therefore borrow \$1 or \$$\frac{4}{4}$ from \$328, leaving \$327, and then subtract. \$$\frac{3}{4}$ from the borrowed \$$\frac{4}{4}$, leaves \$$\frac{1}{4}$, which should be annexed to \$327—\$275, obtaining \$$52\frac{1}{4}$.

3. Subtract \$$425\frac{3}{4}$ from \$$576\frac{2}{3}$.

Solution.

$$\begin{array}{rl} & 12 \\ \$576\frac{2}{3} \ldots & 8+12 \\ 425\frac{3}{4} \ldots & 9 \\ \hline \$150\frac{11}{12} & \end{array}$$

Explanation.—Since \$$\frac{9}{12}$ cannot be taken from \$$\frac{8}{12}$, borrow \$1 or \$$\frac{12}{12}$ from \$576, leaving \$575, and then subtract. \$$\frac{9}{12}$ from (\$$\frac{8}{12}$ + \$$\frac{12}{12}$ =) \$$\frac{20}{12}$ leaves \$$\frac{11}{12}$, which annex to \$575—\$425, obtaining \$$150\frac{11}{12}$.

Rule.—*Subtract the fractions and integers separately and combine the two results.*

EXAMPLES FOR PRACTICE.

Find the difference between

1. \$751 and \$$275\frac{3}{4}$.
2. \$458 and \$$627\frac{5}{8}$.
3. \$291 and \$$525\frac{1}{3}$.
4. \$627 and \$$328\frac{7}{8}$.
5. \$575 and \$$832\frac{5}{7}$.
6. *\$905 and \$$62\frac{3}{7}$.*
7. \$$96\frac{2}{3}$ and \$$75\frac{1}{4}$.
8. \$$873\frac{5}{8}$ and \$$925\frac{9}{10}$.
9. \$$340\frac{1}{6}$ and \$$625\frac{7}{8}$.
10. \$$580\frac{1}{3}$ and \$$316\frac{2}{3}$.
11. \$$671\frac{3}{7}$ and \$$265\frac{5}{6}$.
12. \$$279\frac{5}{8}$ and \$$125\frac{1}{6}$.
13. \$$197\frac{5}{8}$ and \$$327\frac{2}{3}$.
14. \$$281\frac{11}{12}$ and \$$609\frac{7}{8}$.
15. \$$413\frac{1}{2}$ and \$$520\frac{5}{8}$.
16. \$$612\frac{4}{5}$ and \$$417\frac{1}{3}$.
17. 58^1 yd. and 71 yd.
18. 88^1 yd. and 37^3 yd.

112

MULTIPLICATION OF FRACTIONS.

113. Multiplication of Fractions is the process of finding the product of two factors when one or both of them are fractions.

REM.—By referring to the relation of integers (Sol., **61**) it will be seen that one factor of a product (the multiplicand) denotes the weight, measure, value, etc., of one whole addend; and the other factor (the multiplier) the number of such addends. It follows, therefore, that if the multiplier is a proper fraction (less than 1) it denotes that the weight, measure, value, etc., of the required product is less than that of one whole addend (less than the multiplicand); and that if the multiplier is an integer, a mixed number, or an improper fraction (greater than 1) it denotes that the weight, measure, value, etc., of the required product is greater than that of one whole addend (greater than the multiplicand). Hence,

114. Principle.—*Multiplication implies increase of the multiplicand when the multiplier is greater than 1; and decrease of the multiplicand when the multiplier is less than 1.*

NOTE.—Prin. 1, 2, 3, and 4, **31**, relating to integers, are equally applicable to fractions.

115. To multiply a fraction by a fraction.

ILLUSTRATIVE EXAMPLE.

What is the cost of $\frac{4}{15}$ of a yard of cloth at \$$\frac{5}{8}$ per yard?

Explanation.—$\frac{4}{15}$=4 times $\frac{1}{15}$. If 1 yard ($\frac{8}{8}$) cost \$$\frac{5}{8}$, $\frac{1}{15}$ of a yard will cost $\frac{1}{15}$ of \$$\frac{5}{8}$, or \$$\frac{5}{120}$ (Prin. 2, **95**); and $\frac{4}{15}$ of a yard will cost 4 times \$$\frac{5}{120}$, or \$$\frac{20}{120}$ (Prin. 1, **95**), which reduced to lowest terms equals \$$\frac{1}{6}$.

For convenience, cancel all factors common to opposite terms, obtaining the result in lowest terms without further reduction.

Solution.

$$\frac{\$5}{8}\times\frac{4}{15}=\frac{\$20}{120}=\frac{\$1}{6}$$

$$\text{Or, }\frac{\$\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}}=\frac{\$1}{6}$$

Rule.—*Multiply the numerators for a new numerator and the denominators for a new denominator, first canceling all factors which are common to opposite terms.*

NOTE 1.—If one or both factors are *small* mixed numbers, reduce them to improper fractions and employ the above rule.

NOTE 2.—The word *of* written between fractions, or between fractions and integers, indicates that they are to be multiplied. Fractions thus connected are called *Compound Fractions.*

EXAMPLES FOR PRACTICE.

Multiply	**Multiply**	**Multiply**	**Reduce**
1. $\$\frac{5}{7}$ by $\frac{7}{15}$.	6. $\$\frac{4}{11}$ by $\frac{11}{28}$.	11. $\$\frac{7}{8}$ by $1\frac{1}{4}$.	16. $\frac{1}{6}$ of $\$\frac{1}{2}$.
2. $\$\frac{9}{17}$ by $\frac{34}{45}$.	7. $\$\frac{5}{8}$ by $\frac{3}{20}$.	12. $\$\frac{11}{15}$ by $3\frac{2}{3}$.	17. $\frac{2}{3}$ of $\$\frac{3}{4}$.
3. $\$\frac{1}{2}$ by $\frac{1}{3}$.	8. $\$\frac{7}{13}$ by $\frac{1}{12}$.	13. $\$2\frac{1}{4}$ by $1\frac{1}{3}$.	18. $\frac{1}{2}$ of $\frac{3}{4}$ of $\$\frac{4}{5}$.
4. $\$\frac{2}{3}$ by $\frac{3}{4}$.	9. $\$\frac{3}{4}$ by $\frac{2}{5}$.	14. $\$3\frac{1}{2}$ by $5\frac{1}{4}$.	19. $\frac{3}{8}$ of $\frac{6}{7}$ of $\$\frac{7}{8}$.
5. $\$\frac{1}{2}$ by $\frac{4}{5}$.	10. $\$\frac{5}{17}$ by $\frac{2}{5}$.	15. $\$4\frac{1}{3}$ by $2\frac{3}{4}$.	20. $\frac{1}{3}$ of $2\frac{1}{2}$ of $\$5$.

116. Multiplication of fractions and integers.

ILLUSTRATIVE EXAMPLES.

Multiply (1) $\$\frac{3}{4}$ by 71; (2) $\$\frac{5}{16}$ by 8; (3) $\$\frac{5}{8}$ by 32; and (4) $\$\frac{5}{6}$ by 10.

Solutions.

(1.)

$$\frac{\$3\times 71}{4}=\begin{array}{r}71\\ 3\\ \hline 4\,)\,213\\ \hline \$53\frac{1}{4}\end{array}$$

(2.)

$$\frac{\$5}{\underset{2}{\cancel{16}}}\times\overset{1}{\cancel{8}}=\frac{\$5}{2}=\$2\frac{1}{2}$$

(3.)

$$\frac{\$5}{\underset{1}{\cancel{8}}}\times\overset{4}{\cancel{32}}=\$20$$

(4.)

$$\frac{\$5}{\underset{3}{\cancel{6}}}\times\overset{5}{\cancel{10}}=\frac{\$25}{3}=\$8\frac{1}{3}$$

Explanation.—The explanation of these examples is similar to that of Ill. Ex., **115.** This will be evident by writing the integral multipliers in the form of fractions with 1 as their denominators, and then applying Rule, **115.** Thus, (1) $\$\frac{3}{4}\times\frac{71}{1}=?$ (2) $\$\frac{5}{16}\times\frac{8}{1}=?$ (3) $\$\frac{5}{8}\times\frac{32}{1}=?$ (4) $\$\frac{5}{6}\times\frac{10}{1}=?$

Rule.—*Multiply the integer by the numerator of the fraction, and divide the product by the denominator, first canceling all factors common to the integer and the denominator.*

NOTE.—When the integer is seen to be an exact divisor of the denominator of the fraction, apply Prin. 1, **95,** and avoid a written solution involving formal cancellation. Thus, $\$\frac{1}{8}\times 4=\$\frac{1}{8+4}$ or $\$\frac{1}{2}$.

EXAMPLES FOR PRACTICE.

Multiply	**Multiply**	**Multiply**	**Multiply**
1. $\$785$ by $\frac{3}{4}$.	7. $\$\frac{9}{40}$ by 8.	13. $\$925$ by $\frac{3}{7}$.	19. $\$250$ by $\frac{8}{11}$.
2. $\$159$ by $\frac{7}{8}$.	8. $\$44$ by $\frac{4}{11}$.	14. $\$\frac{5}{36}$ by 9.	20. $\$\frac{9}{112}$ by 37.
3. $\$491$ by $\frac{3}{5}$.	9. $\$\frac{8}{15}$ by 45.	15. $\$40$ by $\frac{9}{10}$.	21. $\$4$ by $\frac{7}{8}$.
4. $\$\frac{5}{7}$ by 692.	10. $\$75$ by $\frac{7}{25}$.	16. $\$\frac{9}{35}$ by 14.	22. $\$\frac{13}{14}$ by 56.
5. $\$\frac{8}{9}$ by 185.	11. $\$\frac{11}{63}$ by 14.	17. $\$16$ by $\frac{5}{32}$.	23. $\$13$ by $\frac{7}{39}$.
6. $\$\frac{13}{18}$ by 6.	12. $\$12$ by $\frac{5}{18}$.	18. $\$\frac{7}{16}$ by 28.	24. $\$\frac{4}{5}$ by 25.

116

117. Multiplication of integers and mixed numbers.

ILLUSTRATIVE EXAMPLES.

Multiply (1) \$$179\frac{3}{4}$ by 7; and (2) \$325 by $18\frac{5}{8}$.

Explanation.—(1.) 7 times $\frac{3}{4}=\frac{21}{4}=5\frac{1}{4}$. Write $\frac{1}{4}$ and mentally carry 5. 7 times 9 are 63, and 5 to carry are 68. Write 8, carry 6 to 7 times 7, and continue as with other integers.

(2.) If the integer is large, as in second solution, multiply it by the fractional part of the mixed number as shown in Ill. Ex., **116**, Solution 1; then by the integral part of the mixed number.

Solutions.

$$\begin{array}{r}
\text{(1.)} \\
\$179\frac{3}{4} \\
\underline{7} \\
\$1258\frac{1}{4}
\end{array}
\qquad
\begin{array}{r}
\text{(2.)} \\
\$325 \\
\underline{18\frac{5}{8}} \\
8)\underline{1625} \\
203\frac{1}{8} \\
2600 \\
\underline{325} \\
\$6053\frac{1}{8}
\end{array}$$

Rule.—*Separately multiply the integer by the fractional and the integral part of the mixed number; then add the results.*

EXAMPLES FOR PRACTICE.

Multiply

1. \$$37\frac{1}{5}$ by 15.
2. \$48 by $8\frac{5}{6}$.
3. \$630 by $4\frac{7}{8}$.
4. \$560 by $34\frac{3}{7}$.
5. \$$73\frac{1}{5}$ by 95.

Multiply

6. \$$481\frac{13}{18}$ by 9.
7. \$$394\frac{2}{15}$ by 5.
8. \$471 by $38\frac{4}{5}$.
9. \$$19\frac{5}{8}$ by 251.
10. \$827 by $39\frac{3}{5}$.

Multiply

11. \$987 by $27\frac{3}{4}$.
12. \$$35\frac{5}{9}$ by 71.
13. \$495 by $32\frac{7}{8}$.
14. \$897 by $39\frac{5}{8}$.
15. \$$413\frac{5}{8}$ by 275.

118. Multiplication of mixed numbers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Find the cost of $25\frac{3}{4}$ pounds of coffee at $18\frac{5}{8}$ cents per pound.

Explanation.—Separately multiply the integral and fractional part of the multiplicand (18 and $\frac{5}{8}$) by the fractional part of the multiplier ($\frac{3}{4}$); then separately multiply the fractional and integral part of the multiplicand (18 and $\frac{5}{8}$) by the integral part of the multiplier (25). The sum of the several partial products thus obtained will be the total product.

Solution.

$$\begin{array}{lrl}
 & \$.18\frac{5}{8} & \\
 & \underline{25\frac{3}{4}} & \underline{32} \\
(\frac{5}{8}\times\frac{3}{4}=) & \frac{15}{32} & \ldots 15 \\
(18\times\frac{3}{4}=) & 13\,\frac{1}{2} & \ldots 16 \\
(\frac{5}{8}\times 25=) & 15\,\frac{5}{8} & \ldots \underline{20} \\
(18\times 25=)\left\{\begin{array}{l} \\ \end{array}\right. & \begin{array}{r} 90 \\ 36 \end{array} & \frac{51}{32}=1\frac{19}{32} \\
 & \overline{\$4.79\frac{19}{32}.} &
\end{array}$$

Rule.—I. *Separately multiply the fractional and integral parts of the multiplicand by the fractional part of the multiplier.*

II. *Separately multiply the fractional and integral parts of the multiplicand by the integral part of the multiplier.*

III. *Add the several partial products thus obtained.*

EXAMPLES FOR PRACTICE.

Multiply

1. $45\frac{2}{3}$ oz. by $18\frac{4}{5}$.
2. $16\frac{3}{5}$ ft. by $15\frac{5}{8}$.
3. $56\frac{1}{3}$ yr. by $18\frac{4}{7}$.
4. $438\frac{3}{4}$ yd. by $64\frac{5}{6}$.
5. $79\frac{3}{11}$ bu. by $15\frac{2}{5}$.

Multiply

6. $473\frac{5}{8}$ lb. by $19\frac{3}{4}$.
7. $453\frac{3}{4}$ hr. by $5\frac{2}{3}$.
8. \$$3786\frac{1}{2}$ by $37\frac{1}{2}$.
9. \$$1928\frac{3}{4}$ by $16\frac{2}{3}$.
10. $143\frac{1}{6}$ yr. by $58\frac{2}{7}$.

Multiply

11. \$$375\frac{5}{6}$ by $437\frac{4}{5}$.
12. $75\frac{5}{16}$ bbl. by $64\frac{7}{16}$.
13. \$$1825\frac{3}{4}$ by $25\frac{5}{8}$.
14. \$$519\frac{2}{3}$ by $17\frac{3}{5}$.
15. \$$620\frac{5}{8}$ by $73\frac{5}{6}$.

16. I bought $375\frac{5}{16}$ yards of calico at $5\frac{1}{4}$ cents per yard. $9\frac{3}{4}$ yards being damaged, were sold at $2\frac{1}{8}$ cents per yard, and the remainder at $6\frac{1}{4}$ cents per yard. What was my net profit ?

DIVISION OF FRACTIONS.

119. Division of Fractions is the process of finding the quotient when the dividend, the divisor, or both of them, contain fractions.

REM.—Since the dividend corresponds with the product, and the divisor and quotient to the multiplicand and multiplier (**40**), Rem., **113**, and Prin., **114**, taken reversely, also apply to division; that is, if the divisor (multiplier) is an integer, a mixed number, or an improper fraction whose value is greater than 1, the quotient (multiplicand) will be less than the dividend (product); but if the divisor is a proper fraction, the quotient will be greater than the dividend. Hence,

120. Principle.—*Division implies decrease of the dividend when the divisor is greater than 1, and increase of the dividend when the divisor is less than 1.*

121. The **Reciprocal of a Fraction** is 1 divided by that fraction, or that fraction with its terms inverted.

The Reciprocal of \$$\frac{5}{8}$ is \$1÷\$$\frac{5}{8}$, or 8 *eighths of a dollar* divided by 5 *eighths of a dollar* which equals $\frac{8}{5}$, or the fraction $\frac{5}{8}$ with its terms inverted.

122. The **Reciprocal of an Integer** is 1 divided by that integer.

The Reciprocal of \$8 is $\frac{1}{8}$; of 15 ft. is $\frac{1}{15}$, etc.

123. Division of fractions or integers by fractions.

ILLUSTRATIVE EXAMPLE.

Divide (1) $\$\frac{5}{6}$ by $\$\frac{3}{8}$; and (2) \$14 by $\$\frac{2}{3}$.

Solutions.

(1.)

$$\frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{3} = \frac{20}{9} = 2\frac{2}{9}$$

(2.)

$$\overset{7}{\cancel{14}} \times \frac{3}{\cancel{2}} = 21$$

Explanation.—(1) $\$1 \div \$\frac{3}{8} = \frac{8}{3}$ (**121**); hence, $\frac{5}{6}$ of \$1, or $\$\frac{5}{6} \div \$\frac{3}{8}$ must equal $\frac{5}{6}$ of $\frac{8}{3}$, or $\frac{40}{18} = 2\frac{2}{9}$ (Prin. 2, **47**). The product of $\frac{5}{6} \times \frac{8}{3}$ is obtained by Rule, **115**.

(2) $\$1 \div \$\frac{2}{3} = \frac{3}{2}$; and $\$14 \div \$\frac{2}{3}$ must therefore be 14 times $\frac{3}{2}$, or 21 (Prin. 1, **47**). The product of $14 \times \frac{3}{2}$ is obtained by Rule, **116**.

Rule.—*Multiply the dividend by the reciprocal of the divisor.*

EXAMPLES FOR PRACTICE.

Divide

1. \$1 by $\$\frac{8}{9}$.
2. \$18 by $\frac{6}{11}$.
3. \$5 by $\frac{40}{41}$.
4. $\$\frac{3}{4}$ by $\$\frac{3}{5}$.
5. $\$\frac{6}{7}$ by $\frac{5}{8}$.
6. $\$\frac{1}{6}$ by $\frac{2}{3}$.
7. \$63 by $\frac{35}{36}$.
8. \$15 by $\$1\frac{2}{13}$.

Divide

9. $\$\frac{5}{8}$ by $\$1\frac{7}{8}$.
10. \$69 by $2\frac{7}{8}$.
11. $\$3\frac{1}{2}$ by $1\frac{1}{4}$.
12. $\$2\frac{2}{3}$ by $\$1\frac{1}{6}$.
13. \$225 by $\$\frac{75}{76}$.
14. \$1215 by $\frac{5}{8}$.
15. $\$7\frac{3}{4}$ by $\$\frac{8}{9}$.
16. \$56 by $5\frac{1}{3}$.

Divide

17. \$519 by $\$\frac{6}{7}$.
18. $\$3\frac{3}{5}$ by $4\frac{1}{5}$.
19. $\$2\frac{3}{4}$ by $\$3\frac{1}{2}$.
20. $\$5\frac{1}{3}$ by $\$\frac{7}{12}$.
21. $\frac{2}{3}$ of $\$\frac{7}{8}$ by $\frac{3}{4}$.
22. $\$\frac{14}{37}$ by $\frac{1}{4}$ of $\frac{2}{3}$.
23. $\frac{1}{2}$ of $\$\frac{2}{3}$ by $\frac{1}{3}$ of $\$\frac{1}{4}$.
24. $\frac{5}{6}$ of \$8 by $\frac{1}{2}$ of $\$5\frac{1}{3}$.

124. Division of fractions by integers.

ILLUSTRATIVE EXAMPLES.

Divide (1) $\$\frac{10}{21}$ by 5; (2) $\$\frac{3}{4}$ by 2; (3) $\$\frac{5}{7}$ by 15; (4) $\$\frac{6}{11}$ by 4.

Solutions.

(1.)

$$\frac{\$\overset{2}{\cancel{10}}}{21 \times \cancel{5}} = \frac{\$2}{21}$$

(2.)

$$\frac{\$3}{4 \times 2} = \frac{\$3}{8}$$

(3.)

$$\frac{\$\overset{1}{\cancel{5}}}{7 \times \underset{3}{\cancel{15}}} = \frac{\$1}{35}$$

(4.)

$$\frac{\$\overset{3}{\cancel{6}}}{11 \times \underset{2}{\cancel{4}}} = \frac{\$3}{22}$$

Explanation.—The solution of these examples is simply an application of Rule, **123**, and young or dull pupils should be allowed to use Rule, **123** in lieu of the one following. The only advantage of the following rule is that it avoids the awkwardness of placing integers in the form of fractions, as (1) $\$\frac{10}{21} \div \frac{5}{1} = \$\frac{10}{21} \times \frac{1}{5}$; (2) $\$\frac{3}{4} \div \frac{2}{1} = \$\frac{3}{4} \times \frac{1}{2}$; etc.

Rule.—*Multiply the denominator by the integer and write the product under the numerator, first canceling all factors common to the integer and the numerator.*

Note.—When the integer is seen to be an exact divisor of the numerator of the fraction, apply Prin. 2, **95**, and avoid a written solution involving formal cancellation. Thus, $\$\frac{9}{16}\div3=\$\frac{9+3}{16}$ or $\$\frac{3}{16}$.

EXAMPLES FOR PRACTICE.

Divide	Divide	Divide	Divide
1. $\$\frac{3}{7}$ by 3.	8. $\$\frac{3}{7}$ by $21.	15. $\$\frac{2}{3}$ by $5.	22. $\$1\frac{5}{7}$ by 36.
2. $\$\frac{5}{6}$ by $5.	9. $\$\frac{8}{9}$ by 3.	16. $\$5\frac{1}{4}$ by 7.	23. $\$5\frac{3}{4}$ by 46.
3. $\$\frac{3}{4}$ by 18.	10. $\$\frac{12}{17}$ by $4.	17. $\$\frac{15}{16}$ by $12.	24. $\$2\frac{1}{2}$ by $5.
4. $\$\frac{4}{5}$ by $8.	11. $\$\frac{2}{3}$ by 12.	18. $\$\frac{8}{9}$ by 28.	25. $\$5\frac{5}{8}$ by 9.
5. $\$\frac{3}{4}$ by 7.	12. $\$\frac{9}{17}$ by 5.	19. $\$\frac{35}{36}$ by 7.	26. $\$8\frac{2}{3}$ by 39.
6. $\$\frac{4}{7}$ by 9.	13. $\$1\frac{1}{4}$ by 15.	20. $\$\frac{7}{15}$ by $35.	27. $\$7\frac{3}{5}$ by 4.
7. $\$\frac{8}{9}$ by 4.	14. $\$\frac{9}{10}$ by 27.	21. $\$\frac{4}{5}$ by 3.	28. $\$1\frac{3}{7}$ by 25.

29. $\frac{5}{8}$ of an estate was left to be equally divided among 15 legatees. What part of the estate did each receive?

30. What is the cost of 7 pounds of beef if 4 pounds cost $\$\frac{7}{8}$?

125. Division of mixed numbers by integers.

ILLUSTRATIVE EXAMPLES.

Divide (1) $\$5166\frac{18}{23}$ by 9; and (2) $\$4832\frac{1}{4}$ by $3.

Solutions.

(1.)

$$9\,)\,\underline{\$5166\frac{18}{23}}$$
$$\$574\frac{2}{23}\ \left[\frac{18}{23}\div9=\frac{2}{23}\right]$$

(2.)

$$\$3\,)\,\underline{\$4832\frac{1}{4}}\qquad\left[\begin{array}{c}2\frac{1}{4}=\frac{9}{4}\\ \frac{9}{4}\div3=\frac{3}{4}\end{array}\right]$$
$$1610\frac{3}{4}$$

Explanation.—(1) Divide the integral part of the dividend ($5166) by the divisor (9), obtaining exactly $574. Then divide the fractional part of the dividend ($\frac{18}{23}$) by the divisor (9), obtaining $\frac{2}{23}$. Combine the results.

(2) Dividing $4832 by 3, leaves a remainder of 2, which carried to the fraction ($\frac{1}{4}$) produces $2\frac{1}{4}$ or $\frac{9}{4}$; and $\frac{9}{4}+3=\frac{3}{4}$.

Rule.—*Separately divide the integral and fractional parts of the dividend by the divisor, and combine the two results.*

Note.—Instead of applying this and the two succeeding rules, mixed numbers may be reduced to improper fractions and the division be performed by preceding rules; but this is inconvenient except with *small* mixed numbers.

EXAMPLES FOR PRACTICE.

Divide

1. 987640\frac{3}{4}$ by $8.
2. 5131$\frac{4}{15}$ yd. by 7.
3. 8325$\frac{27}{31}$ bu. by 9.
4. 4185$\frac{3}{5}$ yr. by 5 yr.
5. 191838$\frac{15}{41}$ ft. by 6.

Divide

6. 518321$\frac{1}{2}$ yd. by 3 yd.
7. 917583\frac{4}{5}$ by 10.
8. 761328\frac{5}{7}$ by $12.
9. 28734$\frac{3}{5}$ oz. by 8 oz.
10. 513824\frac{7}{8}$ by 6.

Divide

11. 134827\frac{3}{4}$ by 16.
12. 58372\frac{5}{8}$ by $84.
13. 48273\frac{4}{11}$ by 25.
14. 51513\frac{3}{8}$ by $31.
15. 71854\frac{5}{12}$ by 152.

16. A retail shoe dealer bought 24 pairs of shoes for 72\frac{3}{4}$. What was the average price per pair?

17. If a vessel average 15 miles per hour, in how many hours can she sail 1831$\frac{3}{4}$ miles?

126. Division of integers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Divide $189 by 25$\frac{2}{3}$.

Solution.

$$\begin{array}{rl} 25\frac{2}{3} &)\ \$189 \\ 3 & \quad\ \ 3 \\ \hline 77 &)\ \$567\ (\ \$7\frac{4}{11} \\ & \quad\ 539 \\ \hline & \quad\ \frac{28}{77}=\frac{4}{11} \end{array}$$

Explanation.—To multiply both divisor and dividend by the same number produces no change in the quotient (Prin. 3, **47**). Hence, multiply both divisor and dividend by the denominator of the fractional part of the divisor (3), obtaining an integral divisor and dividend, and then divide as with other integers.

Rule.—*Multiply both divisor and dividend by the denominator of the terminal common fraction of the divisor to obtain an integral divisor and dividend; then divide as with other integers.*

EXAMPLES FOR PRACTICE.

Divide

1. $25316 by 3$\frac{1}{2}$.
2. $57829 by 5\frac{1}{4}$.
3. $91753 by 8\frac{1}{2}$.
4. $21057 by 4$\frac{5}{8}$.

Divide

5. $91873 by 6$\frac{3}{7}$.
6. $41075 by 9\frac{4}{5}$.
7. $29834 by 12$\frac{3}{4}$.
8. $18726 by 28$\frac{3}{8}$.

Divide

9. $2134 by 75\frac{5}{8}$.
10. $5498 by 82$\frac{3}{8}$.
11. $6217 by 125$\frac{6}{7}$.
12. $7275 by 834$\frac{1}{6}$.

13. How many casks containing 36$\frac{3}{8}$ gallons each are capable of holding 29920 gallons of cider?

127. Division of mixed numbers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Divide \$459$\frac{2}{3}$ by \$36$\frac{1}{2}$.

Solution.

$$
\begin{array}{r@{\ )\ }l}
\$36\frac{1}{2} & \$459\frac{2}{3}\ (\\
\underline{\quad 6} & \underline{\quad\ \ 6} \\
\$219 & \$2758\ (\ 12\frac{130}{219} \\
 & \underline{219} \\
 & \ \ 568 \\
 & \ \ \underline{438} \\
 & \ \ 130
\end{array}
$$

Explanation.—Multiply both mixed numbers by the least common denominator (6) of their terminal fractions, obtaining \$2758 as an integral dividend and \$219 as an integral divisor (Prin. 8, **47**). Divide as with other integers, producing $12\frac{130}{219}$ as the required quotient.

Rule.—*Multiply both divisor and dividend by the least common denominator of their terminal common fractions to obtain an integral divisor and dividend; then divide as with other integers.*

EXAMPLES FOR PRACTICE.

Divide	Divide	Divide
1. \$75$\frac{2}{3}$ by 14$\frac{3}{4}$.	5. \$7834$\frac{3}{8}$ by \$28$\frac{1}{2}$.	9. \$517$\frac{2}{15}$ by 92$\frac{4}{25}$.
2. \$465$\frac{1}{2}$ by 8$\frac{5}{8}$.	6. \$5167$\frac{5}{12}$ by 35$\frac{2}{3}$.	10. \$7821$\frac{7}{12}$ by \$38$\frac{5}{18}$.
3. \$219$\frac{4}{5}$ by \$12$\frac{5}{8}$.	7. \$7916$\frac{5}{8}$ by \$16$\frac{5}{6}$.	11. \$591$\frac{6}{7}$ by 128$\frac{2}{3}$.
4. \$926$\frac{1}{4}$ by 9$\frac{5}{6}$.	8. \$4153$\frac{1}{2}$ by 71$\frac{1}{3}$.	12. \$9418$\frac{3}{11}$ by 59$\frac{5}{8}$.

RELATION OF COMMON FRACTIONS.

128. So much of **58, 59, 60, 61,** and **62** as pertains to addition and subtraction of integers also applies without modification to addition and subtraction of fractions; and so much as pertains to multiplication and division of integers will also apply to multiplication and division of fractions, *if the abstract factor* (denoting number of equal addends) *of a given or required product is an integer, a mixed number, or an improper fraction.* If, however, as explained in Rem., **113,** and Rem., **119,** the abstract factor is a proper fraction (denoting one or more *parts* of an addend) the following modification of **61** may be substituted.

129. Fractional multiplication and division compared.

1. Find the cost of $\frac{1}{8}$ of a pound of tea at 48 cents per pound.

2. If $\frac{1}{8}$ of a pound of tea cost 48 cents, what is the price per pound?

3. How much tea at 48 cents per pound can be bought for 6 cents?

Solution.

Factors of dividend. { concrete divisor = $.48 = cost of 1 *whole* pound = equal addend = multiplicand } **Factors of product.**
{ abstract quotient = $\times \frac{1}{8}$ = fractional part of a whole pound = multiplier }
Dividend, if given = $.06 = cost of part of a whole pound = product, if required.

Deduction.—(*a*) The weight, measure, value, etc., of any part of a whole unit or quantity is the product if required, or the dividend if given. (*b*) *One factor* of a required product or given dividend is the weight, measure, value, etc., of *one whole unit or quantity;* and (*c*) the other factor is any considered *fractional part* of that whole unit or quantity. Hence, if a product is required, multiply its two given factors; and if a product is given, divide it by its given factor and the quotient will be its remaining factor.

130. Summary.—(a) *Employ* MULTIPLICATION *if the weight, measure, value, etc., of any part of a whole unit or quantity is* REQUIRED; *or* (b) DIVISION *if the weight, measure, value, etc., of any part of a whole unit or quantity is* GIVEN.

NOTE.—An example may involve *two or more* given or required products, in which case there will be as many separate multiplications as there are products required, and as many separate divisions as there are products given. (An exception is when the weights, measures, values, etc., of *all* the several parts of a whole quantity are given simply for the purpose of adding them to find the weight, measure, value, etc., of the whole quantity.)

ILLUSTRATIVE EXAMPLES.

1. What is the cost of $\frac{3}{4}$ of a pound of tea at \$$\frac{7}{8}$ per pound?

Explanation.—The cost of a fractional part ($\frac{3}{4}$) of a pound is *required* (*a*, **130**), therefore multiply its two given factors as shown in Deduction, **129.**

Solution.

$\$\frac{7}{8} \times \frac{3}{4} = \$\frac{21}{32}$

2. At \$$\frac{3}{4}$ per gallon, how much molasses can be bought for \$$\frac{1}{3}$?

Explanation.—The cost (\$$\frac{1}{3}$) of a fractional part of a gallon is *given* (*b*, **130**), therefore divide it by its given factor (cost of one whole gallon, \$$\frac{3}{4}$) to find its remaining factor (the fractional part of a gallon, $\frac{4}{9}$).

Solution.

$\$\frac{1}{3} \div \$\frac{3}{4} = \frac{4}{9}$

3. If $\frac{3}{8}$ bu. corn can be bought for \$$\frac{5}{8}$, what is the price per bushel?

Solution.

$\$\frac{5}{8} \div \frac{2}{3} = \$\frac{15}{16}$

Explanation.—The cost ($\$\frac{5}{8}$) of a fractional part ($\frac{2}{3}$) of a bushel is *given;* therefore divide it ($\$\frac{5}{8}$) by its given factor (the fractional part of a bushel, $\frac{2}{3}$) to find its other factor (cost of one whole bushel, $\$\frac{15}{16}$).

4. If $\frac{5}{6}$ of a cord of wood contains $106\frac{2}{3}$ cubic feet, how many cubic feet will $\frac{2}{3}$ of a cord contain?

Solution.

$106\frac{2}{3}$ cu. ft. $\div \frac{5}{6} = 128$ cu. ft.

128 cu. ft. $\times \frac{2}{3} = 85\frac{1}{3}$ cu. ft.

Explanation.—The quantity ($106\frac{2}{3}$ cu. ft.) in one fractional part ($\frac{5}{6}$) of a cord is *given,* and the quantity of another fractional part ($\frac{2}{3}$) of a cord is *required;* hence there will be one division of the given product (*b,* **130**); and one multiplication to find the required product (*a,* **130**), as stated in Note, **130.** Both the given and required products have a common factor (the cu. ft. in one whole cord). Therefore,

Divide the given product ($106\frac{2}{3}$ cu. ft.) by its given factor ($\frac{5}{6}$), to find its other factor (128 cu. ft.), which is also a factor of the required product; and multiply the obtained factor of the required product (128 cu. ft.) by its other factor ($\frac{2}{3}$), obtaining $85\frac{1}{3}$ cu. ft. as the required product.

EXAMPLES FOR PRACTICE.

131. 1. At $\$\frac{7}{8}$ per yard, how many yards of cloth can be bought for $\$5\frac{3}{4}$?

2. What is the cost of $\frac{5}{8}$ of a ton of iron at $\$128\frac{4}{15}$ per ton?

3. At $\$\frac{1}{8}$ per pound, how many pounds of sugar can be bought for $\$\frac{13}{16}$?

4. I bought $\frac{3}{4}$ lb. butter for $\$\frac{3}{8}$. What was the price per pound?

5. If a furnace consume $\frac{15}{16}$ of a ton of coal per month, in what time will it consume $3\frac{3}{4}$ tons?

6. How many pounds of cheese at $\$\frac{3}{8}$ per pound can be bought for $\$32\frac{5}{16}$?

7. If a cow eats $\frac{4}{7}$ bu. bran in one day, in what time will she eat $4\frac{5}{7}$ bu.?

8. If I pay $8\frac{1}{4}$ cents for $\frac{3}{4}$ of a pound of sugar, how much of the same kind of sugar can I buy for $60\frac{1}{2}$ cents?

9. What amount of honey worth $30\frac{5}{8}$ cents per pound can be bought for $25\frac{3}{4}$ cents?

10. If a ship can sail $268\frac{13}{14}$ miles in one day, how far can she sail in $3\frac{5}{8}$ days?

11. At the rate of $3\frac{13}{15}$ miles per hour, how long will a man require *to walk* $15\frac{1}{3}$ *miles?*

131

12. How many vegetable beds of $\frac{3}{16}$ of an acre each can be obtained from 1$\frac{1}{3}$ acres of land?

13. A piece of muslin containing 40$\frac{3}{4}$ yards, cost \$2.03$\frac{3}{4}$. At what price per yard was it bought?

14. If there are 9$\frac{3}{5}$ hours in $\frac{2}{5}$ of a day, how many hours are in 4$\frac{3}{5}$ days?

15. I owe \$53$\frac{3}{4}$ and wish to pay the debt with oats worth \$$\frac{5}{8}$ per bushel. How many bushels should I deliver?

16. If 3$\frac{3}{5}$ tons of hay are worth \$62$\frac{1}{20}$, how much hay will be worth \$45$\frac{5}{8}$?

17. If $\frac{5}{8}$ bu. wheat cost \$$\frac{5}{8}$, what is the cost of $\frac{3}{4}$ bu.?

18. If a pupil can write $\frac{3}{5}$ of a composition in $\frac{3}{5}$ of an hour, in what time will he be able to write his entire composition?

19. How much silk can be bought for \$5$\frac{5}{8}$, if $\frac{3}{4}$ of a yard of the same silk cost \$2$\frac{1}{3}$?

20. If 18 men can do a piece of work in $\frac{3}{5}$ of a day, how long will it take one man to do it?

21. If 5 men can do a piece of work in 3$\frac{1}{4}$ days, how long will it take 7 men to do it?

22. A person attempted to walk 37$\frac{1}{2}$ miles, but succeeded in walking only $\frac{3}{5}$ of that distance. How far did he walk?

132. In comparisons of quantities, the word *of* follows the fractional part of the whole quantity (*c*, **129**) and precedes the whole quantity (*b*, **129**), thus facilitating the ready identification of these two factors of a required product or given dividend.

NOTE.—The phrases *more than* and *less than* also precede the whole quantity, but follow fractions which must be respectively added to or subtracted from 1 to find the considered fractional part of the whole quantity.

ILLUSTRATIVE EXAMPLES.

1. What part of \$300 are \$50?

Explanation.—The value (\$50) of a fractional part of the whole quantity (\$300) is *given;* therefore divide \$50 (*b*, **130**) by its given factor (the value of the whole quantity, \$300) to find its other factor (the fractional part of the whole quantity, $\frac{1}{6}$).

Solution.

$$\$50 \div \$300 = \frac{50}{300} = \frac{1}{6}$$

REM.—Applying **132**, *of* is seen to follow the required fractional part (what part) and precedes the whole quantity (\$300), the two factors of the given product, \$50.

132

2. $18\frac{1}{4}$ lb. sugar are $\frac{5}{8}$ of what quantity?

Explanation.—The weight ($18\frac{1}{4}$ lb.) of a fractional part ($\frac{5}{8}$) of a whole quantity is *given;* hence, divide it by its given factor (the considered fractional part of the whole quantity, $\frac{5}{8}$) to find its other factor (the weight of the whole quantity, $29\frac{1}{5}$ lb.).

Solution.

$18\frac{1}{4}$ lb. $\div \frac{5}{8} = 29\frac{1}{5}$ lb.

Rem.—In this example, *of* follows the given fractional part ($\frac{5}{8}$) and precedes the required whole quantity (what quantity), which are the two factors of the given product, $18\frac{1}{4}$ lb.

3. $\frac{5}{6}$ of $3\frac{1}{2}$ yd. silk equals how many yards?

Explanation.—The length of a fractional part ($\frac{5}{6}$) of a whole quantity ($3\frac{1}{2}$ yd.) is *required* (*a*, **130**); therefore multiply its two given factors.

Solution.

$3\frac{1}{2}$ yd. $\times \frac{5}{6} = 2\frac{11}{12}$ yd.

Rem.—In this example, *of* follows the fractional part ($\frac{5}{6}$), and precedes the whole quantity ($3\frac{1}{2}$ yd.), the two given factors of the required product ($2\frac{11}{12}$ yd.).

4. 84 miles is $\frac{3}{4}$ more than what distance?

Explanation.—Add $\frac{3}{4}$ to 1, obtaining $\frac{7}{4}$ as the considered fractional part of the whole distance (Note, **132**).

Solution.

$84 \div \frac{7}{4} = 48$ mi.

The length (84 mi.) of a fractional part ($\frac{7}{4}$) of the whole distance is *given;* therefore divide it (84 mi.) by its given factor (the considered fractional part of the whole distance, $\frac{7}{4}$) to find its other factor (the length of the whole distance, 48 mi.).

Rem.—After deciding, according to **132** and *Note*, what is the whole quantity taken as the standard of comparison, the measure, weight, value, etc., of *more* or *less* than (that is, *any* improper or proper fractional part of) this whole quantity should be regarded as a product if required, or a dividend if given.

133. EXAMPLES FOR PRACTICE.

1. 35 gallons of molasses were sold from a barrel containing 42 gallons. What part of the barrel was sold?

2. If $\frac{3}{8}$ of a patent-right are worth $1500, what is the value of the whole patent-right?

3. What part of 150 gallons of vinegar are 25 gallons of vinegar?

4. How many acres of land in $\frac{3}{4}$ of 750 acres?

5. A man had $8000, and spent $\frac{1}{5}$ of what he had in building a house. What was the cost of the house?

6. A farmer has 18 acres or $\frac{3}{5}$ of all his arable land in oats. How many acres of arable land did he possess?

7. A merchant drew from bank $3000, which was $\frac{5}{13}$ of his deposit. What was his original deposit?

8. I paid \$4500 for a lot, and afterwards sold it for $\frac{11}{20}$ of its cost. What did I obtain for the lot?

9. A farmer sold 75 acres from a farm of 320 acres. What part of the farm did he sell?

10. I sold $\frac{5}{8}$ of a piece of goods containing $39\frac{3}{4}$ yards. How many yards remained in the piece?

11. In a certain battle, a regiment lost $\frac{2}{15}$ of its men in killed, wounded, and prisoners. If its total loss was 160 men, what was its strength before the battle?

12. In a certain school are 375 pupils, of whom 150 are boys and the remainder girls. What part of the school are girls?

13. A gentleman bought \$18500 worth of real estate, paid $\frac{13}{20}$ of the purchase money in cash, and gave his note for the remainder. What was the face of the note?

14. A captain owned $\frac{4}{7}$ of the vessel which he commanded, and valued his share at \$26824. At that rate, what was the value of the vessel?

15. A contributed \$3500 to the capital of a firm, B \$7000, and C \$4500. What part of the total capital did each contribute?

16. A firm has $\frac{1}{3}$ of its capital in merchandise, $\frac{3}{8}$ in real estate, and the remainder, or \$1680, in cash. What is the capital of the firm?

17. If bread weighs $\frac{1}{20}$ more than the flour from which it is made, how much bread can be obtained from 45 pounds of flour?

18. A farm consists of 35 acres in grass, 48 acres in corn, 32 acres in oats, 30 acres in grass, 25 acres in woodland, and the remaining 5 acres in orchard, etc. What part of the farm is in grass?

REVIEW OF COMMON FRACTIONS.

134. 1. A vegetable garden contains $175\frac{3}{28}$ square rods in potatoes, $135\frac{4}{21}$ square rods in cabbage plants, $75\frac{4}{35}$ square rods in onions, and $382\frac{5}{12}$ square rods in other vegetables. What is the area of the garden?

2. A grocer had $75\frac{5}{8}$ pounds of coffee and afterwards bought $87\frac{3}{4}$ pounds at one time and $146\frac{1}{2}$ pounds at another. Of this coffee he sold at one time $95\frac{5}{16}$ pounds, and at another $138\frac{3}{4}$ pounds. How much coffee had he then remaining?

3. A pole was broken off 12 feet from the top, leaving 30 feet still standing. What part of the pole remained standing?

4. A man contracted to dig $1873\frac{1}{4}$ cubic yards of earth. How much of his contract remained after digging $1530\frac{5}{12}$ cubic yards ?

5. The average yield per acre of a field of wheat was $18\frac{3}{4}$ bushels. What was the total yield if the field contained 35 acres ?

6. A farmer sold to a merchant $460\frac{3}{4}$ pounds of pork at $5\frac{1}{8}$ cents per pound, and 50 bushels of corn at $65\frac{1}{8}$ cents per bushel, receiving in part payment 30 yards of calico at $5\frac{3}{4}$ cents per yard, 25 yards of sheeting at $32\frac{5}{8}$ cents per yard, $9\frac{5}{8}$ yards of alpaca at 25 cents per yard, and 75 pounds of sugar at $6\frac{1}{8}$ cents per pound. What is the balance due the farmer ?

7. A boy had $\frac{3}{5}$ of an orange and gave to a companion $\frac{3}{4}$ of what he had. What part of the orange did the boy retain ?

8. A man exchanged with a grocer $\frac{11}{16}$ of a bushel of corn worth 64 cents per bushel for $6\frac{2}{5}$ pounds of bacon. What did the grocer charge per pound for the bacon ?

9. A farm wagon carrying $6\frac{4}{5}$ barrels of corn at a load, delivered all the corn grown in a certain field in 58 loads. What was the total yield of that field ?

10. The distance by rail from Baltimore to Washington is $39\frac{5}{16}$ miles. If a way station on that road is $18\frac{5}{12}$ miles from Baltimore, what is the distance of that station from Washington ?

11. A merchant sold $18\frac{3}{4}$ yards of calico from a piece containing $38\frac{7}{8}$ yards. What part of the piece did he sell ?

12. A factory is worth $12432, and a man owning $\frac{3}{8}$ of an interest in that factory sold $\frac{5}{6}$ of his share at its full value. What sum did he receive?

13. A child received $\frac{7}{8}$ of a quart of milk, but in carrying it home spilled $\frac{5}{8}$ of a quart. What part of what she received did she spill ?

14. A, B, and C counted their money and found that A had $\frac{3}{8}$ of the total amount, B $\frac{1}{4}$, and C the remainder. How much had B and C if A had $30 ?

15. I bought $2\frac{1}{4}$ acres of land and divided it into building lots of $\frac{3}{8}$ of an acre each. How many building lots did I obtain ?

16. A grocer bought 5 bags of coffee, the gross weight of each being respectively $129\frac{3}{4}$, $134\frac{5}{16}$, $131\frac{7}{8}$, $128\frac{1}{2}$, and $132\frac{1}{16}$ pounds. Allowing $1\frac{1}{8}$ pounds for the weight of each bag, what was the total net weight of the coffee ?

17. A merchant sold 75 pounds of butter at $24\frac{3}{8}$ cents per pound, 93 dozen eggs at $16\frac{2}{3}$ cents per dozen, and $48\frac{5}{16}$ gallons of milk at 25 *cents per gallon. What* was the total amount of sales ?

18. A dealer bought a firkin of butter weighing 56 pounds at $22\frac{1}{2}$ cents per pound. If he sells $\frac{3}{8}$ of the firkin for $5.04, and the remainder at a profit of $2\frac{1}{2}$ cents per pound, what is his total gain ?

19. A dealer bought 658 pounds of cheese, and sold a part of it at $15\frac{5}{8}$ cents per pound, obtaining $67.50. How many pounds had he then remaining ?

20. A farmer sold to a merchant 12 pounds of butter at $18\frac{3}{4}$ cents per pound, and 5 hams weighing respectively $12\frac{1}{2}$, $18\frac{5}{8}$, $15\frac{3}{4}$, $14\frac{15}{16}$, and 16 pounds at $11\frac{1}{2}$ cents per pound. He took in part payment 5 pounds of coffee at $22\frac{1}{2}$ cents per pound, 18 pounds of sugar at $6\frac{3}{4}$ cents per pound, 38 yards of muslin at $7\frac{3}{4}$ cents per yard and the remainder in cash. How much money did the farmer receive ?

21. A can do a piece of work in 15 days and B in 18 days. In what time can it be done if both work together ?

22. A by working alone can finish a certain job of work in 9 days, and A and B by working together can finish the same job in 4 days. In what time can B alone finish the job ?

23. A can complete a job of work in 4 days, B in 6 days, and C in 8 days. In what time can the job be completed if all work together ?

24. $18\frac{7}{8}$ gallons were sold from a barrel of molasses containing $45\frac{5}{32}$ gallons. How many gallons remained in the barrel ?

25. I bought 9 bags of wheat weighing respectively $123\frac{1}{2}$, $119\frac{3}{4}$, $125\frac{5}{8}$, $120\frac{7}{8}$, $117\frac{15}{16}$, $119\frac{5}{16}$, $122\frac{1}{4}$, $120\frac{1}{2}$, and $119\frac{1}{4}$ pounds, at $$1\frac{3}{8}$ per bushel of 60 pounds. Allowing 1 pound for the weight of each bag, what was the total cost ?

26. A dealer bought 46 bales of cotton, averaging 435 pounds per bale, at $11\frac{17}{32}$ cents per pound, and sold the entire quantity at $12\frac{3}{4}$ cents per pound. If his expenses for freight, insurance, commission, etc., amounted to $95.30, what was his total net profit ?

27. How many acres does a farm contain if $\frac{3}{8}$ of it is in grass, $\frac{5}{16}$ in corn, $\frac{1}{4}$ in wheat, and the remaining 16 acres in oats ?

28. If a fruit vender buy lemons at the rate of 5 for 3 cents, how many must he sell at the rate of 6 for 5 cents to gain 14 cents ?

29. A and B being $135\frac{3}{4}$ miles apart, started towards each other and met in 4 days, A traveling at the average rate of $3\frac{1}{2}$ miles per hour and $6\frac{3}{4}$ hours per day, and B at the average rate of $2\frac{3}{4}$ miles per hour. How many hours per day did B travel ?

30. A, B, and C engage in partnership, A investing $3000, B $4000, and C $5000. What part of the total capital did each invest ?

31. A barrel has two faucets of different sizes. If only the larger faucet is opened, the barrel will be emptied in 8 minutes; and if both are opened, it will be emptied in 5 minutes. In what time will the barrel be emptied if only the smaller faucet be opened?

32. A pedestrian traveled $32\frac{3}{8}$ miles on Monday, $23\frac{5}{12}$ miles on Tuesday, $37\frac{3}{4}$ miles on Wednesday, $19\frac{5}{8}$ miles on Thursday, $24\frac{1}{6}$ miles on Friday, and $16\frac{1}{3}$ miles on Saturday. What was the average distance traveled per day?

33. A gentleman invested $\frac{3}{4}$ of his money in city property, $\frac{1}{3}$ of the remainder in a farm, $\frac{3}{4}$ of what still remained in improvements upon the farm, and then had $960 left. What did he have at first?

REM.—$(1-\frac{3}{4})$ of $(1-\frac{1}{3})$ of $(1-\frac{3}{4})$=fractional part, or factor, of given product.

34. A house and lot cost $7200, and the house cost $\frac{2}{5}$ more than the lot. What was the cost of each?

35. The capital of a firm is $18248, and the interest of one partner is $\frac{3}{5}$ of the interest of the other. What was each partner's interest?

36. The cost of a carriage and a span of horses is $1260, and the cost of the horses is $\frac{2}{7}$ less than the cost of the carriage. What is the cost of each?

37. A mechanic paid $\frac{1}{4}$ of his weekly wages for board, deposited $\frac{2}{3}$ of the remainder in a savings-bank, and retained $$3\frac{3}{4}$ to defray his expenses through the following week. What were his weekly wages?

38. A man owed $4200 and paid $1575. What part of his original debt did he still owe?

39. A, B, and C, by working together, can do a piece of work in $1\frac{1}{3}$ days. If A can do it alone in 4 days, and B in 6 days, in what time can C do it alone?

40. A man bought a farm, paid $\frac{3}{5}$ of the purchase money in cash, and in payment of the balance, gave his note, secured by mortgage, for $3400. How much did he pay in cash?

41. If 18 boarders consume $\frac{4}{5}$ of a barrel of flour in one week, how much flour per week will be required if 6 additional boarders are received?

42. If a bucket contains 8 pints of water when it is $\frac{2}{5}$ full, what part of the bucket will be filled when 4 pints of water are added?

43. If coffee loses $\frac{1}{15}$ of its weight in roasting, how much green coffee will be required to make 252 pounds of roasted coffee?

44. If a miller charges $\frac{1}{8}$ of a grist for toll, how much corn must I carry to the mill to entitle me to receive 203 pounds of corn meal?

45. If A can cut a cord of wood in $\frac{1}{3}$ of a day, B in $\frac{2}{3}$ of a day, C in $1\frac{1}{2}$ days, and D in $1\frac{1}{3}$ days, in what time will the cord be cut if all work together?

46. A barrel has two spigots. If only the larger spigot is withdrawn, the barrel will be emptied in 15 minutes, and if only the smaller is withdrawn the barrel will be emptied in 20 minutes. In what time will the barrel be emptied if both spigots are simultaneously withdrawn?

47. 8 men could have dug a trench in 9 days; but after doing half the work, one-fourth of the men were discharged. In what time can the remainder finish the job?

48. A merchant sold 468 barrels of a consignment of potatoes and had 312 barrels remaining. What part of the consignment did he sell?

49. A owes B \$30.28$\frac{1}{8}$, and pays it in $32\frac{1}{4}$ yards of dress goods at $82\frac{1}{2}$ cents per yard, and the remainder in calico at $5\frac{1}{4}$ cents per yard. How many yards of calico should B receive?

50. A, B, and C own a mill. If A owns $\frac{1}{4}$ of it, and B $\frac{1}{3}$ of it more than A, what part of the mill does C own?

51. A laborer withdrew $\frac{5}{12}$ of his deposit from a savings-bank, and with $\frac{3}{5}$ of the money thus withdrawn paid his rent, which was \$12. How much money has he still on deposit?

DECIMAL FRACTIONS.

135. A **Decimal Fraction** expresses one or more of the equal decimal parts of a unit.

REM.—The term *decimal* is derived from the Latin *decem*, signifying *ten*; and is applied to this kind of fractions because the denominator is either 10, or the product of two or more tens, as 100, 1000, 10000, etc.

136. The **Decimal Point** is a period (.) written to the left of the numerator of a decimal to indicate its denominator.

REM.—The denominator of a decimal is indicated by writing the decimal point as many places to the left of the right-hand figure of the numerator as there are ciphers in the denominator. Thus, \$$\frac{8}{10}$, \$$\frac{15}{100}$, and \$$\frac{95}{1000}$, by omitting their denominators, may be written \$.8, \$.15, and \$.095.

137. The **Decimal Order** or **Decimal Value** of a figure is its position *with respect to the* decimal point.

Rem.—The *nearer* any figure of a decimal fraction is to its decimal point, the *higher* its value; and the *further* it is removed from the decimal point, the *less* its value.

138. The **Notation of Decimals** is simply an extension, on a descending scale, of the system of notation of integers, in which the value of any order is ten times the value of the next order at the right, and one-tenth of the next order at the left, as is shown in the following

DECIMAL NOTATION TABLE.

Etc., etc.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Etc., etc.
	9	7	5	3	9	1	8	3	9	2	.	5	1	2	4	6	7	9	1	5	
	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	
	Integral Orders.											*Decimal Orders.*									

Explanation.—It will be noticed in the preceding table that, commencing at the decimal point, the integral orders are enumerated from the right to the left, and the decimal orders from the left to the right; that the first integral order at the left of the units' place is *tens*, and the first decimal order at the right of the units' place is *tenths;* that the second integral order at the left of the units' place is *hundreds*, and the second decimal order at the right of the units' place is *hundreths*, etc., etc.; and, generally, that the name of any decimal order corresponds with the name of that integral order which is equally distant from the units' place.

139. A **Complex Decimal** is one which has a common fraction at the right of its lowest order; as \$.5⅔; \$.003¼.

140. A **Simple Decimal** is one which has no common fraction at the right of its lowest order; as \$.25; \$.0098.

Rem.—Complex Decimals are said to be *terminate* when they can be reduced to simple decimals, and *interminate* when they cannot be so reduced.

141. A **Mixed Decimal** is one which is composed of an integer and a decimal fraction; as 5.18 lb.; 98.005 bu.

142. A **Decimal Unit** is *one* of the equal decimal parts into which a whole unit is divided.

Rem.—The name of the decimal unit, or denominator of the decimal, is the same as the name or order of its right-hand figure. Thus, in the decimal $.018, 1 *thousandth of a dollar* is the decimal unit.

143. Principles.—1. *The denominator of a decimal is 1 followed by as many ciphers as it has places to the right of the decimal point.*

Thus, in $.0015, the denominator is 1 followed by four ciphers (10000) or *ten-thousandths;* and the decimal is read 15 ten-thousandths dollars.

2. *Annexing ciphers to, or omitting them from, the right of a decimal does not change its value.*

To annex a cipher to a decimal also annexes a cipher to its denominator (Prin. 1), and hence multiplies both numerator and denominator by 10, and does not change the value of the fraction (Prin. 3, **95**); and to omit a cipher from the right of a decimal also omits a cipher from its denominator, and therefore divides both numerator and denominator by 10.

3. *Prefixing a cipher to a decimal and moving the decimal point to the left of the cipher prefixed, divides the decimal by 10.*

To prefix a cipher to a decimal does not change its numerator, but annexes another cipher to its denominator (Prin. 1), which is equivalent to multiplying the denominator by 10, and dividing the fraction by 10 (Prin. 2, **95**).

4. *Moving the decimal point one place to the right multiplies the decimal by 10; two places to the right multiplies by 100; etc.*

To move the decimal point one place to the right does not change the number of fractional units; but makes the numerator contain one decimal place, and the denominator one cipher, less than before (Prin. 1); which is equivalent to dividing the denominator by 10, and multiplying the fraction by 10 (Prin. 1, **95**).

5. *Moving the decimal point one place to the left divides the decimal by 10; two places to the left divides by 100; etc.*

To move the decimal point one place to the left does not change the number of fractional units; but makes the numerator contain one decimal place, and the denominator one cipher, more than before (Prin. 1); which is equivalent to multiplying the denominator by 10, and dividing the fraction by 10 (Prin. 2, **95**).

NUMERATION OF DECIMALS.

ILLUSTRATIVE EXERCISE.

144. Express, orally, the decimal .00378 of a pound.

Explanation.—First read the numerator as if it were an integer (three hundred and seventy-eight); then, applying Prin. 1, **143**, name the denominator, which is thus found to be 1 followed by five ciphers (100000); or hundred-thousandths.

Rule.—*Read the decimal as if it were an integer; and then state the name of its right-hand order.*

EXAMPLES FOR PRACTICE.

Express orally the following decimals:

1. $.75.
2. .075 oz.
3. .325 lb.
4. .0325 gal.
5. .215 yr.
6. .0215 bbl.
7. .625 bu.
8. .0625 ft.
9. .00625 tons.
10. .937 yr.
11. .0937 yd.
12. .00937 bu.
13. .11625 bbl.
14. .83715 yr.
15. .517382 gal
16. .8267513 oz.
17. .0017834 lb.
18. 5.083291 yd.
19. 7.0056832 bu.
20. 8.0010005 tons.
21. 6.10070082 yd.
22. 1.00015008 ft.
23. 6.005832$\frac{3}{4}$ bu.
24. 4.10075008$\frac{1}{2}$ lb.

NOTATION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

145. **Write decimally twenty-three hundred-thousandths.**

Explanation.—First write the decimal point; then as many ciphers as the numerator 23 (occupying 2 places) lacks of having 5 places, the number of ciphers in the given denominator hundred-thousandths (thus, .000); finally the numerator, obtaining .00023 as the required decimal.

Rule.—*Write the numerator as if it were an integer, and so locate the decimal point that the right-hand figure of the numerator will occupy the decimal order denoted by the denominator.*

NOTE.—The learner should accustom himself to writing the decimal point and prefixed ciphers *before* writing the numerator, as shown in the Illustrative Example. The number of ciphers to prefix equals the number of ciphers in the denominator diminished by the number of orders in the numerator.

Write decimally the following:

1. $\$\frac{7}{10}$.
2. $\$\frac{85}{100}$.
3. $\$\frac{72}{1000}$.
4. $\$\frac{5}{1000}$.
5. $\$\frac{45}{100000}$.
6. $\$\frac{205}{100000}$.
7. $\$\frac{18}{1000000}$.
8. $\$\frac{2001}{1000000}$.
9. $\$\frac{65}{10000000}$.
10. $\$\frac{4}{1000000}$.
11. $\$\frac{23478}{10000000}$.
12. $\$\frac{125678}{10000000}$.
13. $\$\frac{8}{100000000}$.
14. $\$\frac{20107}{100000000}$.
15. $\$\frac{605}{10000}$.
16. $\$\frac{5831\frac{3}{4}}{100000}$.
17. 8 hundredths.
18. 18 hundredths.
19. 375 thousandths.
20. 45 thousandths.
21. *9 thousandths.*
22. 1275 ten-thousandths.
23. 95 ten-thousandths.
24. 75 hundred-thousandths.
25. 1578 hundred-thousandths.
26. 285 millionths.

145

27. One, and two hundred and seventy-five thousandths miles.

28. Five, and two thousand and nine ten-thousandths years.

29. Two, and two hundred and six hundred-thousandths gallons.

30. Two thousand five hundred and nine millionths of a ton.

31. Five thousand three hundred and fifty ten-millionths of a year.

32. Eighty-six, and two million three hundred and eighteen thousand six hundred and seventy-five hundred-millionths miles.

33. Five thousand two hundred eighty, and nineteen thousand three hundred and seventy-nine billionths years.

34. Nine thousand forty, and one million twenty-six thousand eight hundred thirty-seven hundred-millionths acres.

146. Reduction of common fractions to decimals.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{19}{125}$ to a decimal.

Explanation.—$\$\frac{19}{125}$ represents an unexecuted division, $19 being the dividend and 125 the divisor (Rem., **93**). To annex ciphers to the dividend $19 multiplies it by 10 for every cipher annexed; hence, if the numerator with ciphers annexed be divided by the denominator, the quotient will be 10 times the value of the fraction $\$\frac{19}{125}$ for each cipher so annexed (Prin. 1, **47**). Since 3 ciphers are annexed to the numerator, which is equivalent to multiplying the numerator by 1000, the quotient must be 1000 times the true quotient. Rectify this error by dividing the quotient 152 by 1000, effected by cutting off three figures from the right of the quotient (**49**).

Solution.

```
125 ) $19.000 ( $.152
      12 5
      ----
       6 50
       6 25
       ----
         250
         250
         ---
```

Rule.—*Annex ciphers to the numerator, divide by the denominator, and point off from the right of the quotient as many decimal places as there have been ciphers annexed.*

EXAMPLES FOR PRACTICE.

Reduce the following to equivalent decimals:

1. $\frac{3}{8}$ gal.
2. $\frac{7}{8}$ yd.
3. $\frac{5}{16}$ bu.
4. $\frac{3}{16}$ yr.
5. $\frac{15}{32}$ *bbl.*
6. $\frac{17}{64}$ oz.
7. $\frac{13}{80}$ hr.
8. $\frac{7}{20}$ bu.
9. $\frac{24}{25}$ yd.
10. $\frac{1}{125}$ ft.
11. $\frac{1}{16}$ lb.
12. $\frac{5}{64}$ yr.
13. $\frac{1}{125}$ bu.
14. $\frac{3}{128}$ gal.
15. $3\frac{5}{32}$ days.
16. $51\frac{1}{25}$ yd.
17. $47\frac{3}{64}$ bu.
18. $6\frac{14}{25}$ rods.
19. $28\frac{1}{32}$ miles.
20. $76\frac{3}{256}$ gal.

146

NOTE 1.—To avoid long division, separate the denominator into factors, successively divide by each factor, and point off as many decimal places as there have been ciphers annexed to *all* the dividends. Thus, $\$\frac{5}{32}=\$\frac{5}{4\times8}$; and $500÷4=125, and 125000÷8=15625, cutting off 5 places, because 2 ciphers were annexed in the first division and 3 in the second, obtaining $.15625.

Reduce the following to decimals by applying Note 1:

21. $\frac{3}{16}$ lb.	**23.** $\frac{5}{32}$ bu.	**25.** $\frac{7}{125}$ ft.	**27.** $3\frac{5}{256}$ tons.
22. $\frac{11}{64}$ gal.	**24.** $\frac{13}{128}$ yd.	**26.** $\frac{9}{800}$ yr.	**28.** $6\frac{3}{500}$ rods.

NOTE 2.—If the denominator of a common fraction in lowest terms contains other prime factors than 2 or 5, the equivalent decimal will be interminate (Rem., **140**). In such a case, annex to the numerator as many ciphers as there are decimal places required in the result and, after the last cipher has been brought down, discard the remainder if less than half the divisor, or increase the last quotient figure by 1, if the remainder is half or more than half the divisor.

Applying Note 2, reduce the following to decimals of

2 dec. places.	*3 dec. places.*	*4 dec. places.*	*5 dec. places.*
29. $\$18\frac{2}{3}$.	**31.** $\$16\frac{5}{12}$.	**33.** $\$215\frac{1}{64}$.	**35.** $\$185\frac{9}{11}$.
30. $\$75\frac{7}{9}$.	**32.** $\$84\frac{7}{33}$.	**34.** $\$761\frac{7}{9}$.	**36.** $\$940\frac{1}{128}$.

NOTE 3.—To expand a complex decimal to a given number of decimal places, annex to the numerator of the terminal common fraction a cipher for each lacking decimal place, and then divide by the denominator.

Expand the following complex decimals to

5 dec. places.	*6 dec. places.*	*7 dec. places.*
37. $38.278\frac{5}{6}$ lb.	**40.** $97.51\frac{3}{7}$ rods.	**43.** $9.008\frac{2}{3}$ miles.
38. $56.13\frac{15}{32}$ bu.	**41.** $75.4\frac{7}{12}$ tons.	**44.** $17.5001\frac{3}{11}$ acres.
39. $8.4\frac{5}{6}$ yr.	**42.** $1.0005\frac{7}{16}$ yr.	**45.** $.0000\frac{1}{7}$ yr.

147. Reduction of decimals to common fractions.

ILLUSTRATIVE EXAMPLE.

Reduce $.875 and $\$.05\frac{2}{3}$ to common fractions.

First Solution.—Change the decimal to the form of a common fraction by writing its denominator, obtaining $\$\frac{875}{1000}$; then reduce to lowest terms.

Second Solution.—Supplying the denominator produces the complex fraction $\$\frac{5\frac{2}{3}}{100}$; multiplying both terms by the denominator of the terminal common fraction (3) produces the equivalent simple *fraction* $\$\frac{17}{300}$ *(Prin. 3,* ***95****).*

First Solution.

$$\$.875=\frac{\$875\div125=\$7}{1000\div125=\;8}$$

Second Solution.

$$\$.05\tfrac{2}{3}=\frac{\$5\frac{2}{3}\times3=\$17}{100\times3=300}$$

Rule.—*Omit the decimal point and prefixed ciphers, write the denominator underneath, and reduce the resulting fraction to lowest terms.*

EXAMPLES FOR PRACTICE.

Reduce the following decimals to common fractions:

1. .75 bbl.
2. .375 gal.
3. .25 yr.
4. .625 yd.
5. .025 bu.
6. .075 hr.
7. 3.0875 rods.
8. 5.055 acres.
9. .00125 hr.
10. .0005 yr.
11. 8.0064 days.
12. 6.0008 yr.
13. 9.0016 gal.
14. 5.31625 yr.
15. .9375 bu.
16. .34375 hr.
17. $.37\frac{1}{2}$ bbl.
18. $.06\frac{1}{4}$ yd.
19. $3.33\frac{1}{3}$ days.
20. $7.16\frac{2}{3}$ miles.
21. $4.02\frac{2}{9}$ gal.
22. $.08\frac{1}{3}$ yd.
23. $2.98\frac{8}{9}$ rods.
24. $.000\frac{5}{6}$ bu.

ADDITION OF DECIMALS.

ILLUSTRATIVE EXERCISE.

148. 1. Add 5.25 bu., 18.0375 bu., .045 bu., and 9.3125 bu.

Explanation.—Only like orders can be added (Prin. 1, **22**), therefore arrange the addends so that like decimal orders shall fall in the same column. Add as in whole numbers. Since the sum of the tenths' column must be tenths (Prin. 2, **22**), place the decimal point in the sum at the left of 6 tenths to denote its decimal order.

Solution.

$$\begin{array}{r} 5.25 \\ 18.0375 \\ .045 \\ 9.3125 \\ \hline 32.6450 \end{array}$$

2. Find the sum of $9.82\frac{5}{8}$ yr., $5.1\frac{3}{4}$ yr., $7.065\frac{3}{4}$ yr., and $195.004\frac{1}{3}$ yr., to three decimal places.

Explanation.—Arrange the decimals, add, and point off as in Ex. 1; first expanding the complex decimals by Notes 2 and 3, **146**, to 4 decimal places, or one more decimal place than is required in the sum. The extra decimal place in the addends is intended to insure accuracy in the 3 decimal places to be retained in the sum.

Solution.

$$\begin{array}{r} 9.8263- \\ 5.175 \\ 7.0657+ \\ 195.0043+ \\ \hline 217.0713 \end{array}$$

Ans., 217.071 + yr.

Principles.—1. *Only similar decimal orders can be added; hence the decimal points of the addends should fall in a column.*

2. *The sum expresses decimal units of the same order as the columns added; hence the decimal point of the sum should fall beneath the decimal points of the addends.*

148

Rule.—*Arrange the addends so that their decimal points shall fall in a column. Add as in whole numbers; and place the decimal point in the result directly beneath the decimal points of the addends.*

EXAMPLES FOR PRACTICE.

What is the sum of

1. 3.18 lb., 54.375 lb., 18.3 lb., 1.00486 lb., 216.75 lb. ?

2. 419.04 bu., 8.0032 bu., 75.865 bu., 475.5 bu., 5.073 bu. ?

3. 7253.25 yr., 45.0056 yr., 392.275 yr., 4.03157 yr., .05 yr. ?

4. 625.475 gal., 297.35 gal., 4.0056 gal., .8 gal., 52.08 gal. ?

5. 6.75 yd., .0013 yd., .09 yd., .5 yd., 48.0537 yd., .15 yd. ?

6. $18.45\frac{3}{4}$ rods, $412.196\frac{5}{8}$ rods, $6.3\frac{4}{5}$ rods, $19.413\frac{3}{16}$ rods, $16.81\frac{7}{32}$ rods, and $4.05\frac{3}{80}$ rods, as a simple decimal ?

7. $8.15\frac{3}{8}$ miles, $16.038\frac{7}{25}$ miles, $230.9\frac{3}{500}$ miles, $89.72\frac{1}{3}\frac{2}{3}$ miles, $418.875\frac{1}{3}$ miles, .8 of a mile, and 4.25 miles, as a simple decimal ?

8. 425.85 acres, $25.38\frac{3}{4}$ acres, $17.009\frac{3}{400}$ acres, 326 acres, 7.32 acres, 182 acres, and .45 of an acre, as a simple decimal ?

9. $45.08\frac{1}{4}$ cords, $690.8\frac{5}{8}$ cords, 23 cords, $78.13\frac{3}{16}$ cords, .65 of a cord, and .8 of a cord, as a simple decimal ?

10. $3.8\frac{1}{200}$ tons, $58\frac{7}{800}$ tons, $.37\frac{3}{4}$ tons, $16\frac{4}{5}$ tons, .0072 of a ton, and $.51\frac{1}{16}$ of a ton, as a simple decimal ?

11. 4.318 yr., $473.1\frac{3}{16}$ yr., $85.48\frac{3}{8}$ yr., 125.19 yr., $76.34\frac{1}{3}\frac{2}{3}$ yr., 50 yr., and $5\frac{6}{7}$ yr., to 3 decimal places (See Ill. Ex. 2) ?

12. $\frac{7}{16}$ bu., $\frac{5}{8}$ bu., $.02\frac{5}{64}$ bu., $.31\frac{1}{32}$ bu., to 4 decimal places ?

13. $$.58\frac{3}{8}$, $$7.39\frac{5}{8}$, $$250.81\frac{5}{6}$, $$12\frac{3}{8}$, to 3 decimal places ?

14. $$582\frac{7}{8}$, $$25\frac{3}{16}$, $$129.4\frac{2}{3}$, $$48.37\frac{5}{8}$, $$72.64\frac{4}{7}$, $$43.25\frac{1}{3}$, $$182.36\frac{5}{11}$, and $$.78\frac{1}{3}$, to 3 decimal places ?

15. 3080.015 tons, $375.009\frac{3}{4}$ tons, 52.01876 tons, $3782.07\frac{5}{16}$ tons, $490.809\frac{3}{8}$ tons, and $34.58\frac{5}{12}$ tons, to 5 decimal places ?

16. Nineteen, and forty-nine ten-thousandths acres; seventy-three, and one hundred fifty-six millionths acres; thirty-four, and eight hundred-thousandths acres; and three thousand nine hundred thirty-seven, and two hundred ninety-five ten-thousandths acres ?

17. Three hundred four, and thirty-two thousandths miles; eighteen, and two thousand seventy-five hundred-thousandths miles; three, and fifteen ten-thousandths miles; and five thousand eighty-two, and *one thousand nineteen* hundred-thousandths miles.

SUBTRACTION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

149. Subtract 575.15$\frac{5}{8}$ gal. from 921.0$\frac{5}{9}$ gal. to 3 decimal places.

Explanation.—Expand the minuend and subtrahend as directed in Explanation, Ill. Ex. 2, **148,** for addends. Arrange the numbers so that the orders of the subtrahend shall fall under like orders of the minuend. As the difference of the tenths' orders must also be tenths, place the decimal point in the remainder at the left of 8 tenths to denote its decimal order.

Solution.

921.0556—
575.1563—
345.8993

Ans., 345.899+ gal.

Principles.—1. *Only similar decimal orders can be subtracted, one from the other; hence the decimal point of the subtrahend should fall directly under that of the minuend.*

2. *The difference expresses decimal units of the same name as the orders subtracted; hence the decimal point of the remainder should fall directly under the decimal points of the minuend and subtrahend.*

Rule.—*Arrange the decimals so that the decimal point of the subtrahend shall fall directly under the decimal point of the minuend. Subtract as in whole numbers; and place the decimal point in the remainder directly beneath those of the minuend and subtrahend.*

EXAMPLES FOR PRACTICE.

Find the difference between

1. 32.7509 oz. and 85.5 oz.
2. 5.103 rods and 7.08091 rods.
3. 18.09 acres and 13.0513 acres.
4. 39.05137 yr. and .18 yr.
5. 135.08346 yd. and 500 yd.
6. 83.712 miles and 25 miles.
7. 358.0015 bu. and 71.21008 bu.
8. 25.01005 ft. and 40.5807 ft.
9. 3751.048 hr. and 413.7182 hr.
10 69.715 tons and 52.3 tons.
11. 41.5167 bu. and 83.7 bu.
12. .91 yr. and .4173 yr.
13. 18.72$\frac{3}{4}$ bbl. and 25$\frac{7}{8}$ bbl., as a simple decimal.
14. 1.005$\frac{3}{8}$ miles and 8.0$\frac{4}{5}$ miles, as a simple decimal.
15. 52.0082$\frac{5}{16}$ acres and 2.728$\frac{3}{32}$ acres, as a simple decimal.
16. 92.48$\frac{1}{16}$ cords and 150.3$\frac{5}{64}$ cords, as a simple decimal.
17. 518.38$\frac{2}{3}$ months and 753.5683 months, to 5 decimal places.
18. *8.071$\frac{4}{7}$ rods* and 3.25 rods, to 6 decimal places.

149

Find the difference between

19. $125\frac{3}{4}$ weeks and $68.513\frac{1}{11}$ weeks, to 4 decimal places.

20. $68.78\frac{1}{13}$ ft. and $319.286\frac{1}{3}$ ft., to 5 decimal places.

21. Thirty-eight and two-tenths years *and* two hundred forty-six and thirty-five thousandths years.

22. From nine hundred and forty-eight years take three hundred and seventy-five ten-thousandths of a year.

23. From eight hundred fifty-four, and one hundred ninety-six hundred-thousandths fathoms take three hundred sixty-two, and one thousand two hundred thirty-nine millionths fathoms.

24. One thousand five hundred eighteen, and one thousand and three hundred-thousandths tons *and* nine thousand two hundred fifty-six, and thirty-five thousandths tons.

25. The sum of two quantities is 352.018 cords, and one of the quantities is 18.00359 cords. What is the other quantity?

MULTIPLICATION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

150. Multiply 57.83 years by .315.

Explanation.—$57.83=\frac{5783}{100}$, and $.315=\frac{315}{1000}$. Multiply the numerator of one fraction (5783) by the numerator of the other fraction (315), obtaining 1695645 as the numerator of the required product (**115**). Since the denominator of $\frac{5783}{100}$ is 1 followed by *two* ciphers, and the denominator of $\frac{315}{1000}$ is 1 followed by *three* ciphers, the product of these denominators (100000) or 1 followed by 2+3, or 5 ciphers, must be the denominator of the required product (**115**); and to denote this denominator, point off 5 decimal places (Rem., **136**).

Solution.

$$\begin{array}{r} 53.83 \\ .315 \\ \hline 26915 \\ 5383 \\ 16149 \\ \hline 16.95645 \text{ yr.} \end{array}$$

Rule.—*Multiply as in whole numbers; and from the right of the product point off as many decimal places as are contained in both factors.*

NOTE 1.—If the product contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

NOTE 2.—Terminate Complex Decimals (Rem., **140**) are usually expanded to simple decimals before applying the rule; and Interminate Complex Decimals are multiplied as they stand by Rule, **117** or **118.**

NOTE 3.—Decimals are multiplied by 10, 100, 1000, etc., by moving the decimal *point of the multiplicand* as many places to the right as there are ciphers *in the multiplier.*

150

EXAMPLES FOR PRACTICE.

Multiply

1. 579.35 ft. by 32.
2. 67.815 yd. by 4.15.
3. 5.0179 acres by .065.
4. 28.0415 rods by 2.58.
5. 71.475 miles by .3146.
6. 829.5 gal. by 62.75.
7. .0078 months by .058.
8. .000532 bu. by .0086.
9. 75.38$\frac{3}{4}$ yd. by .57.
10. .658$\frac{4}{5}$ bbl. by 4.65$\frac{4}{5}$.

Multiply

11. 45.783\frac{2}{25}$ by .62$\frac{4}{35}$.
12. 6.0081\frac{5}{64}$ by .000$\frac{7}{8}$.
13. 14.37\frac{3}{8}$ by .085.
14. 5.26\frac{3}{4}$ by .042$\frac{4}{5}$.
15. 18.35\frac{4}{7}$ by 2.41$\frac{1}{5}$.
16. $.391$\frac{3}{8}$ by .65$\frac{5}{8}$.
17. $18.345 by 100 (Note 3).
18. $62.78 by 10000.
19. $.0451 by 500 (100 × 5).
20. $5.17 by 18000.

DIVISION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

151. Divide $11.842 by 3.82.

Solution.

$$\begin{array}{r} 3.82\ )\ \$11.842\ (\ \$3.1 \\ 11\ \dot{4}6 \\ \hline 382 \\ 382 \\ \hline \end{array}$$

Explanation.—382 *hundredths* are contained in 1184 *hundredths* 3 (whole) times; and if the first quotient figure (3) expresses units, the second quotient figure (1) must express the next lower order to units, or tenths. Hence, place a decimal point in the quotient at the left of 1, to make it denote tenths.

Principles.—1. *The dividend contains as many decimal places as both divisor and quotient.*

This is evident when it is considered that the dividend is a product, and the divisor and quotient are its factors (**40**); and every product contains as many decimal places as both its factors (Rule, **150**).

From Prin. 1 are deduced the following:

2. *The dividend should contain at least as many decimal places as the divisor.*

3. *The number of decimal places in the quotient equals the number of decimal places in the dividend minus the number of decimal places in the divisor.*

Rule.—*Divide as in whole numbers; and from the right of the quotient point off as many figures as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.*

151

NOTE 1.—If the quotient contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

NOTE 2.—To obtain a given number of decimal places in the quotient, use as many decimal places of the dividend as will equal the number of decimal places in the divisor *plus* the number of decimal places required in the quotient, annexing decimal ciphers to the dividend, if necessary.

NOTE 3.—Decimals are divided by 10, 100, 1000, etc., by moving the decimal point of the dividend as many places to the left as there are ciphers in the divisor.

NOTE 4.—Complex decimals, if terminate, are usually expanded to simple decimals before applying the rule; and if interminate, are usually divided by Rule, **125**, **126**, or **127**.

EXAMPLES FOR PRACTICE.

Divide

(*Obtaining simple decimals as quotients*).

1. $44.65 by .005.
2. 31 cords by .0004.
3. 43.2 acres by .0016.
4. .000875 yr. by 1.75 yr.
5. .0009 bu. by .003.
6. 18.72 yd. by .4.
7. 10 miles by .001.
8. .001 rods by 5 rods.
9. 18.57 months by 1.5.
10. 423 fathoms by .18.
11. .00018 yr. by .0075 yr.
12. 5 tons by .0512.

Divide

(*Obtaining simple decimals as quotients*).

13. 5 bbl. by .005 bbl.
14. $72.0372 by $41.64.
15. $.31 by .0005.
16. $90.288 by $4.18.
17. 246.75 gal. by 32.9 gal.
18. 23204.88 yd. by .0324.
19. .0002784 ft. by .032 ft.
20. $25.375 by 5.33\frac{1}{3}$.
21. $198.455 by $.66$\frac{2}{3}$.
22. 2.72$\frac{1}{8}$ months by .000$\frac{1}{16}$.
23. 457.62 rods by .058$\frac{1}{3}$.
24. 13.702$\frac{1}{3}$ miles by .16$\frac{2}{3}$.

(*Carrying the quotients to as many decimal places as are specified in the parenthesis*).

25. 19 fathoms by .06 (3 places).
26. 578.3175 lb. by 53 lb. (4 pl.).
27. 672.51 miles by 34 (3 pl.).
28. 76.28128 yr. by .352 (3 pl.).
29. $35.14 by $7.1835 (2 pl.).
30. 35 months by 17 months (3 pl.).
31. $475.8 by .148 (3 pl.).
32. .00176 yd. by 11 yd. (5 pl.).
33. 1 mile by 756 miles (3 pl.).
34. $783 by $.043 (4 pl.).
35. 58.1753 pt. by 1.9 (2 pl.).
36. *.068134 ft. by 6.75* (3 pl.).
37. 1.482917 yd. by 372.6 (3 pl.).
38. 41$\frac{1}{3}$ cords by .05 (3 pl.).
39. 74.18$\frac{5}{16}$ rods by 12.5 (3 pl.).
40. 5.004$\frac{3}{8}$ fathoms by 38 (4 pl.).
41. .0$\frac{2}{3}$ bbl. by 75 (4 pl.).
42. 15\frac{3}{4}$ by 18\frac{5}{16}$ (2 pl.).
43. $\frac{4}{7}$ yr. by $\frac{1}{3}$ (3 pl.).
44. .00$\frac{1}{3}$ bu. by .04$\frac{1}{6}$ (2 pl.).
45. .018$\frac{4}{7}$ pt. by .1$\frac{2}{3}$ pt. (3 pl.).
46. .5$\frac{1}{4}$ gal. by .00$\frac{2}{25}$ (2 pl.).
47. 3.182\frac{4}{5}$ by 1.71\frac{2}{3}$ (3 pl.).
48. 5.18372$\frac{1}{3}$ yr. by .8$\frac{1}{7}$ (2 pl.).

151

[When the divisor terminates in one or more ciphers, apply Note 8.]

Divide

49. $18.475 by $10.
50. 345.1298 acres by 100.
51. 41523.85 miles by 1000.
52. ¾ bu. by 100.
53. 2135\frac{3}{16}$ by 10 (3 pl.).

Divide

54. $375.82 by $500 (2 pl.).
55. $172.81 by 300 (3 pl.).
56. 1867.8 acres by 7000 (4 pl.).
57. $18.25 by 1600 (5 pl.).
58. 18 yr. by 1100 (3 pl.).

REVIEW OF DECIMALS.

152. Decimal fractions have the same relation to each other as other fractions. Hence, **128, 129,** and **130,** relating to common fractions, are equally applicable to decimals.

1. A farmer owns five tracts of land containing respectively eight hundred seventy-six, and eighteen thousandths; twenty-eight and seven-tenths; four hundred fifty-six, and five hundred thirty-nine ten-thousandths; seventy-two, and thirteen thousandths; and nine thousand three hundred twenty-four, and seven hundred sixteen hundred-thousandths acres. How much land does the farmer own?

2. I lack 15.035 bushels of having 24.08 bushels of wheat. How many bushels of wheat have I?

3. What is the cost of 48.625 cords of wood at $7.06¼ per cord?

4. A farmer's yield of potatoes averaged 98.94 bushels per acre, of oats 34.65 bushels per acre, and of wheat 13.65 bushels per acre. If 110 acres were planted in potatoes, 25.2 acres in oats, and 17.8 acres in wheat, what was the total number of bushels harvested?

5. Multiply the difference between 5 cents and 18 dimes by .0012.

6. If a man buys 750 yards of goods and sells 615 yards, how many hundredths of his purchase will he have left?

7. How many bushels of wheat at $1.20375 per bushel can be exchanged for 1926 bushels of apples at 40 cents per bushel?

8. A dealer in produce bought four wagon loads of wheat, containing respectively 30.015, 31.5, 29.05, and 33.1 bushels. What was the total cost at $1.05625 per bushel?

9. A contractor bargained to remove an embankment at $.425 per cart load, and in settlement received $5378.3325. How many *loads had he removed?*

152

10. If .34 of a barrel of vinegar cost $3.06, what will 5.3 barrels of the same vinegar cost?

11. A man's salary is $650 per annum, of which he pays $200 for board, $125 for clothes, and $97.50 for sundry other expenses. What decimal part of his salary does he save?

12. A grocer buys four bags of coffee which average 132.08 pounds per bag. If three of the bags weigh respectively 128.05, 131.75, and 134.15 pounds, what is the weight of the fourth bag?

13. A and B are 382.0135 miles apart and walk towards each other, A traveling at the rate of 3.75 miles per hour and 8.50 hours per day, and B at the rate of 4.15 miles per hour and 6 hours per day. How far apart will they be at the end of the sixth day?

14. If a man has one hundred eighty-five and seventeen ten-thousandths acres; then buys sixty-eight and seventy-five thousandths acres; then sells ninety-five and forty-seven ten-thousandths acres; then buys forty-two and one hundred thirty-two hundred-thousandths acres; then buys one hundred twelve and thirty-nine thousandths acres; and then sells forty-eight and five hundredths acres; how much land will he have remaining?

15. What is the total cost of 22.75 tons of coal bought at $6.875 per ton, 15.625 tons at $6.375 per ton, 18.25 tons at $6.125 per ton, and 5.0625 tons at $5.1875 per ton?

16. From 17 dollars and 3 cents + 95 dollars and 5 dimes + 32 dollars and 15 cents take 5 dollars and 4 cents + 8 dollars and 35 cents + 16 dollars, 9 dimes and 8 cents.

17. What is the cost of 58 barrels of cider, averaging 38.125 gallons per barrel, at $.18¼ cents per gallon?

18. A, B, and C own a tract of land, A owning .258 of the tract, B .425, and C the remainder. What part of the tract does C own?

19. Divide $1 into two parts so that one part will be $.0625 more than the other.

20. A farmer owned 140 sheep and sold .05 of the flock to his neighbor. How many sheep had he remaining?

21. If .174 of a job of work can be done in 6.09 days, in what time can the whole of it be done?

22. A and B at first had farms of equal size, but after A added 75.003 acres to his farm and B sold 15.043 acres from his, it was found that *together they had* 360.12 acres. How many acres had each at *first?*

23. The sum of five addends is 375.045 years. If four of the addends are 72.05, 31.009, .4158, and 8.3 years, what is the fifth?

24. The product of two factors is 2285.035 yards, and one of the factors is 318¼. What is the other factor?

25. The remainder is 18.2675 miles, and the subtrahend 375.1 miles. What is the minuend?

26. What is the cost of 18.375 cubic yards of stone at $1.60 per yard?

27. A clerk spent .45 of his monthly salary and had $39.60 remaining. What was his salary per month?

28. A merchant withdrew .375 of his capital to invest in a farm which cost him $6890.25. What was his original capital?

29. How many times can a measure holding .0625 of a gallon be filled from .75 of a cask which when full holds 64 gallons?

30. A merchant deposited a certain amount of money in bank, and afterwards withdrew, at different times, .012, .034, .45, and .215 of his original deposit. What part of his original deposit had he still remaining in bank?

31. I bought an invoice of coffee, and disposed of .25 of the purchase at one sale, .014 of the purchase at a second sale, and .75 of what was left of the preceding sales at a third sale. What part of the invoice remained after the third sale?

32. I bought 6000 yards of muslin at 5 cents per yard, sold .018 of the purchase at one sale at 6 cents per yard, .25 of what was left after the first sale at 7 cents per yard, and the remainder at 8 cents per yard. What was my total gain?

33. A retail grocer sold .128 of an invoice of sugar on Monday, .098 on Tuesday, .11 on Wednesday, .106 on Thursday, .2 on Friday, and .25 on Saturday. He then had 175½ pounds remaining. How many pounds of sugar did he sell on each day?

34. I paid $5280 for a house and .03125 as much for a lot. What was the cost of both?

35. A man bequeathed .125 of his property to an orphan asylum, $\frac{5}{16}$ to his son, and the remaining $24120 to his wife. What was the value of his property?

36. A man sold .75 of his farm and had 115 acres remaining. How many acres did he sell?

37. A man sold off 204 acres from his farm and had 221 acres *remaining. What decimal* part of his farm did he sell?

38. After making a payment of .475 of his debt, a man finds that he still owes $210. How much did he originally owe?

39. A grocer bought sugar and coffee amounting to $114.80, the cost of the sugar being .435 of the cost of the coffee. What did he pay for each?

COUNTING-HOUSE PRACTICE.

153. Counting-House Practice, as here treated, embraces the principal contractions employed in business computations.

154. Quantity is the total number of pounds, yards, dozens, etc., involved in a business transaction.

155. Price is the money value of *one* pound, *one* yard, *one* dozen, or *one* measuring unit of any kind of commodity involved in a business transaction.

156. Cost is the money value of the total number of units involved in a business transaction.

Rem.—*Cost* is the product, and its two factors are *Price* and *Quantity* (Deduction, **61**). Hence, *if cost is required, multiply its two given factors; and if cost is given, divide it by its given factor and the quotient will be its other factor.*

157. Calculations involving a given or required cost can often be simplified by changing the *decimal* price to its equivalent *common fraction* of a dollar.

Rem.—An **Aliquot Part** of a number is such a part as will exactly divide that number. Hence, a decimal price is an aliquot part of a dollar if the numerator of its equivalent common fraction is 1, showing that the decimal price is an exact divisor of $1.

Aliquot Parts of One Dollar.

50¢ = $$\frac{1}{2}$.
$33\frac{1}{3}$¢ = $$\frac{1}{3}$.
25¢ = $$\frac{1}{4}$.

20¢ = $$\frac{1}{5}$.
$16\frac{2}{3}$¢ = $$\frac{1}{6}$.
$12\frac{1}{2}$¢ = $$\frac{1}{8}$.

10¢ = $$\frac{1}{10}$.
$8\frac{1}{3}$¢ = $$\frac{1}{12}$.
$6\frac{1}{4}$¢ = $$\frac{1}{16}$.

5¢ = $$\frac{1}{20}$.
4¢ = $$\frac{1}{25}$.
2¢ = $$\frac{1}{50}$.

Aliquot Parts of Aliquot Parts of One Dollar.

25¢ = $\frac{1}{2}$ of 50¢.
$12\frac{1}{2}$¢ = $\frac{1}{4}$ of 50¢.

$6\frac{1}{4}$¢ = $\frac{1}{8}$ of 50¢.
5¢ = $\frac{1}{10}$ of 50¢.

$12\frac{1}{2}$¢ = $\frac{1}{2}$ of 25¢.
$6\frac{1}{4}$¢ = $\frac{1}{4}$ of 25¢.

153—157

158. To find the cost when the price is an aliquot part of $1.

ILLUSTRATIVE EXAMPLE.

What is the cost of 751 bushels of wheat at $1.75 per bushel?

Explanation.—At $1 per bu., the cost is as many dollars as bushels, or $751; at 50¢ per bu., the cost is half as many dollars as bushels, or $375.50; at 25¢ per bu., the cost is half as much as at 50¢ per bu., or $187.75; hence, at $1 + 50¢ + 25¢, or $1.75 per bu., the cost is $751 + $375.50 + $187.75, or $1314.25.

Solution.

$751 = cost at $1.
(½ of $751) $375.50 = " " 50¢.
(½ of $375.50) $187.75 = " " 25¢.
$1314.25 = " " $1.75

Or, $1.75 = $7/4.
$(751 \times 7) \div 4 = \$1314.25$.

Rule.—I. *Divide the quantity by the number of its units that will cost $1.* Or,

II. *Multiply the quantity by the price expressed as a common fraction.*

EXAMPLES FOR PRACTICE.

Find the cost of

1. 8271 lb. coffee, at 33⅓¢.
2. 7512 " sugar, " 12½¢.
3. 918 " honey, " 16⅔¢.
4. 1215 yd. bunting, at 20¢.
5. 324 brushes, " 25¢.
6. 7824 bu. oats, " 62½¢.

Find the cost of

7. 340 bu. rye, at 75¢.
8. 560 lb. tobacco, " $1.25.
9. 318 bu. wheat, " $1.12½.
10. 712 " corn, " $1.18¾.
11. 625 bbl. flour, " $5.66⅔.
12. 2712 " apples, " $2.87½.

159. To find the quantity when the price is an aliquot part of $1.

ILLUSTRATIVE EXAMPLE.

How many yards of dress goods can be bought for $75, at 25 cents per yard? At 66⅔ cents per yard?

First Solution.—At 25¢ per yard, $1 will buy 4 yards, and $75 will buy 75 times 4 yards, or 300 yards.

Second Solution.—Divide the cost ($75) by the price *expressed as a common fraction* (66⅔¢ = $⅔), obtaining *112½ yards.*

First Solution.
$75 \times 4 = 300$ yd.

Second Solution.
$75 \div \frac{2}{3} = 112\frac{1}{2}$ yd.

Rule.—I. *Multiply the cost by the quantity that $1 can buy.*

II. Or, *divide the cost by the price expressed as a common fraction.*

EXAMPLES FOR PRACTICE.

Find the number of yards.			Find the number of gallons.		
	Cost.	*Price per yd.*		*Cost.*	*Price per gal.*
1.	$275	12½¢.	7.	$753	37½¢.
2.	$348	25¢.	8.	$1456	87½¢.
3.	$267	16⅔¢.	9.	$135	62½¢.
4.	$1908	33⅓¢.	10.	$375.50	$1.25.
5.	$514	6¼¢.	11.	$920.80	$1.66⅔.
6.	$628.14	75¢.	12.	$615.30	$1.75.

13. How many pounds of coffee at 16⅔ cents per pound should be given in exchange for 320 pounds of sugar at 10 cents per pound, and 140 gallons of molasses at 75 cents per gallon?

14. I bought 50 yards of cloth at $2.50 per yard, 182 yards at $3.25 per yard, and 153 yards at $1.33⅓ per yard, and in payment agreed to deliver 500 yards of muslin at 12½ cents per yard, 800 yards of jeans at 25 cents per yard, and the remainder in cottonades at 20 cents per yard. How many yards of cottonades should I deliver?

160. To find cost when the price is per 100 or per 1000.

ILLUSTRATIVE EXAMPLE.

What is the cost of 8256 feet of boards at $3.45 per 100 feet?

Explanation.—Multiply the quantity (8256) by the price ($3.45), obtaining $28483.20 as the cost, if the price had been $3.45 *per foot;* but as the price is $3.45 *per* 100 *feet,* this product is 100 times too great. Therefore move the decimal point *two additional places* to the left, which is equivalent to dividing the product by 100, to obtain the true product ($284.83+).

Solution.

```
      8256
      3.45
    ------
     41280
    33024
   24768
 ---------
 $284.8320
```

Rule.—I. *Multiply the price per 100 by the quantity, and point off two more decimal places than are found in both factors.*

II. *Multiply the price per 1000 by the quantity, and point off three more decimal places than are found in both factors.*

EXAMPLES FOR PRACTICE.

Find the cost.

	Quantity.		*Price per* 100.
1.	7183 ft. boards	@	$2.75.
2.	5196 lb. freight	"	$1.35.
3.	875 pine-apples	"	$12.50.
4.	3275 cabbages	"	$3.12.
5.	7450 cocoa-nuts	"	$6.25.
6.	8215 lb. beef	"	$11.25.

Find the cost.

	Quantity.		*Price per* 1000.
7.	6835 ft. scantling	@	$11.25.
8.	340 railroad ties	"	$275.50.
9.	75382 bricks	"	$7.50.
10.	1875 shingles	"	$8.25.
11.	9870 melons	"	$183.00.
12.	3500 cigars	"	$84.00.

161. To find the cost when the price is per ton of 2000 pounds.

ILLUSTRATIVE EXAMPLE.

What is the cost of 7564 pounds of coal at $6.40 per ton?

Solution.

$$\begin{array}{r} 7564 \\ 3.20 \\ \hline 151280 \\ 22692 \\ \hline \$24.20480 \end{array}$$

Explanation.—If $6.40 is the price per ton of 2000 lb., ½ of $6.40, or $3.20 must be the price of ½ of 2000 lb., or 1000 lb. Hence, applying **160**, multiply the quantity (7564 lb.) by the price per 1000 lb. ($3.20) and point off 5 decimal places, obtaining $24.20+ as the required cost.

Rule.—*Multiply the quantity by one-half the price per ton of 2000 pounds, and point off three more decimal places than are contained in both factors.*

NOTE.—If more convenient, the price *per ton* may be multiplied by one-half the number of pounds; or the price per ton by the number of pounds and the product divided by 2.

EXAMPLES FOR PRACTICE.

Find the cost.

	Quantity.			*Price per ton.*
1.	3456 lb.	coal	@	$5.75.
2.	58763 "	iron	"	$65.30.
3.	7134 "	hay	"	$15.75.
4.	5387 "	guano	"	$53.45.
5.	35872 "	quartz	"	$375.00.

Find the cost.

	Quantity.			*Price per ton.*
6.	976 lb.	ice	@	$6.45.
7.	1574 "	coal	"	$7.50.
8.	6358 "	freight	"	$3.25.
9.	7150 "	straw	"	$9.50.
10.	312 "	lime	"	$6.75.

11. What is the cost of five loads of hay weighing respectively 1916, 1591, 2100, 1759, and 1698 pounds, at $17.50 per ton?

161

12. A dealer bought 8375 pounds of coal at $5.25 per ton, and retailed it at 35 cents per 100 pounds. What was his gain?

162. Grain, seeds, and other farm produce are sold by the bushel of a certain number of pounds. The weight of a bushel in pounds as prescribed by law in most States, is as follows:

Barley	48	Corn, in the ear	70	Rye	56
Beans	60	Corn Meal	50	Rye Meal	50
Buckwheat	48	Oats	32	Timothy Seed	45
Clover Seed	60	Peas	60	Wheat	60
Corn, shelled	56	Potatoes	60	Wheat Bran	20

163. To find the cost of a given number of pounds when the price is per bushel.

ILLUSTRATIVE EXAMPLE.

What is the cost of 874 lb. corn at 65 cents per bushel?

Explanation.—874 times 65 cents, or $568.10 would be the cost if the price were 65 cents *per pound;* but as the price is 65 cents per bushel of 56 pounds, the obtained cost is 56 times the true cost. Divide by 56, obtaining $10.14+ as the true cost.

Solution.

```
        874
        .65
       4370
      5244
56 ) 568.10 ( $10.14+
     56
      81
      56
      250
      224
       26
```

Rule.—*Multiply the weight in pounds by the price, and divide the product by the number of pounds in a bushel.*

EXAMPLES FOR PRACTICE.

Find the cost.

Quantity.		*Price per bu.*
1. 3125 lb.	rye	@ 85¢
2. 4116 "	shelled corn	" 62¢
3. 728 "	wheat	" 95¢
4. 275 "	timothy seed	" $2.25
5. 948 "	corn meal	" 75¢

Find the cost.

Quantity.		*Price per bu.*
6. 2350 lb.	beans	@ $1.75.
7. 5739 "	buckwheat	" $1.15
8. 7134 "	potatoes	" 58¢
9. 1725 "	wheat bran	" 24¢
10. 7293 "	corn in ear	" 63¢

11. I bought 5836 lb. corn at 65¢ per bushel, sold 2354 lb. at 70¢ per bushel, and the remainder at 73¢ per bushel. What was my total profit?

BILLS AND ACCOUNTS.

164. An **Account** is a record of one or more sums or items due from business transactions; such as purchases or sales, services rendered, etc.

165. The **Ledger** is the principal book of accounts, in which the several items of each account are systematically arranged under the head of debit or credit, or both.

166. The **Debit** side of an account (marked Dr.) exhibits the amounts of the items due *to* the party keeping the account for goods sold, services rendered, money paid, etc.

167. The **Credit** side of an account (marked Cr.) exhibits the amounts of the items due *from* the party keeping the account for goods bought, services or money received, etc.

REM.—The difference between its debit and credit side is called the *Balance* of the account.

168. A **Bill** or **Invoice** is a detailed statement of merchandise sold or of services rendered.

REM. 1.—A Bill or Invoice should state the place and date of sale; the names of buyer and seller; the terms of the sale; the identifying mark of the case, bale, barrel, etc., in which the merchandise has been shipped; the distinguishing characters or numbers which are marked on the smaller packages of a case of miscellaneous articles so that each may be readily found; the quantity, name, and price of each item, with its aggregate amount; and other details, if any, of the sale.

REM. 2.—When paid, the Bill or Invoice is usually receipted by writing at the bottom, under the words "Received Payment," the signature of the party or firm selling the goods.

REM. 3.—An authorized employé may receipt a Bill by first signing his own name and, in the next line, that of his employer, preceded by the word *for*, as in Bill No. 3; or what is more customary, by writing his employer's name first and, in the next line, his own name, preceded by *per* or *by*, as in Bill No. 2.

169. A **Statement of Account** is a written exhibit of the *sum* of the items charged at different dates of purchase.

REM.—A *Statement* is not itemized as a Bill, but embraces the date and the aggregate of each sale, the itemized Bills of which had been previously rendered, as in No. 7.

170. In making out Bills, etc., many terms, characters, and abbreviations are used, the principal of which will be found in the following list:

COMMERCIAL ABBREVIATIONS.

@	at.
A 1	First quality.
Acct. or a/c	Account.
Agt	Agent.
Amt	Amount.
Apr	April.
Arr	Arrived.
Ass'd.	Assorted.
Aug.	August.
Bal.	Balance.
Bbl	Barrel.
Bdls	Bundles.
Bds	Boards.
Bgs	Bags.
Bkts	Baskets.
Blk	Black.
Bls	Bales.
Bot	Bought.
Br	Brig.
Bro	Brown, or Brother.
Brot	Brought.
Bu	Bushels.
Bxs	Boxes.
C	Hundred.
c. or cts	Cents.
Chgd	Charged.
Chts	Chests.
Cks	Casks.
Co	Company.
C. O. D	Collect on delivery.
Col'd	Colored.
Com	Commission.
Cr	Creditor.
Cs	Cases.
Cwt	Hundredweight.
d	Pence.
Dec	December.
Dft	Draft.
Disct	Discount.
Do., ditto, or "	The same.
Doz	Dozen.
Dr	Debtor.
Dryge	Drayage.
Ds. or da	Days.
Ea	Each.
E. & O. E.	Errors and omissions excepted.
Emb'd	Embroidered.
Exch	Exchange.
Feb	February.
Fig'd	Figured.
Fir	Firkin.
Fol	Folio or page.
Fr't	Freight.
Ft	Foot or feet.
Gal	Gallon.
Gr. or gro	Gross.
Guar	Guarantee.
Hlf	Half.
Hhd	Hogshead.
i. e	That is.
In	Inches.
Ins	Insurance.
Inst	Present month.
Int	Interest.
Jan	January.
Jr. or Jun	Junior.
Lbs	Pounds.
Ledg	Ledger.
M	Thousand.
Mar	March.
Mdse	Merchandise.
Mem	Memorandum.
Messrs	Gentlemen or Sirs.
Mo	Month.
Mr	Master or Mister.
Mrs	Mistress.
N. B	Take particular notice.
No. or #	Number.
Nov	November.
Oct	October.
Oz	Ounces.
P	Page.
Pp	Pages.
Pay't or Paym't	Payment.
Pd	Paid.
Per, p., or ⅌	By, or by the.
Pkgs	Packages.
Plts	Plates.
P. O	Post Office.
Pr	Pair.
Prox	Next month.
Ps	Pieces.
Qr	Quarter.
Rec'd paym't	Received payment.
Rec't	Receipt.
R. R	Railroad.
Rs. or rls	Rolls.
Schr	Schooner.
Sh	Ship.
Shipt	Shipment.
Stbt	Steamboat.
Stor	Storage.
Str	Steamer.
Sunds	Sundries.
Super	Superfine.
Trcs	Tierces.
Ult	Last month.
Wt	Weight.
Yds	Yards.
Yr	Year.

COMMERCIAL CHARACTERS.

@	At.
a/c	Account.
¢	Cents.
c/o	Care of.
√	Check mark.
$	Dollars.
"	Ditto, or the same.
%	Hundredths.
n/c	New account.
#	Number.
o/c	Old account.
⅌	Per, or by the.
£	Pounds sterling.
×	By, as 7 × 8 inches.

REM. 1.—5^1 yards, 8^2 yards, 13^3 yards, respectively mean 5¼, 8½, and 13¾ yards; 12^{34} bushels and 75^{18} bushels respectively mean 12 bushels 34 pounds and 75 bushels 18 pounds. 15^{12} pounds and 18^{11} pounds respectively mean 15 pounds 12 ounces and 18 pounds 11 ounces.

REM. 2.—24 doz., $\frac{8}{\$7,}$ $\frac{14}{\$8.25,}$ $\frac{2}{\$8.75,}$ mean 24 doz.—8 doz. @ $7 per dozen, 14 doz. @ $8.25 per dozen, and 2 doz. @ $8.75 per dozen, the quantity being written above a horizontal line and its price below.

REM. 3.—16 doz., $\frac{5}{1}$ @ 80¢, $\frac{7}{2}$ @ 70¢, $\frac{4}{3}$ @ 62¢, mean 16 dozen—5 doz. of No. 1 @ 80¢ per doz., 7 doz. No. 2 @ 70¢ per doz., and 4 doz. No. 3 @ 62¢ per doz., the quantity being written above a horizontal line, and the class or distinguishing number of the goods below.

REM. 4.—2 bbls. sugar $\frac{247-19}{258-20}$ means 2 barrels sugar, one barrel weighing 247 lbs. gross, from which has been deducted 19 lbs. for the weight of the barrel, the other barrel 258 lbs. gross, minus 20 lbs. for weight of barrel, leaving (247—19)+(258—20) or 466 lbs. as the net weight of the sugar charged.

171. The following embraces several forms of Bills, Invoices, Statements, etc. The pupil should carefully and neatly copy each with pen and ink, make the necessary extensions, and "foot up" the several columns.

Folio 843. ***Bill No. 1.***

BALTIMORE, *Dec. 18, 1886.*

MR. GEO. W. HAMMOND,

CITY.

Terms Cash. *Bought of* COURTNEY & BRO.

4	lbs. Old Govt. Java Coffee,	25¢.	$*	**		
½	" Oolong Tea,	70¢.		**		
15	" Granulated Sugar,	7½¢.	*	**		
10	" A Sugar,	6½¢.		**		
8	" P. R. Rice,	6¼¢.		**		
2	gal. Syrup,	75¢.	*	**		
½	" N. O. Molasses,	70¢.		**		
$\frac{5}{12}$	doz. Cans Tomatoes,	$1.50.		**		
$\frac{7}{12}$	" " Peaches,	$1.80.	*	**		
½	" " Pineapples,	$2.50.	*	**		
¼	" " Strawberries,	$2.25.		**		
3	lbs. Cheese,	25¢.		**		
2½	" Butter,	40¢.	*	**		
7	" Bro. Sugar,	6¢.		**	$**	**

171

Bill No. 2.

PHILADELPHIA, *Sept.* 1, 1886.

A. J. BRADSHAW,
Per Express.

Bought of J. FAGAN & SON.

4	Doz. Hatchets, ...	@ $6	$**	**
3	" "	" 7	**	**
2	" Doorsprings, ...	" 8	*	**
5	" Chains, ...	" $2^{\underline{10}}$	**	**
12	" "	" $2^{\underline{15}}$	**	**
10	" A bits, ...	" $3^{\underline{75}}$	**	**
10	Iron Awls, ...	" 2	**	**
6	Doz. Grindstone Fixtures, . .	" $7^{\underline{20}}$	**	**
			$***	**

Received Payment,
J. FAGAN & SON,
Per W. S. HILL.

Bill No. 3.

BRADFORD, PA., *Sept.* 19, 1886.

GEO. W. BIRD & CO.,

Bought of THE BRADFORD PAPER CO.

No. of Case.	*No. of Reams.*	*Kind of Paper.*	*Weight per Ream.*	*Net Weight Case. lbs.*	*Price.*	*Amount.*	
7466	15	Congress Note, . . .	5	75	18½	$**	**
	10	" " . . .	6	60	18½	**	**
	10	" Letter, . .	10	100	16½	**	**
	5	" " ¼ pk., .	12	60	16¼	**	**
	20	" Cap, . . .	20	400	17½	**	**
7581	25	Capital City Broad Bill, .	12	300	17	**	**
	25	Baltimore Cap, . .	14	350	18¼	**	**
	64	Dixie Note plate fin., .	7	448	19	**	**
7583	10	Congress Legal Cap, .	18	180	19	**	**
	10	" Broad Bill, . .	14	140	17	**	**
7587	30	Druid Mills Cap, . .	14	420	18	**	**
						$***	**

Received Payment,
J. R. JAMES,
for The Bradford Paper Co.

Sept. 29, '86.

Bill No. 4.

S. B. 102, Fol. 36. BALTIMORE, *June* 3, 1886.

GEO. B. WESTERFIELD,

Bought of CANBY, GILPIN & CO.

Terms: 30 days. Shipped by Western Md. R. R.

1	Keg Crystal Chlorate of Potash,	112 lbs.	16¼	$**	**
1	bbl. Powd. Alum, . .	358 "	5½	**	**
3	" Ground " . .	1200 "	2	**	**
10	" Glauber Salts, . .	3260 " 170	¼	**	**
1	Bale Gum Myrrh, .	6 = 164 "	25	**	**
100	oz. Quinine, ...		1.40	***	**
200	lb. Carb. Magnesia, ...		30	**	**
1	Gross Rheumatic Cure, . . .		72.00	**	**
1	" Oatmeal Soap, ...		14.00	**	**
1	" Silver "		9.00	*	**
2	" Bath "		11.00	**	**
4	" Glycerine "		21.00	**	**
1	bbl. Alcohol,	46 gal.	2.18	***	**
				$***	**
	Insurance, . , . .			3	30
				$***	**

Bill No. 5.

NEW YORK, *July* 20, 1886.

MESSRS. HOOVER & CO.,

Folio 923, S. B. 127. *Bought of* A. T. STEWART & CO.

Case.			*No. of Yards.*	*Price.*	*Amount.*	
#27	24	Pcs. Merrimac Prints. 22^1 28^2 26^3 32^1 36^2 38^1 42 29^1 27 29^1 37^3 44^1 28^2 41^1 37^1 42 43^1 27^2 37^1 39^2 41^1 32 26 37^2	***			
#28	23	Pcs. Standard Prints. 36^1 38^2 37 39^2 41^3 46 29^1 42^3 46^2 43^3 44^1 41 39^1 36^2 28^1 33^2 29^1 28^2 27^1 35^2 36 37^1 35	***			
			****	5¢	$**	**

Bill No. 6.

BALTIMORE, *July* 3, 1886.

MR. T. W. JOHNSON,
City,

Folio 313. *Bought of* ELLIOTT & CARR.

Terms: No claims allowed unless made within ten days from date of invoice.

12	M 9960—5 Envelopes, ...	1.60	$**	**		
6	" 6 "	1.75	**	**		
3	" 7684 3 "	1.85	*	**		
3	" 5½ "	2.10	*	**		
3	Rms. Treasury 14 Legal, . .	2.80	*	**		
½	Doz. 066 Frames, ...	9.00	*	**		
¼	" 688 "	10.80	*	**		
½	" 501 "	6.00	*	**		
1	" 031 "		3	50		
¼	" 449 " small, . .	6.75	*	**		
¼	" 449 " large, . .	8.90	*	**		
3	" No. 3 Pads, ...	85	*	**		
1	" " 10 "			40		
1½	Gro. Penholders, ...	3.50	*	**		
1	Doz. Photos 124 (12.50) 219 (18.00) 160 (27.00) .		**	**		
¼	" " 462 (1.00) 77 (2.00) 459 (1.20) 428 (2.25)		*	**		
½	" Arnold's Ink, quarts, . .	5.50	*	**		
1½	" Spencerian Ink, quarts, .	6.25	*	**		
¼	" " " pints, . .	3.75		**		
¾	" " " hlf-pints, .	2.10	*	**		
					$***	**

No. 7. Monthly Statement.

BALTIMORE, *July* 1, 1886.

MR. W. E. SADLER,
Carroll, Md.

Folio 919. *Debtor to* ALLEN S. WILL.

1884.						
June	10	To Invoice rendered, ...	$15	62		
"	15	" " "	23	85		
"	17	" " "	17	92		
"	25	" " "	2	91		
"	27	" " "	17	22	$**	**

8. W. H. Smith bought of Geo. S. Tyler & Co., July 3, 5 doz. W. I. bolts, 6 in., @ $\$2^{50}$; $3\frac{1}{6}$ doz. 10 in. drawing-knives @ $\$9^{75}$; $2\frac{1}{4}$ doz. socket chisels, $1\frac{1}{2}$ in., @ $\$7^{50}$; $2\frac{3}{4}$ doz. 10 in. screw-drivers @ $\$6^{00}$; $2\frac{7}{12}$ doz. Ames cast-steel shovels @ $\$13^{50}$; $3\frac{3}{4}$ doz. nail gimlets @ 50¢; $2\frac{1}{12}$ doz. spike gimlets @ 75¢; $1\frac{3}{4}$ doz. claw-hammers @ $\$5^{75}$; $\frac{3}{4}$ doz. smoothing-planes @ $\$9^{00}$; $\frac{3}{4}$ doz. jack-planes @ $\$10^{80}$; $1\frac{1}{2}$ doz. garden rakes @ $\$6^{00}$; and $\frac{3}{4}$ doz. lawn rakes @ $\$4^{50}$. What is the amount of bill (Form of Bill No. 2)?

9. Geo. H. Lewis bought of the Mt. Holly Paper Co., July 15, 1884, case # 8322: 22 reams Congress note, 5 lbs., @ $16\frac{3}{4}$¢; 24 reams same, 6 lbs., @ $16\frac{3}{4}$¢; 18 reams Congress letter, 10 lbs., @ 17¢; 8 reams same, $\frac{1}{2}$ packets, 12 lbs., @ $17\frac{1}{2}$¢. Case # 8391: 28 reams Druid Mills cap, 12 lbs., @ 19¢; 32 same, 14 lbs., @ 19¢; 16 reams Druid Mills broad bill, 12 lbs., @ $18\frac{3}{4}$¢; 12 reams same, 10 lbs., @ $18\frac{3}{4}$¢; $16\frac{3}{4}$ reams Congress legal, 12 lbs., @ $19\frac{1}{2}$¢; 8 reams same, 14 lbs., @ $19\frac{1}{2}$¢. What is the amount of bill (Form 3)?

10. T. W. Jenkins bought of D. Leonard & Co., Aug. 9, $\frac{1}{3}$ doz. O. K. shirts @ $33; $\frac{7}{12}$ doz. linen hdkfs. @ $\$9^{50}$; $1\frac{5}{12}$ doz. O. K. collars @ $\$2^{75}$; $\frac{1}{6}$ doz. neckties @ $\$6^{75}$; $\frac{1}{2}$ doz. undershirts, # 42, @ $\$17^{75}$; $\frac{1}{2}$ doz. cotton hose @ $\$2^{10}$; $\frac{3}{4}$ doz. mixed do. @ $\$4^{50}$; $\frac{1}{4}$ doz. silk do. @ $\$13^{50}$. What is the amount of bill (Form 4)?

11. Frank Church bought of W. H. Patrick & Co., Sept. 9, 5 lbs. O.G. Java coffee @ 32¢; 3 lbs. Mocha do. @ 28¢; 25 lbs. granulated sugar @ $8\frac{1}{4}$¢; 20 lbs. coffee A do. @ $7\frac{1}{2}$¢; 5 lbs. crushed do. @ 9¢; 1 gal. N. O. molasses @ 65¢; 3 lbs. Heno tea @ 50¢; $4\frac{1}{2}$ lbs. butter @ 45¢; $1\frac{1}{2}$ oz. nutmegs @ 6¢; 7 lbs. Oswego starch @ $17\frac{1}{2}$¢; 8 bars soap @ $7\frac{1}{2}$¢; $2\frac{1}{4}$ lbs. cheese @ 22¢; $\frac{3}{4}$ doz. canned corn @ $\$2^{25}$; $\frac{3}{4}$ doz. canned tomatoes @ $\$1^{75}$; $\frac{1}{2}$ gal. maple syrup @ $\$4^{50}$; 2 lbs. sperm candles @ 35¢; 3 bottles yeast powder @ 13¢. What is the amount of bill (Form 6)?

12. M. S. Chapman bought of H. B. Claflin & Co., Sept. 1, case # 143,—pieces American fancy prints, 28^2 26^1 33^3 36^1 38^2 41 42^3 44^1 45 38 37^3 36^2 39^1 28^1 34^2 27^1 29^3 36^2 35 37 39^2 40 41 43^2 44^1 36^1 29^2 37^2 43^1 28^2 29^1 34^2 37^1 38 40 42^1 43^2 37^1 28^1 @ $5\frac{1}{4}$¢;

Case # 149,—pieces Manchester delaines, 36^1 32 34^1 37^2 38^1 37 42^1 43^2 44^1 47^3 37^2 31^1 41^2 28^1 33^2 29^1 26^3 38^1 37 35^1 41^2 46^3 29^1 39^2 45^1 @ $17\frac{3}{4}$¢;

Case # 157,—pieces Amoskeag sheetings, 28^1 29^2 31^3 27^2 28^1 33^1 37 36^2 41^3 29^2 28^1 27^3 26^2 33^1 @ $11\frac{7}{8}$¢. What is the amount of bill (Form 5)?

13. Jos. S. Sinclair bought of Jno. A. Harrison & Co., the following goods: Mar. 3, 1885, 5 doz. plates: 2 doz. 5 in. @ 85¢, 2 doz. 6 in. @ 95¢, 1 doz. 7 in. @ $\$1^{\underline{12}}$; 5 sets tea ware @ $\$3^{\underline{15}}$; and 3 doz. pitchers: 1 doz. 4 qt. @ $\$3^{\underline{50}}$; 2 doz. 6 qt. @ $\$4^{\underline{50}}$. Mar. 18, 1885, 4 doz. tureens: 1 doz. # 1 @ $\$3^{\underline{25}}$, 1 doz. # 2 @ $\$3^{\underline{15}}$, 2 doz. # 3 @ $\$4^{\underline{25}}$; and 5 doz. bowls @ $\$2^{\underline{30}}$. Mar. 23, 1885, 7 doz. dishes @ $\$4^{\underline{20}}$; 8 sets dinner ware @ $\$6^{\underline{25}}$; and 5 bowls and covers @ 95¢. Mar. 27, 1885, 5 doz. syrup cans @ $\$3^{\underline{50}}$. Render separate bill for each sale, and monthly statement for March (Form 7).

DENOMINATE NUMBERS.

172. A **Denominate Number** is a concrete number whose units are used to measure, to weigh, or to value.

5 yards (units of measure), 3 pounds (units of weight), and $16 (units of value) are denominate numbers.

173. A **Simple Denominate Number** is one which expresses its units in a single denomination.

2 feet, 7 gallons, 8 hours, are each simple denominate numbers.

174. A **Compound Denominate Number,** usually called a *Compound Number,* is one which expresses its units in two or more denominations.

(5 feet, 8 inches), (8 gallons, 3 quarts, 1 pint), and (7 dollars, 58 cents) are compound numbers.

175. **Similar Denominate Numbers** are such simple or compound denominate numbers as express units of the *same class* of measures, weights, or values.

The simple denominate numbers 3 pecks and 19 bushels are called *similar,* because both denominations belong to the *same class* of measures (dry measure); and the compound numbers (5 miles, 18 rods, 1 yard) and (4 yards, 2 feet, 7 inches) are also similar because the denominations of both are of the same class of measures (linear measure).

176. **Similar Denominations,** whether of simple or compound numbers, are those of their terms which are of the *same name,* as well as of the same class.

While 3 pecks and 8 bushels are similar denominate numbers, they do not express similar denominations; but 3 pecks and 8 pecks are similar denominations, for they are of the same name (pecks) and of the same class (dry measure).

MEASURES OF VALUE.

177. The **Value** of a thing, in a commercial sense, is its worth, or what it will produce when sold. *Money* is the measure of value.

Rem. 1.—*Specie* consists of gold, platinum, silver, nickel, copper, or bronze coins, which are stamped by the government which issues them, to authorize their circulation and to denote the number of monetary units they represent.

Rem. 2.—*Alloy* is a baser metal, which is compounded with gold and silver in making coin, to render such coins harder and more durable. In estimating the value of coins, the alloy is not considered.

Rem. 3.—*Bullion* is uncoined gold or silver in bars, or in the mass. The term bullion is sometimes applied to coined gold and silver which has been clipped, punched, or otherwise mutilated, or when abraded to such an extent that it cannot longer remain in circulation. The value of such coins is then estimated by weight.

UNITED STATES MONEY.

178. United States Money is the legal currency of the United States. The standard unit is the gold dollar which weighs 25.8 grains.

TABLE.

10 mills (m.) = 1 cent (¢.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

Rem. 1.—The *mill* is not a coin. It is used simply as a matter of convenience to denote tenths of a cent. In expressing final results, mills are usually omitted if less than 5, and regarded as an additional cent if 5 or more.

Rem. 2.—All gold coins of the United States are a legal tender at their nominal value for any amount, unless mutilated or abraded below $\frac{199}{200}$ of their original weight. Currency dollars (silver) are also a full legal tender for any amount; but silver coins of lower denominations than one dollar are only a legal tender for sums not exceeding $10. Nickel and bronze coins are a legal tender for sums not exceeding 25¢.

Rem. 3.—United States money is expressed and read decimally. Eagles and dimes are not specifically named in reading, all at the left of the decimal point being read as dollars; the two orders following the decimal point, as cents; and the lower orders, if read at all, as mills and fractions of a mill.

ENGLISH OR STERLING MONEY.

179. Sterling Money is the legal currency of Great Britain. The standard unit is the pound or sovereign, the value of which, in U. S. money, is $4.8665.

TABLE.

4 farthings (far.) = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound (£)

Rem. 1.—Other denominations of English money are the *florin* (=2 shillings), the crown (=5 shillings), and the guinea (=21 shillings).

Rem. 2.—The pound is not a coin, but simply a denomination of currency represented in the coinage by the sovereign, and in the paper currency by the £1 note of the Bank of England.

Rem. 3.—In expressing sterling money, shillings are usually written at the left of an incline plane, pence at the right, and farthings as fractions of a penny. Thus, 6 shillings is written 6/-; 7 shillings and 6 pence, 7/6; 8 pence, -/8; and 3 shillings, 5 pence, 1 farthing, 3/5¼.

CANADA MONEY.

180. Canada Money is the legal currency of the Dominion of Canada. The standard unit is the dollar.

Rem. 1.—The denominations of Canada money are the mill, the cent, and the dollar, all of which have the same nominal value as the corresponding denominations of U. S. money; but their intrinsic value is a little less, the fifty-cent piece of Canada being worth about 46¼ cents in U. S. coin, and the smaller silver coins in the same proportion. The gold coins of Canada are the English sovereign and half-sovereign.

Rem. 2.—In Canada, accounts are now usually kept by the decimal notation in dollars and cents, the sovereign being estimated at $5.

GERMAN MONEY.

181. German Money is the legal currency of the Empire of Germany. The standard unit is the mark, worth $.2385 in U. S. money.

Rem.—The denominations of German money are the mark (*reichsmark*) and the penny (*pfennig*), 100 pennies equaling 1 mark. German money is expressed decimally, 20 marks and 9 pennies being written 20.09 marks.

FRENCH MONEY.

182. French Money is the legal currency of France. The standard unit is the franc, worth $.193 in U. S. money.

Rem.—French money is expressed in francs and decimals of a franc. 18 francs, 5 decimes, and 3 centimes are written 18.53 francs, and read 18 francs and 53 centimes.

TABLE.

10 millimes (m.) = 1 centime (ct.)
10 centimes = 1 decime (dc.)
10 decimes = 1 franc (fr.)

183. The following Table will show the units of currency of the principal nations of the world and their intrinsic value in U. S. money, as proclaimed by the Secretary of the Treasury on Jan. 1, 1886.

TABLE.

Country.	*Unit of Currency.*	*Standard.*	*Value* in U. S. money.
Argentine Republic	Peso of 100 centavos. . . .	Gold and Silver.	$.965
Austria	Florin of 100 kreutzers . . .	Silver	.371
Belgium	Franc of 100 centimes . . .	Gold and Silver.	.193
Bolivia	Boliviano of 100 centavos . .	Silver	.751
Brazil	Milreis of 1000 reis	Gold	.546
British America .	Dollar of 100 cents	Gold	1.00
Chili	Peso of 100 centavos. . . .	Gold and Silver.	.912
Cuba	Peso of 100 centavos. . . .	Gold and Silver.	.932
Denmark	Crown of 100 öre	Gold	.268
Ecuador	Peso of 100 centavos. . . .	Silver	.751
Egypt	Piaster of 40 paras	Gold	.049
France	Franc of 100 centimes . . .	Gold and Silver.	.193
Great Britain . .	Pound sterling of 20 shillings	Gold	4.8665
Greece	Drachma of 100 lepta . . .	Gold and Silver.	.193
German Empire .	Mark of 100 pfennige . . .	Gold	.238
Hayti	Gourde	Gold and Silver.	.965
India	Rupee of 16 annas	Silver	.357
Italy	Lira of 100 centesimi . . .	Gold and Silver.	.193
Japan	Yen of 100 sen.	Silver	.81
Liberia	Dollar of 100 cents	Gold	1.00
Mexico	Dollar of 100 centavos . . .	Silver	.816
Netherlands . . .	Florin or Guilder of 100 cents	Gold and Silver.	.402
Norway	Crown of 100 öre	Gold	.268
Peru	Sol of 100 centavos	Silver	.751
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks . . .	Silver	.601
Sandwich Islands .	Dollar of 100 cents	Gold	1.00
Spain	Peseta of 100 centimes . . .	Gold and Silver.	.193
Sweden	Crown of 100 öre	Gold	.268
Switzerland . . .	Franc of 100 centimes . . .	Gold and Silver.	.193
Tripoli	Mahbub of 20 piasters . . .	Silver	.677
Turkey	Piaster of 40 paras	Gold	.044
U. S. of Colombia .	Peso of 100 centavos. . . .	Silver	.751
Venezuela . . .	Bolivar	Gold and Silver.	.193

MEASURES OF WEIGHT.

184. Weight is the measure of the force of gravity.

REM. 1.—All terrestrial bodies are drawn towards the centre of the earth by an invisible force called the *attraction of gravitation.* By weight is meant the measure or extent of this force. Thus, if a quantity of sugar, placed in one dish of a balance, be drawn towards the centre of the earth with a force which will cause it to raise and balance a pound weight placed in the opposite dish, such quantity of sugar is said to weigh one pound.

REM. 2.—The standard unit of weight is the *Troy Pound,* and is the weight of 22.794377 cubic inches of distilled water at the temperature of 39.83° Fahrenheit, barometer at 30 inches.

REM. 3.—The kinds of weight in common use are distinguished as *Troy Weight, Diamond Weight, Apothecaries' Weight,* and *Avoirdupois Weight.*

TROY WEIGHT.

185. Troy Weight is used in weighing gold, silver, jewels, and also in philosophical experiments.

TABLE.

24 grains (gr.)	=	1 pennyweight	(pwt.)
20 pennyweights	=	1 ounce	(oz.)
12 ounce	=	1 pound	(lb.)

DIAMOND WEIGHT.

186. Diamond Weight is used in weighing diamonds and other precious stones. The standard unit is the carat, which equals 3.168 Troy grains.

TABLE.

16 parts = 1 carat grain.
4 carat grains = 1 carat.

REM.—The carat of diamond weight should not be confounded with the assay carat. The latter is not a denomination of weight, but simply denotes the *relative purity* of the gold in coins and jewelry. Thus, pure gold is said to be 24 carats fine. If $\frac{22}{24}$ of a coin is gold and the remaining $\frac{2}{24}$ is of any baser metal (termed *alloy*), such coin is said to be 22 carats fine; and if only $\frac{18}{24}$ of it is pure gold and the remaining $\frac{6}{24}$ alloy, it is said to be 18 carats fine.

APOTHECARIES' WEIGHT.

87. Apothecaries' Weight is used by physicians and apothecaries in compounding medicines which are not liquid.

Rem. 1.—Physicians, in prescribing, express numbers by the Roman notation, employing *small letters* instead of the usual capitals, substituting j for the final i of the Roman numeral, and writing the symbol of the denomination *before* the numeral expressing its quantity. Thus, 6 ounces, 7 drams, 2 scruples, are written ℥ vj, ʒ vij, ℈ij.

TABLE.

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (℔)

Rem. 2.—The pound, ounce, and grain of apothecaries' weight are the same as the corresponding denominations of Troy weight, the only difference being in the subdivisions of their ounce weights.

Rem. 3.—Dry medicines are weighed by avoirdupois weight except when compounding physicians' prescriptions, formulæ, recipes, etc.

AVOIRDUPOIS WEIGHT.

188. Avoirdupois Weight is the customary weight of commerce, and is used in all cases except when compounding medicines, and weighing precious metals and precious stones.

Rem. 1.—At the United States custom houses, in ocean freights, and in wholesale transactions in coal, the *long ton* of 2240 pounds is used.

TABLE.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

Rem. 2.—The following denominations are in common use: 100 pounds of fish =1 quintal; 100 pounds of produce=1 cental; 100 pounds of nails=1 keg; 200 pounds of beef, pork, or fish=1 barrel; 196 pounds of flour=1 barrel. For the weight per bushel of grain, seeds, etc., see **162.**

Rem. 3.—While the corresponding denominations of Troy weight and Apothecaries' weight are the same (Rem. 2, **187**), both differ from the corresponding denominations of Avoirdupois weight as shown in the following

COMPARISON OF WEIGHTS.

7000 Troy grains = 1 lb., Avoir.
5760 " " = 1 " Troy

437½ Troy grains = 1 oz., Avoir.
480 " " = 1 " Troy

MEASURES OF EXTENSION.

189. Measures of Extension are used to measure lines, surfaces, or solids. The standard unit is the yard.

Rem.—Every body has three dimensions, *length*, *breadth*, and *thickness*. Extension has reference to one or more of these dimensions.

LINEAR MEASURE.

190. Linear or **Long Measure** is used in measuring lengths, breadths, depths, heights, or distances.

Rem. 1.—The following denominations are in common use: 1 size = ⅓ of an inch (used by shoemakers); 1 palm = 3 inches; 1 hand = 4 inches (used in measuring the height of horses); 1 span = 9 inches; 1 pace = 3 feet; 1 fathom = 6 feet.

TABLE.

12	inches (in.)	= 1 foot	(ft.)
3	feet	= 1 yard	(yd.)
5½	yards	= 1 rod	(rd.)
320	rods	= 1 mile	(mi.)

SURVEYORS' LINEAR MEASURE.

191. Surveyors' Linear Measure is used by surveyors in measuring boundaries of land, length of roads, etc. The standard unit is the *Gunter's chain*, which is 4 rods or 66 feet long.

TABLE.

7.92	inches	= 1 link	(l.)
100	links	= 1 chain	(ch.)
80	chains	= 1 mile	(mi.)

MEASURES OF SURFACE.

192. Measures of Surface are used in measuring all surfaces by determining the number of square units* which they contain.

Rem.—A surface is the *exterior* of a solid or body.

SURFACE OR SQUARE MEASURE.

193. Surface or **Square Measure** is used in measuring surfaces, such as land, boards, plastering, painting, paving, etc.

TABLE.

144	square inches (sq. in.)	= 1 square foot	(sq. ft.)
9	square feet	= 1 square yard	(sq. yd.)
30¼	square yards	= 1 square rod	(sq. rd.)
160	square rods	= 1 acre	(A.)
640	acres	= 1 square mile	(sq. mi.)

Rem. 1.—The unit of measure in estimating land is the *acre;* and in estimating other surfaces, the *square yard.*

---

* For a definition of "square unit," and for other information relating to surface measurements, see 261, 262, 263, and 264.

Rem. 2.—A square rod is sometimes called a *Pole* or *Perch;* and 40 square rods, a *Rood.*

SURVEYORS' SQUARE MEASURE.

194. Surveyors' Square Measure is used by surveyors in measuring land.

TABLE.

10000 square links (sq. l.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Rem. 1.—Measurements are usually recorded by surveyors in square miles, acres, and decimals of an acre—square chains being written as *tenths* of an acre, and square links as *hundred-thousandths* of an acre.

Rem. 2.—In the West, one square mile of land is called a *section;* and 36 sections, arranged in the form of a square, are called a *township.* Each section is subdivided into *half-sections* (320 acres), *quarter-sections* (160 acres), *half-quarter-sections* (80 acres), and *quarter-quarter-sections* (40 acres).

MEASURES OF VOLUME.

195. Measures of Volume are used in measuring the contents of hollow bodies or the volume (bulk, size) of solid bodies, by determining the number of cubic units * which they contain.

TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Also, in measuring wood,

16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet, or 128 cu. ft. = 1 cord (cd.)

Rem. 1.—A cubic yard of earth is called a *load.*

Rem. 2.—In measuring cargoes, in the United States a ton is estimated to each 40 cubic feet of occupied space; and in England to each 42 cubic feet.

Rem. 3.—In computing the tonnage of vessels, a *registered ton* is 100 cubic feet of their internal capacity.

Rem. 4.—A *perch* of masonry is one perch or rod (16½ feet) long, 1 foot high, and 1½ feet thick, and is therefore equal to 16½ × 1 × 1½ or 24¾ cubic feet, or practically 25 cubic feet.

---

* For a definition of "cubic unit," and for other information relating to the measurement of volumes, see **268, 269, 270,** and **271.**

Rem. 5.—In measuring timber, ⅕ of the solidity of round timber is allowed for waste in hewing or sawing it. Thus, a round log whose solidity is 50 cubic feet will, when hewed or sawed square, contain only 40 cubic feet. Hence, round timber which will make 40 cubic feet when squared, will weigh as much as 50 cubic feet of timber already squared, and each is estimated at *one ton* or *load*.

Rem. 6.—The standard cord-wood pile is 8 feet long, 4 feet high, and 4 feet wide; and one foot of the length of the standard cord-wood pile is called a cord foot.

MEASURES OF CAPACITY.

196. Measures of Capacity are used to measure quantities by determining how often they can fill a measuring vessel of known capacity. The standard unit is the gallon for liquids, and the bushel for dry substances.

Rem.—There are three kinds of measures of capacity: one for measuring dry substances, and called *Dry Measure;* and two for measuring liquids, one called *Liquid Measure*, used by the general public, and the other called *Apothecaries' Fluid Measure*, used by physicians and apothecaries.

LIQUID MEASURE.

197. Liquid Measure is used to measure the various kinds of liquids, or in estimating the capacity of vessels intended to hold them.

Rem. 1.—The standard liquid gallon of the United States contains 231 cubic inches. The imperial gallon of Great Britain is one-fifth larger, and contains 277.274 cubic inches.

TABLE.

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Rem. 2.—In estimating the capacity of cisterns, wells, tanks, etc., 31½ gallons are sometimes called a *barrel;* and 63 gallons, a *hogshead*.

Rem. 3.—The following denominations are sometimes used: 42 gallons = 1 tierce; 84 gallons = 1 puncheon; 126 gallons = 1 pipe or butt; 252 gallons = 1 tun. These denominations, in reality, have no uniform capacity. When used in business, they simply indicate the *names* of the various kinds of casks or their approximative size, without reference to their actual capacity. In practice, each cask is gauged and its exact capacity in gallons is marked upon it.

APOTHECARIES' FLUID MEASURE.

198. Apothecaries' Fluid Measure is used by physicians in prescribing and by apothecaries in selling or compounding liquid medicines.

TABLE.

60 minims (♏) = 1 fluiddrachm (f ʒ).
8 fluiddrachms = 1 fluidounce (f ℥).
16 fluidounces = 1 pint (O).
8 pints = 1 gallon (Cong.).

Rem. 1.—The peculiar notation of Apothecaries' Weight (Rem. 1, **187**) is also extended to Apothecaries' Fluid Measures.

Rem. 2.—The gallon and pint of this measure are of the same capacity as those of Liquid Measure.

DRY MEASURE.

199. Dry Measure is used in measuring grain, fruit, vegetables, and small quantities of coal.

TABLE.

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Rem. 1.—In the U. S., the standard bushel is a cylindrical measure 18½ inches in diameter, and 8 inches in depth. It contains 2150.42 cubic inches.

Rem. 2.—The half-peck of this measure is sometimes called a *gallon*, and contains 268.8 cubic inches.

Rem. 3.—The gallon, the quart, and the pint of Dry Measure have about ⅙ greater capacity than the denominations of the same name in Liquid Measure.

Rem. 4.—The standard imperial bushel of Great Britain contains 2218.192 cubic inches, and is equal to 1.03+ standard bushels of the United States. Eight English bushels are sometimes called a *quarter*.

CIRCULAR MEASURE.

200. Circular Measure is used by surveyors in determining the direction in which to take their linear measurements; by navigators in finding the latitude and longitude of vessels or places; and by astronomers in making observations.

TABLE.

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle (c.)

Rem. 1.—Since a degree is $\frac{1}{360}$ of a circle, it is evident that all the degrees of the same circle are of equal length, and that the degrees of different circles must vary in length with the size of such circles.

Rem. 2.—A *minute* of the circumference of the earth is called by sailors a knot or nautical mile, and by geographers a geographic mile. It is equal to a small fraction more than 1.15 common miles. Hence, 60 geographic miles or 69.16 common miles equal 1 degree of latitude on a meridian, or of longitude on the equator.

MEASURE OF TIME.

201. Time is a measurable part of duration.

TABLE.

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
365 days = 1 common year (yr.)
366 days = 1 leap year (yr.)

Also,

7 days = 1 week.
12 months = 1 year.
100 years = 1 century.

Rem. 1.—The *Civil Day* begins and ends at 12 o'clock, midnight.

Rem. 2.—A *Solar Day* is the time required for any point on the earth's surface which has the sun in its meridian, to be carried round by the diurnal revolution of the earth until such point again has the sun in its meridian.

Rem. 3.—A *Solar Year* is the exact time required by the earth to make a complete revolution around the sun. It consists of 365 days, 5 hours, 48 minutes, and 49.7 seconds.

Rem. 4.—A *Common Year*, for convenience, is made to consist of 365 days; hence 4 common years are nearly one day less than 4 solar years, and 1 day is added to the fourth common year to rectify the error. The extra day is added to the month of February, and the year so increased is called a *leap year*.

Rem. 5.—*Years divisible by 4, except centennial years, are leap years. Centennial years divisible by 400 are leap years.*

Rem. 6.—The order and name of each month and its length in days are:

1st,	January	31 days.	7th,	July	31 days.
2d,	February	28 or 29 "	8th,	August	31 "
3d,	March	31 "	9th,	September	30 "
4th,	April	30 "	10th,	October	31 "
5th,	May	31 "	11th,	November	30 "
6th,	June	30 "	12th,	December	31 "

MISCELLANEOUS MEASURES.

202. In counting certain classes of articles, the following is used:

TABLE.

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (g. gro.)
20 things = 1 score.

203. Paper is measured by the following

TABLE.

24 sheets = 1 quire (qu.)
20 quires = 1 ream (rm.)
2 reams = 1 bundle (bdl.)
5 bundles = 1 bale (bl.)

204. To indicate the comparative size of the pages of books, if the leaves of a book are formed by folding a sheet of paper of the standard size into

2	leaves,	or	4	pages,	the	book	is	called	a	*Folio.*
4	"	"	8	"	"	"	"	"	a	*Quarto,* or *4to.*
8	"	"	16	"	"	"	"	"	an	*Octavo,* or 8*vo.*
12	"	"	24	"	"	"	"	"	a	*Duodecimo,* or 12*mo.*
16	"	"	32	"	"	"	"	"	a	16*mo.*
18	"	"	36	"	"	"	"	"	an	18*mo.*
24	"	"	48	"	"	"	"	"	a	24*mo.*

REDUCTION OF DENOMINATE NUMBERS.

205. Reduction of Denominate Numbers is the process of changing them from one denomination to another without altering their value.

There are two kinds of reduction, which are distinguished as *Reduction Descending* and *Reduction Ascending.*

206. Reduction Descending is the process of changing a denominate number to an equivalent one of a *lower denomination.*

Thus, the process of changing 8 bushels to an equivalent denominate number expressing pecks, a lower denomination than bushels, is called reduction descending.

207. Reduction Ascending is the process of changing a denominate number to an equivalent one of a *higher denomination.*

Thus, the process of changing 72 inches to an equivalent denominate number expressing feet, a higher denomination than inches, is called *reduction ascending.*

208. Reduction of compound numbers to a required lower denomination.

ILLUSTRATIVE EXAMPLE.

Reduce 19 bu., 2 pk., 5 qt. to quarts.

Explanation.—First reduce the highest denomination (19 bu.) to the next lower denomination (pk.) as follows: Since 1 bu. = 4 pk., 19 bu. must equal 19 times 4 pk., or 76 pk., and 19 bu. 2 pk. must equal 76 + 2, or 78 pk.

Then reduce the obtained result (78 pk.) to the next lower denomination (qt.), as follows: Since 1 pk. = 8 qt., 78 pk. must equal 78 times 8 qt., or 624 qt., and 78 pk. 5 qt. must equal 624 + 5, or 629 qt.

Solution.

$$\begin{array}{rcc} \text{bu.} & \text{pk.} & \text{qt.} \\ 19 & 2 & 5 \\ \underline{4} & & \\ 78 & \multicolumn{2}{c}{(19 \times 4 + 2)} \\ \underline{8} & & \\ 629 & \multicolumn{2}{c}{(78 \times 8 + 5)} \end{array}$$

Principles.—1. *Compound numbers are reduced to lower denominations by successive multiplications, in which the highest denomination is taken as the first multiplicand; and the units of the next lower denomination which equal one unit of the highest denomination is taken as the first multiplier.*

2. *The first and each successive product is of the same denomination as the multiplier which produced it.*

3. *Each successive multiplier is the number of units of the next lower denomination which equals 1 unit of the denomination of the last product.*

Rule.—I. *Multiply the highest denomination by the number of units of the next lower denomination, which will equal one unit of the highest denomination, and to the product add the given number, if any, of such lower denomination.*

II. *Similarly reduce the result thus obtained to units of the next lower denomination; and so continue until the given compound number has been reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

MEASURES OF VALUE.

Reduce

1. $5 and 3¢ to mills.
2. $18 and 15¢ to cents.
3. £75 15s. 9d. to farthings.
4. 19 marks 9 pennies to pennies.
5. 183 fr. 7 ct. to millimes.
6. $13 and 18¢ to mills.
7. $20 and 7 dimes to cents.
8. £125 3d. to farthings.
9. $25 and 9 mills to mills.
10. £47 12s. to pence.

Find value in U. S. Money.

11. 953 fr. (France).
12. £547 (England).
13. 35 crowns (Norway).
14. 437 sovereigns (England).
15. £78 15s. (England).
16. 915 marks (Germany).
17. £143 10s. (England).
18. 325.25 marks (Germany).
19. 3175 roubles (Russia).
20. 628 yen (Japan).

MEASURES OF EXTENSION.

Reduce

21. 5 mi. 137 rd. 1 ft. to inches.
22. 35 A. 73 sq. rd. 4 sq. yd. to sq. ft.
23. 7 cd. 5 cd. ft. to cubic feet.
24. 19 cd. 32 cu. ft. to cu. ft.
25. 195 A. 15 sq. rd. 18 sq. yd. to sq. in.
26. 19 rd. 4 yd. 2 ft. to feet.

Reduce

27. 5 sq. mi. 15 A. to sq. ch.
28. 132 cu. yd. 18 cu. ft. to cu. in.
29. 5 cd. 52 cu. ft. to cu. ft.
30. 15 hands to inches.
31. 19 mi. 13 rd. to feet.
32. 7 mi. 30 ch. 75 l. to inches.

208

MEASURES OF WEIGHT.

Reduce

33. 8 T. 3 cwt. 7 lb. to pounds.
34. 75 lb. 8 oz. to pennyweights.
35. 3 ℔. 5 ℥ 7 ʒ to ℈.
36. 195 T. 18 lb. to ounces.
37. ʒ vij ℈ ij to ℈.
38. 5 lb. 17 pwt. to grains.

Reduce

39. 15 T. 14 cwt. 83 lb. to ounces.
40. 3 lb. 7 oz. 12 pwt. to grains.
41. 325 long tons to pounds.
42. 75 bbl. flour to pounds.
43. 928 bu. corn to pounds.
44. 168 bu. wheat to pounds.

MEASURES OF CAPACITY.

Reduce

45. 5 gal. 3 pt. to fluidounces.
46. 31 bu. 1 pk. 7 qt. to pints.
47. 47 gal. 1 pt. to gills.
48. 38 bu. 3 pk. 1 pt. to pints.

Reduce

49. 12 gal. 3 qt. 1 pt. 3 gi. to gills.
50. f ℥ iij f ʒ vj to ♏.
51. 75 bu. 1 pt. to pints.
52. 315 tuns wine to gallons.

OTHER MEASURES.

Reduce

53. 2 yr. 15 da. 7 hr. 34 sec. to sec.
54. 15° 32′ 47″ to seconds.
55. 1 leap year to minutes.
56. The year 1886 to sec.
57. 5 reams to sheets.

Reduce

58. The month of August to sec.
59. 2 bales of paper to quires.
60. 1 yr. 271 da. 45 min. to sec.
61. 19 reams to pages 8vo.
62. 6 reams to pages 4to.

209. Reduction of denominate numbers to higher denominations.

ILLUSTRATIVE EXAMPLE.

Reduce 5749 pence to the higher denominations.

Solution.

$$\begin{array}{r|l} 12 & 5749 \\ \hline 20 & 479 + 1\text{d.} \\ \hline & £23 + 19\text{s.} \end{array}$$

Ans. £23 19s. 1d.

Explanation.—First reduce the given denomination (d.) to the next higher denomination (s.), as follows: if 12d. = 1s., there are as many shillings in 5749d. as 12d. are contained times in 5749d., or 479s. and 1d. remainder.

Then reduce the obtained result (479s.) to the next higher denomination (£), as follows: if 20s. = £1, there are as many £'s in 479s. as 20s. are contained times in 479s., or £23 and 19s. remainder. Hence, 5749d. equal £23 19s. 1d.

Principles.—1. *Denominate numbers are reduced to higher denominations by successive divisions;* in which

2. *Each successive divisor should be of the same denomination as its dividend.*

3. *Each remainder is of the same denomination as the dividend from which it came.*

4. *Each quotient is of the denomination which its divisor equals* (the denominate units of the divisor being equal to 1 denominate unit of the quotient).

Rule.—I. *Divide the given number by the units of its denomination which equal 1 unit of the next higher denomination, and reserve the remainder, if any, as a part of the answer.*

II. *Proceed in like manner with this and each succeeding quotient until the required higher denomination is reached.*

III. *The last quotient with the several remainders annexed will be the answer.*

NOTE.—A denominate number may be reduced to a required higher denomination in one operation by dividing it by the number of its units which equal 1 unit of that higher denomination.

EXAMPLES FOR PRACTICE.

MEASURES OF VALUE.

Reduce

1. 5678 mills to dollars.
2. 17365 farthings to £'s.
3. 38375 millimes to fr.
4. 17532 pence to £'s.
5. 6138 pennies to marks.

Reduce

6. $92.4635 to £'s.
7. $112.572 to marks.
8. $148.224 to francs.
9. $517.75 to roubles.
10. $2345 to marks.

MEASURES OF WEIGHT.

Reduce

11. 153812 oz., Avoir., to tons.
12. 58376 gr. to pounds, Troy.
13. 7183 ʒ to pounds, Apoth.
14. 5273 pwt. to pounds, Troy.
15. 57186 oz., Avoir., to cwt.
16. 28351 gr. to pounds, Apoth.

Reduce

17. 2016 lb., Avoir., to lb., Troy.
18. 720 pwt. to lb., Apoth.
19. 1575 ℔., Apoth., to lb., Avoir.
20. 1568 lb. flour to barrels.
21. 4817 lb. wheat to bushels.
22. 5318 lb. oats to bushels.

MEASURES OF EXTENSION.

Reduce

23. 2183 cd. ft. to cords.

24. 183712 in. to miles.

25. 357182 sq. ft. to acres.

26. 4918 cu. ft. to cords.

27. 960 rd. to chains.

28. 37821 sq. yd. to acres.

Reduce

29. 3458192 cu. in. to cu. yd.

30. 173815714 sq. l. to acres.

31. 938713 ft. to rods.

32. 753218 sq. ch. to townships.

33. 893621 l. to miles.

34. 517865 cu. in. to cords.

MEASURES OF CAPACITY.

Reduce

35. 71835 ♏ to gallons.

36. 5178 pt. to bushels.

37. 31473 gi. to gallons.

Reduce

38. 716 qt. to bushels.

39. 38173 f℥ to gallons.

40. 8276 pt. to gallons.

OTHER MEASURES.

Reduce

41. 3812718 min. to com. years.

42. 1034162″ to degrees.

43. 7318 sheets to reams.

44. 93784 min. to weeks.

45. 78 years to scores.

Reduce

46. 5127 bottles to dozens.

47. 81372 pens to gross.

48. 7183 com. miles to degrees.

49. 1325173 sec. to com. years.

50. 217582 sheets of paper to bales.

210. Reduction of denominate common fractions to integers of lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce £$\frac{5}{6}$ to integers of the lower denominations.

Solution.

5
20
6) 100 (16s.
6
40
36
4
12
6) 48 (8d.
48

Explanation.—208, relating to denominate integers, is equally applicable to denominate fractions. Hence, applying Rule, **116,** for multiplication of fractions by integers, reduce £$\frac{5}{6}$ to the next lower denomination (s.) by multiplying its numerator (5) by 20, and dividing the product by its denominator (6), obtaining 16 integral shillings and $\frac{4}{6}$ of a shilling.

In the same manner, reduce $\frac{4}{6}$s. to the next lower denomination (d.) by multiplying by 12, obtaining 8d.

Hence, the required answer is 16s. 8d.

Rem.—The rules for the reduction of denominate fractions are the same as for denominate integers (**208, 209**); no additional rules are therefore deemed necessary.

210

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations.

1. $\frac{7}{16}$ bu.
2. $\frac{5}{12}$ mi.
3. $\frac{3}{4}$ gal., Apoth.
4. $\frac{2}{3}$ lb., Troy.
5. $\frac{4}{5}$ ℔., Apoth.
6. $\frac{2}{9}$ da.
7. £$\frac{3}{8}$.
8. $\frac{3}{11}$ A.
9. $\frac{3}{8}$ gal.
10. $\frac{7}{15}$ cd.
11. $\frac{5}{16}$ T.
12. $\frac{3}{5}$ yr.
13. $\frac{5}{8}$ pk.
14. $\frac{4}{7}$ yd.
15. $\frac{5}{6}$ rd.
16. £$\frac{5}{16}$.

211. Reduction of denominate decimals to integers of lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce .359375 of a bushel to integers of lower denominations.

Explanation.—Applying **208**, reduce .359375 bu. to the next lower denomination (pk.), obtaining 1.4375 pk. Reserve the integral part of this result (1 pk.) as the first term of the answer, and reduce the decimal part (.4375 pk.) to the next lower denomination (qt.), obtaining 3.5 qt. Reserve the integral part of this result (3 qt.) as the second term of the answer, and reduce the decimal part (.5 qt.) to the next lower denomination, obtaining 1 pt. as the third term of the answer.

Solution.

$$\begin{array}{rr} & .359375 \\ & 4 \\ \hline \text{pk.} & 1.437500 \\ & 8 \\ \hline \text{qt.} & 3.5000 \\ & 2 \\ \hline \text{pt.} & 1.0 \end{array}$$

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations.

1. .46875 gal.
2. .4375 oz., Troy.
3. .875 lb., Troy.
4. .1375 bu.
5. .14225 yr.
6. £.375.
7. .1875 gal.
8. .15915625 T.
9. .4609375 cd.
10. .378125 A.
11. .45 yr.
12. £.00625.

212. Reduction of a denominate fraction to a specified lower denomination.

ILLUSTRATIVE EXAMPLE.

Reduce $\frac{7}{1920}$ of a £ to an equivalent fraction of a penny.

Explanation.—Applying **208**, first reduce £$\frac{7}{1920}$ to the next lower denomination (s.) by multiplying by 20, obtaining $\frac{7}{1920}$ of $\frac{20}{1}$ of a shilling; then reduce this compound fraction of a shilling to the next and required lower denomination (d.), obtaining $\frac{7}{1920}$ of $\frac{20}{1}$ of $\frac{12}{1}$ of a penny. Applying **115**, reduce to a simple fraction of a penny, obtaining $\frac{7}{8}$d.

Solution.

$$\frac{7}{\underset{\underset{8}{96}}{\cancel{1920}}} \times \frac{\cancel{20}}{1} \times \frac{\overset{1}{\cancel{12}}}{1} = \frac{7}{8}\text{d.}$$

EXAMPLES FOR PRACTICE.

What fraction of	**What decimal of**
1. A grain is $\frac{1}{1280}$ of an oz.?	5. A penny is £.001525?
2. A day is $\frac{5}{4032}$ of a yr.?	6. A mill is $.000917?
3. A pint is $\frac{1}{160}$ of a bu.?	7. An ounce is .0000125 T.?
4. A cubic foot is $\frac{1}{192}$ of a cd.?	8. A peck is .0625 bu.?

213. Reduction of a denominate fraction to a specified higher denomination.

ILLUSTRATIVE EXAMPLE.

What fraction of a mile is $\frac{7}{8}$ of a foot?

Solution.

$$\frac{7}{\overset{}{\underset{4}{\cancel{8}}}} \times \frac{1}{3} \times \frac{\cancel{2}}{11} \times \frac{1}{320} = \frac{7}{42240} \text{ mi.}$$

Explanation.—209, relating to denominate integers is equally applicable to denominate fractions. Hence, applying Rule, **124,** for division of fractions by integers, reduce $\frac{7}{8}$ ft. to the next higher denomination (yd.) by dividing $\frac{7}{8}$ ft. by 3 ft. (Prin. 2, **209**), obtaining $\frac{7}{8}$ of $\frac{1}{3}$ yd. (Prin. 4, **209**). Then reduce to the next higher denomination (rd.) by dividing $\frac{7}{8}$ of $\frac{1}{3}$ yd. by $5\frac{1}{2}$ yd., obtaining $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{2}{11}$ rd. Then reduce to the next higher and specified denomination (mi.) by dividing $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{2}{11}$ rd. by 320 rd., obtaining $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{2}{11}$ of $\frac{1}{320}$ mi. Reduce to a simple fraction.

EXAMPLES FOR PRACTICE.

What fraction of	**What decimal of**
1. A bushel is $\frac{3}{4}$ pt.?	5. A gallon is .25 gi.?
2. A pound, Troy, is $\frac{6}{7}$ pwt.?	6. A yard is .027 in.?
3. A day is $\frac{4}{5}$ min.?	7. A dollar is .235 of a mill?
4. A ton is $\frac{5}{6}$ cwt.?	8. A £ is .018 d.?

214. Reduction of a compound number to a common fraction of a specified higher denomination.

ILLUSTRATIVE EXAMPLE.

Reduce 2 pk. 3 qt. 1 pt. to a common fraction of a bushel.

Solution.

2 pk. 3 qt. 1 pt. = 39 pt.

1 bu. = 64 pt.

$39 \div 64 = \frac{39}{64}$ bu.

Explanation.—Reduce the given compound number to a simple denominate number of its lowest denomination (39 pt.), and divide the result by the number of units of this lowest denomination which equal 1 unit of the required higher denomination (64 pt. = 1 bu.)

EXAMPLES FOR PRACTICE.

What common fraction of

1. A £ are 17s. 6d.?
2. A pound are 4 oz. 18 pwt. 8 gr.?
3. A gallon are 2 qt. 1 pt.?

What common fraction of

4. A yard are 2 ft. 6 in.?
5. An hour are 15 min. 12 sec.?
6. A mile are 8 rd. 4 yd.?

215. Reduction of a compound number to a decimal of a specified higher denomination.

ILLUSTRATIVE EXAMPLE.

Reduce 16s. 3d. to a decimal of a £.

Explanation.—Commence with the lowest denomination (3d.) and reduce it to the next higher denomination (s.) by dividing it by 12d. (Prin. 2, **209**), obtaining .25s. (Prin. 4, **209**). Prefix 16s. and reduce the result (16.25s.) to the next higher and specified denomination (£) by dividing it by 20s. (Prin. 2, **209**), obtaining £.8125.

Solution.

12) 3.00

20) 16.2500

£.8125

EXAMPLES FOR PRACTICE.

What decimal of

1. A bushel are 3 pk. 5 qt. 1 pt.?
2. A gallon are 1 qt. 1 pt. 2 gi.?
3. A £ are 1s. 3d.?
4. A ton are 1 cwt. 20 lb.?
5. A yard are 2 ft. 3 in.?

What decimal of

6. An hour are 4 min. 30 sec.?
7. A day are 21 hr. 54 min.?
8. A £ are 13s. 9d.?
9. A yr. are 5 da. 13 hr. (4 dec. pl.)?
10. A mile are 78 rd. 2 yd. (5 dec. pl.)?

ADDITION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

216. Add 19 lb. 10 oz. 17 pwt. 12 gr.; 31 lb. 11 oz. 16 pwt. 15 gr.; 8 lb. 10 oz. 23 gr.; 12 lb. 7 oz. 15 pwt.; and 13 lb. 14 pwt. 16 gr.

Explanation.—Arrange the addends so that their similar denominations shall fall in the same column.

Beginning with the lowest denomination, add the column of grains, obtaining 66 gr., or 2 pwt. 18 gr. Write 18 gr. under the column of grains, and add the carrying figure (2 pwt.) to the column of pennyweights, obtaining 64 pwt., or 5 oz. 4 pwt., etc. Write 4 pwt. under the column of pennyweights, etc., etc.

Solution.

lb.	oz.	pwt.	gr.
19	10	17	12
31	11	16	15
8	10	0	23
12	7	15	0
13	0	14	16
86	5	4	18

Rem.—The processes in addition, subtraction, multiplication, and division of compound numbers are based upon the same principles as the corresponding processes in simple numbers. In simple numbers, however, the carrying figure is obtained by the use of a uniform scale of 10, but in denominate numbers by a varying scale. Special rules are therefore deemed unnecessary.

EXAMPLES FOR PRACTICE.

(1.)

£	s.	d.	far.
75	5	8	3
13	16	5	1
96	8	11	2
52	13	9	1
7	2	4	2
33	18	8	3

(2.)

lb.	oz.	pwt.	gr.
5	9	12	9
13	4	16	8
41	6	8	15
71	11	18	22
56	7	13	19
72	8	14	16

(3.)

mi.	rd.	yd.	ft.	in.
5	175	3	1	8
17	248	4	2	6
25	315	1	1	11
41	214	3	1	7
50	16	2	2	9
48	296	4	1	5

4. Add 5 gal. 3 qt. 1 pt.; 7 gal. 2 qt.; 2 qt. 1 pt.; and 8 gal. 1 pt.

5. An importer paid £375 8s. 6d. for an invoice of cutlery; £251 5s. 8d. for an invoice of files; and £73 17s. 6d. for an invoice of saws. The charges for transportation amounted to £19 2s. 4d. What was the total cost of the goods in sterling?

6. Add 5 T. 13 cwt. 19 lb. 4 oz.; 7 T. 18 cwt. 39 lb. 5 oz.; 2 T. 12 cwt. 78 lb. 11 oz.; and 5 T. 12 cwt. 94 lb. 7 oz.

7. What is the sum of $\frac{3}{4}$ bu., $\frac{7}{8}$ pk., and $\frac{1}{2}$ qt.?

8. Add £.832, .0125s., 5.275s., and .17d.

9. What is the sum of .9375 gal., 2.125 gal., .5 qt., and .25 pt.?

SUBTRACTION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

217. Subtract 5 bu. 3 pk. 5 qt. from 12 bu. 2 pk. 7 qt.

Explanation.—Write the several denominations of the subtrahend under similar denominations of the minuend (Prin. 1, **26**). Beginning with the lowest denomination, subtract 5 qt. from 7 qt., obtaining a remainder of 2 qt.; then, since 3 pk. cannot be subtracted from 2 pk., borrow 1 bu. or 4 pk., and 3 pk. from 2+4, or 6 pk., leaves 3 pk. Next subtract 5 bu. from 12—1, or 11 bu., obtaining 6 bu.

Solution.

bu.	pk.	qt.
12	2	7
5	3	5
6	3	2

EXAMPLES FOR PRACTICE.

(1.)				(2.)				(3.)			
lb.	oz.	pwt.	gr.	mi.	rd.	yd.	ft.	bu.	pk.	qt.	pt.
75	6	13	12	12	75	3	2	18	1	3	1
42	9	18	23	8	318	4	1	16	3	2	1

4. A merchant sold 25 gal. 2 qt. 1 pt. 3 gi. of vinegar from a barrel containing 46 gal. 1 qt. 1 pt. 2 gi. What quantity remained?

5. A farmer engaged to deliver 15 T. 16 cwt. 75 lb. of hay, and subsequently delivered 9 T. 18 cwt. 34 lb. How much hay had he yet to deliver?

6. An English merchant's sales for a year amounted to £15396 5s. 8d. 2 far., and his purchases £18352 2s. 9d. 1 far. How much greater were his purchases than his sales?

7. Find the difference between .522 yd. and .02345 mi.

8. Subtract $\frac{3}{4}$ sq. yd. from $\frac{5}{8}$ sq. rd.

218. To find the interval between two dates.

ILLUSTRATIVE EXAMPLE.

Find the interval between Oct. 19, 1848, and July 28, 1868.

Explanation.—The later date expresses a *greater* period of the Christian era, and should therefore be written as the minuend; under which place the subtrahend and subtract by **217**.

The minuend denotes the 1868th *year* of the Christian era, the 7th *month* of the year, and the 28th *day* of the month; the subtrahend denotes the 1848th *year* of the Christian era, the 10th *month* of the year, and the 19th *day* of the month.

Solution.

yr.	mo.	da.
1868	7	28
1848	10	19
19	9	9

NOTE.—In compound subtraction it is usual to consider the borrowed month as equal to 30 days.

EXAMPLES FOR PRACTICE.

By compound subtraction find the interval between

1. Feb. 3, 1875, and Aug. 1, 1883.
2. June 7, 1853, and Mar. 11, 1881.
3. Jan. 25, 1815, and Sept. 3, 1836.
4. Mar. 3, 1848, and Mar. 2, 1861.
5. Oct. 15, 1825, and Apr. 5, 1886.
6. Nov. 3, 1849, and Aug. 15, 1880.
7. Apr. 15, 1871, and Feb. 2, 1879.
8. Dec. 5, 1843, and Jan. 19, 1847.
9. Oct. 25, 1875, and Oct. 15, 1884.
10. May 19, 1807, and Feb. 13, 1887.

11. A debt contracted Feb. 23, 1851, was not paid until Nov. 15, 1863. How long did it remain unpaid?

12. What is your age in years, months, and days?

13. What is the interval between 30 min. past 5 o'clock, P. M., on July 18, 1838, and 20 min. past 5 o'clock, A. M., on June 15, 1884?

14. Find the interval between 7 o'clock, A. M., Aug. 9, 1879, and 3 o'clock, P. M., Apr. 4, 1886.

219. To find the exact number of days between two dates.

ILLUSTRATIVE EXAMPLE.

Find the exact number of days between Jan. 18, 1883, and June 5, 1883.

Explanation.—The required interval will include the remainder of January (31—18=13 da.), the whole of February (28 da.), the whole of March (31 da.), the whole of April (30 da.), the whole of May (31 da.), and 5 days in June, amounting to 138 days.

Note.—In finding the exact number of days, the first day of the interval is omitted and the last day counted.

Solution.

13 days, included part of Jan.
28 " in February.
31 " " March.
30 " " April.
31 " " May.
5 " included part of June.
138 " in the interval.

EXAMPLES FOR PRACTICE.

Find the exact number of days between

1. May 13,1885, and Nov.12,1885.
2. Aug. 25, 1885, and Feb. 3, 1886.
3. Jan.20,1886, and Sept. 15,1886.
4. Feb. 2, 1884, and June 28, 1884.
5. Mar. 9, 1883, and July 7, 1883.
6. Oct. 25, 1884, and May 21,1885.
7. July 12,1885, and Dec. 25,1885.
8. Apr. 22, 1886, and Oct. 3, 1886.
9. Mar.15,1883, and Sept. 6, 1884.
10. June 9, 1884, and Nov. 1, 1885.
11. May 20,1882, and June 7, 1883.
12. Feb. 3, 1884, and July 17, 1885.

13. A note dated Jan. 18, 1883, was paid May 25, 1883. How many days did it remain unpaid?

14. In how many days will a note nominally mature if dated Aug. 13, 1883, and made payable Mar. 12, 1884?

15. What is the exact number of days from Oct. 17, 1884, to June 12, 1885?

16. An interest-bearing note dated Sept. 8, 1885, was paid Apr. 15, 1886. How many days' interest did it draw?

MULTIPLICATION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

220. Multiply £17 13s. 7d. by 9.

Explanation.—Place the multiplier (9) under the lowest denomination of the multiplicand (7d.).

7d. × 9=63d.=5s. 3d. Write 3d. as a term of the product, and carry 5s. to the product of shillings.

13s. × 9=117s., to which add the carried 5s., obtaining 122s., or £6 2s. Write 2s. as the second term of the product, and carry £6 to the product of £'s.

(£17 × 9)+£6 to carry=£159, the last term of the product.

Solution.

£17	13s.	7d.
		9
£159	2s.	3d.

EXAMPLES FOR PRACTICE.

Multiply

1. 3 da. 12 hr. 15 min. 38 sec. by 8.
2. 19 bu. 3 pk. 5 qt. 1 pt. by 7.
3. 7 lb. 9 oz. 15 pwt. 18 gr. by 5.
4. 5 gal. 3 qt. 1 pt. 2 gi. by 11.
5. 17 A. 112 P. 18 sq. yd. by 9.

Multiply

6. 16 mi. 275 rd. 4 yd. 1 ft. by 12.
7. ℔2 ℥7 ʒ6 ℈1 gr. 15 by 6.
8. £75 15s. 8d. by 132 (=12 × 11).
9. 15° 16′ 34″ by 64 (=8 × 8).
10. 5 rd. 3 yd. 2 ft. 5 in. by 91.

11. What is the cost of 58 yd. of cloth at £1 3s. 8d. per yard?

12. What is the cost of 12 barrels of vinegar, averaging 41 gal. 3 qt. 1 pt. 3 gi. per barrel, at 18 cents per gallon?

13. I paid $35 for 8 bbl. of chestnuts, averaging 2 bu. 3 pk. 2 qt. per bbl., and retailed them at 5 cents per pt. What was my gain?

14. In a certain voyage of 28 days, a vessel averaged 12 mi. 56 rd. 4 yd. per hour. What was the total distance traversed?

15. The average weight of 37 barrels of pork is 197 lb. 8 oz. The full weight of each barrel should be 200 pounds. How much less than the required quantity of pork did the 37 barrels contain?

16. A farm of 173 A. 19 P. 18 sq. yd. is divided into four fields, of which the first contains 8 A. 125 P. 17 sq. ft.; the second, 4 times as much as the first; and the third, twice as much as the first and second. What is the area of the fourth field?

17. An Englishman's daily income is £2 5s. 8d. What was his annual income during the year 1886?

18. Find the product of $\frac{3}{8}$ mi. × 7. Of $\frac{5}{16}$ bu. × 31.

19. Find the product of .011875 bu. × 125. Of .1832 T. × 2.5.

DIVISION OF COMPOUND NUMBERS.

221. Division of compound numbers by abstract numbers.

ILLUSTRATIVE EXAMPLE.

Divide 275 bu. 2 pk. 4 qt. by 6.

Solution.

$$\begin{array}{r|rrr} 6 & 275 \text{ bu.} & 2 \text{ pk.} & 4 \text{ qt.} \\ \hline & 45 \text{ bu.} & 3 \text{ pk.} & 6 \text{ qt.} \end{array}$$

Explanation.—$\frac{1}{6}$ of 275 bu. = 45 bu. and a remainder of 5 bu. (Prin. 4, **41**). Write 45 bu. as the first term of the quotient, and reduce the remainder (5 bu.) to the next lower denomination (pk.), obtaining 20 pk., to which add 2 pk. in the dividend, obtaining 22 pk. as the next partial dividend.

$\frac{1}{6}$ of 22 pk. = 3 pk. and a remainder of 4 pk. Write 3 pk. as the second term of the quotient, reduce the remainder (4 pk.) to the next lower denomination (4 pk. = 32 qt.) to which add 4 qt. in the dividend, obtaining 36 qt. as the next partial dividend. $\frac{1}{6}$ of 36 qt. = 6 qt., which write as the third term of the quotient.

EXAMPLES FOR PRACTICE.

Divide

1. £18 16s. 9d. by 9.
2. 96 bu. 3 pk. 2 qt. by 4.
3. 117 lb. 2 oz. 16 pwt. by 8.
4. ℔3 ℥7 ʒ6 ℈2 gr. 13 by 6.
5. 59° 18′ 36″ by 6.

Divide

6. 8 T. 5 cwt. 8 lb. by 7.
7. 131 yr. 13 hr. by 5.
8. 75 mi. by 9.
9. 1147 gal. 2 qt. by 72 (= 8 × 9).
10. 37 gal. 3 qt. 1 pt. by 25 (5 × 5).

11. Divide 385 yd. 2 feet 8 in. by 41.

Solution.

41) 385 yd. 2 ft. 8 in. (9 yd.
369
16 yd. Remainder.
3
41) 50 (1 ft.
41
9 ft. Remainder.
12
41) 116 ($2\frac{34}{41}$ in.
82
$\frac{34}{41}$

Explanation. — If the divisor is greater than 12 and not composite, it is usually more convenient to obtain the quotient by long division.

385 yd. ÷ 41 = 9 yd. and a remainder of 16 yd. or 48 ft., to which add 2 ft. in the dividend, obtaining 50 ft. as the next partial dividend.

50 ft. ÷ 41 = 1 ft., and a remainder of 9 ft. or 108 in., to which add 8 in. in the dividend, obtaining 116 in. as the next partial dividend.

116 in. ÷ 41 = $2\frac{34}{41}$ in.

Complete answer: 9 yd. 1 ft. $2\frac{34}{41}$ in.

221

Divide

12. 51 da. 18 hr. 15 min. 35 sec. by 13.

13. 34 T. 17 cwt. 83 lb. 13 oz. by 19.

14. 115 A. 130 sq. rd. 18 sq. yd. 37 sq. in. by 31.

15. £718 17s. 5d. by 325.

16. 342 cd. 39 cu. ft. by 537.

17. If a farm containing 325 A. 115 sq. rd. be equally divided into 6 fields, what will be the area of each field ?

18. If 18 yards of cloth are bought for £23 6d., what is the price per yard ?

19. If a pedestrian walked 375 miles in 18 days, what was the average distance traveled per day ?

20. If 18 silver spoons weigh 2 lb. 7 oz. 10 pwt., what is the weight of each spoon ?

21. Divide $1\frac{5}{8}$ yr. by 5 ; $\frac{5}{11}$ rd. by $\frac{4}{7}$; $6\frac{3}{8}$ T. by $\frac{5}{8}$.

22. Divide 3.09375 gal. by .15 ; .3 T. by .625 ; .015625 mi. by 2.5.

222. Division of one compound number by another.

ILLUSTRATIVE EXAMPLE.

Divide 8 bu. 5 qt. by 2 pk. 3 qt. 1 pt.

Solution.

8 bu. 5 qt. = 522 pt.
2 pk. 3 qt. 1 pt. = 39 pt.

$$\begin{array}{r|l} 39\,)\,522\,(\,13\frac{5}{13}\text{ times} \\ \underline{39} \\ 132 \\ \underline{117} \\ \frac{15}{39} = \frac{5}{13} \end{array}$$

Explanation.—Reduce both divisor and dividend to simple denominate numbers of the lowest denomination mentioned in either, that is, to pints, obtaining 522 pt. as the equivalent dividend and 39 pt. as the equivalent divisor.

39 pt. are contained in 522 pt. $13\frac{5}{13}$ times.

EXAMPLES FOR PRACTICE.

Divide

1. £226 9s. 5d. by £17 8s. 5d.

2. 29 bu. 3 pk. 3 qt. by 5 bu. 3 pk. 7 qt.

3. £44 2s. by £1 11s. 6d.

4. 5 A. 140 sq. rd. 5 sq. yd. by 96 sq. in.

5. 3 mi. by 5 yd. 2 ft. 5 in.

6. 2 lb. 2 oz. 5 pwt. 16 gr. by 3 oz. 5 pwt. 17 gr.

7. At 3s. 4d. per yard, how many yards of dress goods can be bought for £2 6s. 8d.?

8. If a horse is given 5 qt. 1 pt. of oats three times a day, in how many days will he consume 7 bu. 2 pk. 7 qt. 1 pt. of oats?

9. If the average speed of a train is 18 mi. 180 rd. 4 yd. 2 ft. per hour, in how many hours can it make 352 mi. 236 rd. 2 ft.?

10. How many bottles, each holding 1 pt. 3 gi., will be required to empty a cask containing 31 gal. 2 qt. 1 pt. 3 gi. of wine?

11. Divide 6¼ gal. by 2⅓ pt.; 18¼ yd. by 1⅓ ft.; $\frac{1}{16000}$ T. by $\frac{1}{24}$ lb.

12. Divide 3.35 cd. by .67 cu. ft.; .0625 mi. by .53 rd.; £.0375 by 1.5s.

REVIEW OF DENOMINATE NUMBERS.

223.—**1.** A merchant retailed 57 gal. 3 qt. 1 pt. of wine at $1.25 per pint. How much did he receive for the quantity?

2. At 8¢ per quart, what is the cost of 5 bu. 1 pk. 3 qt. 1 pt. of beans?

3. At $6.25 per barrel, how much should be paid for 4704 lb. flour?

4. An apothecary paid 22¢ per pound for 140 pounds, avoirdupois, of drugs, and sold the same at 80¢ per pound, Troy. What was his gain?

5. A grocer sold 7 gal. 2 qt. 1 pt. of syrup daily for 25 days at the rate of 5¢ per pint. How much did he receive for the syrup in the given time?

6. A lady paid £5 for 40 yd. linen. How much did she pay per yard?

7. At $20 per ounce, what is the value of 24 gold rings, each weighing 10 pwt., and 4 gold bracelets, each weighing 3 oz. 16 pwt.?

8. If 2 square miles of land be equally divided among 24 settlers, how much land will each receive?

9. A farmer sold 3 loads of wheat containing respectively 1574, 1643, and 1583 lb., at $1.32 per bushel. How much did he receive?

10. What is the cost of 3 hhd. of sugar containing respectively 6 cwt. 5 lb., 5 cwt. 90 lb., and 5 cwt. 83 lb., at 7¢ per pound?

11. The American Civil War commenced April 11, 1861, and closed April 19, 1865. How long did it continue?

12. A note drawn May 15, 1885, was not paid until Jan. 5, 1886. How many days did it remain unpaid?

13. From a cask containing 42 gallons of water, 1 gal. 3 qt. leak out daily. In what time will the cask be emptied ?

14. A dealer bought 220 long tons of coal at $4.80 per ton, and sold it at $5.10 per short ton. What was his gain ?

15. What quantity of potatoes will 145 A. yield, if the average per acre is 119 bu. 2 pk. 5 qt. ?

16. An apothecary bought 3 gallons of medicine at $6 per gallon, and retailed it at 10¢ per f ℥. What was his total gain ?

17. How many gallons of paregoric are required to fill 75 gross of vials holding f ℥ 1 each ?

18. A peddler bought 7 bushels of peanuts at $1.30 per bushel, and retailed them at 8¢ per quart. What was his total gain ?

19. A ring is 16 carats fine. What proportion of pure gold does it contain ?

20. How many carats fine is a ring containing ⅙ alloy ?

21. A man is engaged to work at $1.80 per day of 10 hours. What should be his week's wages, if on Monday he worked 8 hr. 15 min., on Tuesday 9 hr. 30 min., on Wednesday 5 hr., on Thursday 7 hr. 45 min., on Friday 8 hr. 20 min., and on Saturday 6 hr. 20 min. ?

22. If ¾ of a gill of water be mixed with 1 quart of alcohol, what part of the mixture will be alcohol ?

23. How many inches higher is a horse measuring 16¾ hands than one measuring 14½ hands in height ?

24. What is the total cost of 9875 lb. coal at $4.80 per ton, 12360 lb. at $5.25 per ton, and 7240 lb. at $5.45 per ton ?

25. A farm laborer was employed on the morning of March 15, 1884, at 45¢ per day, Sundays and holidays included, and discharged on the evening of Jan. 18, 1885. What was the balance due him if he had previously drawn $75.30 ?

26. If the average capacity per barrel is 2 bu. 2 pk. 7 qt., how many barrels will a farmer need to ship 230 bu. potatoes ?

27. What is the cost of 5229 lb. corn at 80¢ per bushel ?

28. In going 14 miles, how many revolutions will be made by a wheel which is 13 ft. 9 in. in circumference ?

29. A clock loses time at the rate of ½ second every 5 minutes. What time will it lose in 24 da. 7 hr. 18 min. ?

30. Add £45 19s. 6¾d. ; £234 4s. 5½d. ; £218 4s. 6d. ; £45 3s. 8d. ; 13s. 7d. ; £18 9s. 9¼d. ; and £147 17s. 6d.

31. What is the cost of 3 qt. 1 pt. molasses at 56¢ per gallon ?

32. In what time will a locomotive go 85 mi. 36 rd. 2 yd. 1 ft., if its driving-wheels are 9 ft. 6 in. in circumference, and average 6 revolutions per second?

33. What decimal of a mile is the depth of a bay which when sounded was found to be 165 fathoms deep?

34. What is the total cost of 15 tons of wheat at $1.20 per bushel and 75 centals of rye at 95¢ per bushel?

35. At the rate of a mile in 6 min. 30 sec., in what time will a ship sail 143 miles?

36. How many times is $\frac{7}{8}$ of a yard contained in $\frac{3}{4}$ of a mile?

37. A farmer has three meadows. From the first he raised 12 T. 17 cwt. 16 lb. hay; from the second, 14 T. 5 cwt. 4 lb.; and from the third, 7 T. 8 cwt. 13 lb. He reserved for his own use 12 T. 5 cwt. 8 lb., and sold the remainder at $18 per ton. How much did he receive?

38. If a team of horses eat 6 bu. 3 pk. 2 qt. of oats per week, what quantity will the team consume in 27 weeks?

39. A milkman adulterated 32 gallons of milk by adding a pint of water to every gallon of milk, and sold the entire quantity at the rate of 8 cents per quart. How much more than the correct amount did the milkman dishonestly obtain?

40. How many forks averaging 2 oz. 9 pwt. each can be made from 5 lb. 1 oz. 5 pwt. of silver?

41. If a pedestrian walk 15 mi. 240 rd. in 4 hr. 12 min., what distance can he walk in 1 hr. 40 min.

42. At $4.8665 to the £, what is the value in U. S. money of £3256 17s. 6d.?

43. At $.193 to the franc, what is the value in U. S. money of 437 francs, 5 centimes?

44. How many marks can be bought for $98.77 at $.238 per mark?

45. What is the cost of 17.0625 gallons of wine at 50¢ per pint?

46. Find the cost of $\frac{4}{5}$ of a pound of gold at 90¢ per pennyweight?

47. What is the cost of 3 pk. 5 qt. of clover seed at $4 per bushel?

48. What part of an inch is $\frac{1}{528}$ of a rod?

49. What decimal of a ton is 5 cwt. 8 lb. 14 oz.?

50. Reduce .00625 Cong. to integers of the lower denominations.

51. Reduce $\frac{7}{12}$ of a mile to integers of the lower denominations.

52. What part of a day is 1 min. 15 sec.?

53. What is the cost of 5 cwt. 18 lb. iron at $140 per ton?

LATITUDE, LONGITUDE, AND TIME.

224. The **Latitude** of a place is its distance north or south of the equator, estimated in degrees, minutes, and seconds.

Rem.—Distance north of the equator is called *North Latitude;* and distance south of the equator, *South Latitude.*

225. The **Longitude** of a place is its distance east or west from a given prime meridian, estimated in degrees, minutes, and seconds.

Rem.—Distance east of the prime meridian is called *East Longitude;* and distance west of the prime meridian, *West Longitude.*

226. To find the difference of latitude or longitude between two places.

ILLUSTRATIVE EXAMPLE.

The longitude of New York is 74° 0′ 3″ west, of Philadelphia 75° 10′ west, and of Paris 2° 20′ 22″ east. What is the difference of longitude between New York and Philadelphia, and between New York and Paris?

First Solution.—Since New York and Philadelphia are both *west* of the common prime meridian of Greenwich, their difference of longitude must be the distance which Philadelphia is further west than New York; and this distance is found by subtracting the less longitude of New York from the greater longitude of Philadelphia.

Second Solution.—Since the distance from the meridian of Paris to the prime meridian is 2° 20′ 22″, and from the prime meridian to the meridian of New York is 74° 3″ further, the total distance or difference of longitude between Paris and New York must be the *sum* of these two distances.

First Solution.

75°	10′	0″
74	0	3
1°	9′	57″

Second Solution.

74°	0′	3″
2	20	22
76°	20′	25″

Rule.—I. *If the latitudes of the two given places are both north or both south, subtract the less latitude from the greater; but if one is north and the other south, add the two latitudes.*

II. *If the longitudes of the two given places are both east or both west, subtract the less longitude from the greater; but if one is east and the other west, add the two longitudes.*

Note.—If the sum of two longitudes exceeds 180°, such sum should be subtracted from 360° to obtain the correct difference of longitude.

EXAMPLES FOR PRACTICE.

1. The latitude of Washington is 38° 53′ 39″ north, and of Montreal 45° 35′ north. What is their difference of latitude?

2. The longitude of Boston is 71° 3′ 30″ west, and of Cincinnati 84° 29′ 31″ west. What is their difference of longitude?

3. The latitude of Philadelphia is 39° 56′ 39″ north, and of Cape Horn 55° 58′ 30″ south. What is their difference of latitude?

4. The longitude of Berlin is 13° 23′ 45″ east, and of St. Louis 90° 15′ 15″ west. What is their difference of longitude?

5. The longitude of Calcutta is 88° 19′ 2″ east, and of San Francisco 122° 26′ 45″ west. What is their difference of longitude?

227. The solar time of any place depends upon the sun's relative position. It is 12 o'clock, M., or *noon*, at a place when the sun crosses its meridian; hence, one hour *after* the sun crosses its meridian, it is 1 o'clock, P. M., or afternoon, at that place; and one hour *before* the sun crosses its meridian, it is 11 o'clock, A. M., or forenoon, at that place. If it is noon at our meridian, it must be afternoon at all places east of us, and forenoon at all places west.

Principle.—*A place is west of other places which have later clock time, and east of all places which have earlier clock time.*

228. The daily revolution of the earth on its axis causes the sun *to appear* to pass from east to west over the 360° of the earth's longitude in 24 hours. Hence, in 1 hour the sun will appear to pass over $\frac{1}{24}$ of 360°, or 15° of longitude; in 1 minute of time $\frac{1}{60}$ of 15°, or 15′ of longitude; and in 1 second of time $\frac{1}{60}$ of 15′, or 15″ of longitude. Hence,

15° long. = 1 hr. time. | 15′ long. = 1 min. time. | 15″ long = 1 sec. time.

229. In 1883, the principal cities and railroads of the United States adopted what is called "*standard time*," by which four meridians of longitude (the 75th, 90th, 105th, and 120th) are taken as *time centres*, the difference between adjacent centres being 15°, or *exactly* 1 *hour*. The 75th meridian is the time centre for all the New England and Middle States, the District of Columbia, and Maryland, Virginia, West Virginia, North Carolina, and South Carolina. The 90th meridian is the time centre for Michigan, Wisconsin, Minnesota, Nebraska, Kansas, Iowa, Missouri, Indiana, Illinois, Ohio, Kentucky, Tennessee, Georgia, Florida, Alabama, Mississippi, Louisiana, Texas, Arkansas, and Indian Territory. The 105th meridian is the time centre for Colorado, Dakota, Montana, Wyoming, Idaho, Utah, New Mexico, and Arizona. The 120th meridian is the time centre for Oregon, California, Nevada, and Washington Territory. By this convenient arrangement, all places having the 75th meridian as their time centre take as their clock time the solar time of the 75th meridian; all places having the 90th meridian as their time centre take as their clock time the solar time of the 90th meridian, which is exactly

1 hour earlier than the solar time of the 75th meridian, and 1 hour later than the solar time of the 105th meridian, etc. Hence, throughout the United States, the minute and second hands of all watches should be at the same point of the dial, *only the hour hands* varying with their respective time centres.

230. To find the difference of time between two places.

ILLUSTRATIVE EXAMPLE.

The longitude of St. Louis is 90° 15′ 15″ west, and of Richmond 77° 25′ 45″ west. What is their difference of solar time?

Explanation.—First find the difference of longitude by **226**, obtaining 12° 49′ 30″; and $\frac{1}{15}$ of this result will be the required difference of time (**228**).

Solution.

	90°	15′	15″
	77	25	45
15)	12°	49′	30″
		51 min.	18 sec.

Rule.—*Divide the difference of longitude in degrees, minutes, and seconds, by 15, and the quotient will be the difference of solar time respectively in hours, minutes, and seconds.*

EXAMPLES FOR PRACTICE.

1. The longitude of San Francisco is 122° 26′ 45″ west, and that of New Orleans 90° 2′ 30″ west. What is their difference of solar time?

2. What is the solar time in New Orleans when the solar time in San Francisco is 35 minutes past 8 o'clock, A. M.?

3. The longitude of Rome is 12° 27′ 14″ east, and that of Albany 73° 44′ 50″ west. What is their difference of solar time?

4. What is the solar time in Albany when it is half-past 2 o'clock, P. M., in Rome?

5. What is the solar time in Rome when the solar time in Albany is 18 minutes past 11 o'clock, A. M.?

6. What is the standard time in San Francisco when the standard time in New York is 35 min. 18 sec. past 1 o'clock, P. M.?

7. What is the standard time in Baltimore when the solar time is 20 min. past 2 o'clock, A. M., in St. Petersburg, long. 30° 19′ east?

8. What is the solar time in Shanghai, China, longitude 121° 28′ east, when the solar time is 4 min. 36 sec. past 5 o'clock, P. M., in Baltimore, longitude 76° 37′ west?

231. To find the difference of longitude between two places when their difference of solar time is known.

ILLUSTRATIVE EXAMPLE.

The difference of solar time between Berlin and Paris is 44 min. 13⅗ sec. If the longitude of Berlin is 13° 23′ 45″ east, what is the longitude of Paris ?

Explanation.—Multiply the difference of time by 15, to find the difference of longitude (**228**), obtaining 11° 3′ 24″. Since Paris is west of Berlin, the longitude of Paris must be 11° 3′ 24″ nearer the prime meridian of Greenwich, which is also west of Berlin. Hence, subtract the difference of longitude from the longitude of Berlin to find the longitude of Paris.

Solution.

$$\begin{array}{rrl} & \text{44 min.} & 13\frac{3}{5}\text{ sec.} \\ & & 15 \\ \hline 11^\circ & 3' & 24'' \\ 13 & 23 & 45 \\ \hline 2^\circ & 20' & 21''\text{ east.} \end{array}$$

Rule.—*Multiply the difference of solar time in hours, minutes, and seconds, by 15, and the product will be the difference of longitude respectively in degrees, minutes, and seconds.*

EXAMPLES FOR PRACTICE.

1. When the solar time in Washington is 48 min. past 11 o'clock, A. M., the solar time in Portland is 15 min. past 12 o'clock, P. M. If the longitude of Washington is 77° 15″ west, what is the longitude of Portland ?

2. The longitude of Jefferson City is 92° 8′ west. What is the longitude of Boston if, when the solar time in Jefferson City is 20 min. 30 sec. past 11 o'clock, A. M., the solar time in Boston is 44 min. 48 sec. past 12 o'clock, P. M. ?

3. When the solar time in San Francisco is 10 min. past 8 o'clock, P. M., the solar time in Calcutta is 13 min. 3$\frac{2}{15}$ sec. past 10 o'clock, A. M. If the longitude of San Francisco is 122° 26′ 45″ west, what is the longitude of Calcutta ?

4. A sailor observed that when the sun crossed the meridian of his ship, it was 18 min. 3 sec. past 5 o'clock, A. M., by his chronometer set to Greenwich time. In what longitude was his ship ?

5. If the standard time of Trenton is 40 min. past 9 o'clock, P. M., Thursday, and at the same moment the solar time of another place is 45 min. 16 sec. past 3 o'clock, A. M., Friday, what is the longitude of the latter place ?

231

METRIC SYSTEM.

232. The **Metric System** is a decimal system of weights and measures, which has for its basis a certain unit of length called the *meter*.

Rem.—The *meter* is $\frac{1}{10000000}$ of the distance from the equator to either pole, as ascertained by the measurement of that part of the meridian of Dunkirk and Barcelona which is included between those cities. The length of the meter is 39.37079 inches.

233. The **Primary Units** of the metric system are:

Kind of Measure.	**Primary Unit.**	**Pronunciation.**
Measure of Length,	*Meter,*	(mee′-ter).
Measure of Capacity,	*Liter,*	(lee′-ter).
Measure of Weight,	*Gram,*	(gram).
Land Measure,	*Ar,*	(air).
Ordinary Surface Measure,	*Square Meter,*	(square mee′ter).
Ordinary Cubic Measure,	*Cubic Meter,*	(cubic mee′-ter).
Wood Measure,	*Ster,*	(stair).

234. Higher Denominations than the above primary units are denoted by prefixing the following Greek numerals: *deka* (10 primary units); *hekto* (100 primary units); *kilo* (1000 primary units); *myria* (10000 primary units).

235. Lower Denominations than the primary units are denoted by prefixing the following Latin numerals: *deci* ($\frac{1}{10}$ of a primary unit); *centi* ($\frac{1}{100}$ of a primary unit); *milli* ($\frac{1}{1000}$ of a primary unit).

Rem.—The above numeral prefixes indicate the decimal value of the derivative units as compared with the primary units.

236. METRIC LINEAR MEASURE.

Rem. 1.—The primary unit of Linear Measure is the meter, which equals 39.37079 in.

Rem. 2.—The *meter* is used in measuring dry goods and short distances; the *centimeter* and millimeter in measuring minute distances; and the kilometer in measuring long distances.

TABLE.

10 millimeters (mm.)	=	1 centimeter	(cm.)
10 centimeters	=	1 decimeter	(dm.)
10 decimeters	=	1 *meter*	(M.)
10 meters	=	1 dekameter	(Dm.)
10 dekameters	=	1 hektometer	(Hm.)
10 hektometers	=	1 kilometer	(Km.)
10 kilometers	=	1 myriameter	(Mm.)

237. METRIC LAND MEASURE.

Rem. 1.—The primary unit of Land Measure is the ar, a square each side of which is 10 meters. The ar is therefore equal to 100 sq. meters or 119.6034 sq. yd.

TABLE.

100 centars (ca.) = 1 *ar* (A.)
100 ars = 1 hektar (Ha.)

Rem. 2.—The hektar equals very nearly 2½ acres, and is the metric denomination ordinarily employed in expressing the quantity of land in farms.

238. ORDINARY SURFACE MEASURE.

Rem.—The square meter (1.196 sq. yd.) is the denomination generally used in expressing surfaces other than land, such as the area of floors, ceilings, dry goods, etc.

TABLE.

100 sq. centimeters (sq. cm.) = 1 sq. decimeter (sq. dm.)
100 sq. decimeters = 1 *sq. meter* (Sq. M.)

239. METRIC WOOD MEASURE.

Rem.—The primary unit of Wood Measure is the ster, a cube each edge of which is 1 meter. The ster therefore equals 1 cu. meter or 35.317 cu. ft.

TABLE.

10 decisters (ds.) = 1 *ster* (S.)
10 sters = 1 dekaster (Ds.)

240. ORDINARY CUBIC MEASURE.

TABLE.

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.)
1000 cubic centimeters = 1 cubic decimeter (cu. dm.)
1000 cubic decimeters = 1 *cubic meter* (Cu. M.)

Rem.—The cubic meter (35.317 cu. ft. or 1.308 cu. yd.) is the denomination employed in expressing the contents of ordinary volumes or solids other than firewood; but minute volumes are expressed in cu. centimeters or cu. millimeters.

241. METRIC MEASURE OF CAPACITY.

Rem. 1.—The metric measure of capacity is used to measure all liquids and such dry substances as salt, grain, fruits, vegetables, etc. The primary unit is the liter, a cylinder whose volume is equal to that of a cube each edge of which is $\frac{1}{10}$ of a meter. The liter equals 1.05678 qt., liquid measure, or .9081 qt., dry measure.

TABLE.

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 *liter* (L.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 hektoliter (Hl.)
10 hektoliters = 1 kiloliter (Kl.)

Rem. 2.—Of liquids, the dekaliter is used to measure or express *large* quantities; the liter, *moderate* quantities; and the centiliter or milliliter, *minute* quantities.

Rem. 3.—Of grain, fruits, vegetables, etc., the hektoliter, like the bushel, is used for large quantities (wholesale); and the dekaliter, liter, or deciliter for small quantities (retail).

242. METRIC MEASURE OF WEIGHT.

Rem. 1.—The gram is the primary unit of weight. It equals 15.432348 Troy grains, or the weight, in a vacuum and at its greatest density, of a cube of distilled water, each edge of which cube is $\frac{1}{100}$ of a meter.

Rem. 2.—The *tonneau*, like our ton, is the denomination used to express the weight of very large quantities; the *kilogram* (generally abbreviated to kilo) like our pound, to express the weight of moderate quantities; and the *gram*, like our pennyweight and grain, to express the weight of minute quantities, such as jewels, medicines, letters, etc.

TABLE.

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 *Gram* (G.)
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 kilogram (Kg.)
10 kilograms = 1 myriagram (Mg.)
10 myriagrams = 1 quintal (Q.)
10 quintals = 1 tonneau (Ton.)

TABLE OF EQUIVALENTS.

243. The equivalents here given are such as have been established by Act of Congress. In the United States, reductions from the metric system to the common system, or the reverse, must be made in accordance therewith to be legal.

EQUIVALENTS OF LINEAR MEASURE.

1 inch = 2.54 centimeters.
1 foot = .3048 of a meter.
1 yard = .9144 of a meter.
1 rod = 5.029 meters.
1 mile = 1.6093 kilometers.

1 centimeter = .3937 of an inch.
1 decimeter = .328 of a foot.
1 meter = 1.0936 yards.
1 dekameter = 1.9884 rods.
1 kilometer = .62137 of a mile.

EQUIVALENTS OF SQUARE MEASURE.

1 sq. inch = 6.452 sq. centimeters.
1 sq. foot = .0929 of a sq. meter.
1 sq. yard = .8361 of a sq. meter.
1 sq. rod = 25.293 sq. meters.
1 acre = 40.47 ars.
1 sq. mile = 259 hektars.

1 sq. centimeter = .155 of a sq. inch.
1 sq. decimeter = .1076 of a sq. foot.
1 sq. meter = 1.196 sq. yards.
1 ar = 3. 954 sq. rods.
1 hektar = 2.471 acres.
1 sq. kilometer = .3861 of a sq. mile.

EQUIVALENTS OF CUBIC MEASURE.

1 cu. inch = 16.387 cu. centimeters.
1 cu. foot = 28.317 cu. decimeters.
1 cu. yard = .7645 of a cu. meter.
1 cord = 3.624 sters.

1 cu. centimeter = .061 of a cu. inch.
1 cu. decimeter = .0353 of a cu. foot.
1 cu. meter = 1.308 cu. yards.
1 ster = .2759 of a cord.

EQUIVALENTS OF CAPACITY.

1 liquid quart = .9463 of a liter.
1 dry quart = 1.101 liters.
1 liquid gallon = .3785 of a dekaliter.
1 peck = .881 of a dekaliter.
1 bushel = .3524 hektoliters.

1 liter = 1.0567 liquid quarts.
1 liter = .908 of a dry quart.
1 dekaliter = 2.6417 liquid gallons.
1 dekaliter = 1.135 pecks.
1 hektoliter = 2.8375 bushels.

EQUIVALENTS OF WEIGHT.

1 ounce, Avoir. = 28.35 grams.
1 ounce, Troy = 31.104 grams.
1 pound, Avoir. = .4536 of a kilogram.
1 pound, Troy = .3732 of a kilogram.
1 grain, Troy = .0648 of a gram.
1 ton (of 2000 lb.) = .9072 of tonneau.

1 gram = .03527 of an ounce, Avoir.
1 gram = .03215 of an ounce, Troy.
1 gram = 15.432 grains, Troy.
1 kilogram = 2.2046 pounds, Avoir.
1 kilogram = 2.679 pounds, Troy.
1 tonneau = 1.1023 tons (of 2000 lb.).

NOTATION AND NUMERATION OF METRIC QUANTITIES.

244. Any metric quantity, except ordinary surfaces or solids, is written according to the decimal system of notation by letting its primary units occupy the units' order, and by letting each of the other denominations occupy that order of the decimal scale which is denoted by its prefix.

To write 2 Hg. 6 Dg. 8 G. 3 dg. 7 mg., consider 2 Hg. as 2 *hundreds* of grams, 6 Dg. as 6 *tens* of grams, and 8 G. as 8 *units* of grams, making the integral expression 268 grams; and consider 3 dg. as 3 *tenths* of grams and 7 mg. as 7 *thousandths* of grams, making the decimal expression .307 of a gram, and the entire expression 268.307 grams.

245. Any metric quantity except ordinary surfaces or solids, is usually read like any other decimal; but frequently the fractional part is read as an integer of the denomination of its right-hand order.

Thus, 318.75 L., instead of being read 3 Hl. 1 Dl. 8 L. 7 dl. 5 cl., is usually read 318 and 75 hundredths liters; or, frequently, 318 liters and 75 centiliters.

246. Ordinary surfaces are based upon the scale of 100, and ordinary solids upon the scale of 1000. Each denomination of these measures must therefore occupy as many decimal orders as there are 0's in the scale of its measure. Ordinary surfaces and solids are written by letting the primary unit occupy the units' order, and letting the position of each of the other denominations of surface measure extend *twice* as far, and of cubic measure *three* times as far, from the decimal point as that order of the scale which is denoted by its prefix.

Thus, 18 Sq. M. 5 sq. dm. are written 18.05 Sq. M.; 5 Ha. 3 A. 7 ca. are written 503.07 A., or 5.0307 Ha.; and 95 Cu. M. 7 cu. dm. 18 cu. cm. 157 cu. mm. are written 95.007018157 Cu. M.

247. If the number of digits in the decimal part of a metric quantity of ordinary surface is not divisible by 2, or of ordinary solids is not divisible by 3, sufficient ciphers are understood to be annexed to such decimal part to make it so divisible, before it can be read in terms of its lowest denomination.

248. EXERCISE IN NUMERATION AND NOTATION.

ILLUSTRATIVE EXERCISE.

15.327 G. are read 15 and 327 thousandths Grams; or 1 Dg. 5 G. 3 dg. 2 cg. 7 mg.; or 15 Grams and 327 milligrams.

Read the following metric quantities:

1. 718.5134 Ton.
2. 157.814 M.
3. 325.817 G.
4. 563.108 A.
5. 184.253 L.
6. 751.51 A.
7. 38.2515 Sq. M.
8. 15.183 Sq. M.
9. 98.5 S.
10. 18.315718415 Cu. M.
11. 72.41900885 Cu. M.
12. 569.0031759 Cu. M.
13. 8.5198 Cu. M.
14. 31.2817 Ha.
15. 175.1873 Kilos.

16. Write 5 Ha. 15 ca. as a decimal of an ar.

17. Write 5 Km. 8 Hm. 4 Dm. 8 M. 7 dm. 3 mm. as a mixed decimal of a meter.

18. Write 1 Kg. 5 G. 8 cg. as a mixed decimal of a gram.

19. Write 5 Cu. M. 18 cu. dm. 123 cu. cm. 9 cu. mm. as a mixed decimal of a cubic meter.

20. Write 15 Ds. 9 ds. as a mixed decimal of a ster.

21. Write 9 Hl. 3 Dl. 7 cl. 4 ml. as a mixed decimal of a liter.

22. Write 5 Sq. M. 9 sq. dm. 84 sq. cm. as a mixed decimal of a sq. decimeter.

249. REDUCTION OF METRIC QUANTITIES.

ILLUSTRATIVE EXAMPLE.

1. Reduce 51.783214 Kg. to dg.

Solution.

51.783214 Kg. = 517832.14 dg.

Explanation.—Since the integer (51) expresses Kilograms, the following order (7) must express the next lower denomination (hektograms), the next order (8) dekagrams, the next order (3) grams, and the next order (2) the required denomination, decigrams. Hence, remove the decimal point and place it to the right of the order of decigrams.

2. Reduce 2651837.24 cu. mm. to cu. dm.

Solution.

2651837.24 cu. mm. = 2.65183724 cu. dm.

Explanation.—Since the integer (2651837) expresses cu. mm., and cubic measure is based on the scale of 1000, the *three* orders to the left of the decimal point (837) must express cu. mm.; the next *three* orders (651), the next higher denomination (cu. cm.); and the remaining order (2), the next higher and required denomination (cu. dm.). Hence, remove the decimal point and place it to the right of the order of cubic decimeters.

Rule.—*Remove the decimal point and place it at the right of the order which expresses the required denomination; and annex the abbreviation of the required denomination.*

EXAMPLES FOR PRACTICE.

Reduce

1. 132.7182 Hg. to dg.
2. 37.28146 Km. to cm.
3. 157183.26 cl. to Hl.
4. 178242.1 dg. to Dg.
5. 8175131.52 cu. cm. to Cu. M.
6. 5185415.31 sq. cm. to Sq. M.
7. 38713.41 Sq. M. to sq. dm.

Reduce

8. 51.71834 Cu. M. to cu. cm.
9. 135.8 A. to Ha.
10. 987.1625 Ds. to ds.
11. 317.6 ds. to S.
12. 187531.28 ca. to Ha.
13. 718632.5 dm. to Hm.
14. 8418673.21 mg. to Kg.

250. Reductions from the common system to the metric, or the reverse, are effected according to the Table of Equivalents (**243**).

Thus, to find how many sters equal 75 cords of wood, refer to the Table of Equivalents to find how many sters equal 1 cord (3.624 sters), and 75 cords must equal 75 times 3.624 sters, or 271.8 sters.

Reduce

1. 385.25 A. to acres.
2. 375 Hl. of wheat to bushels.
3. 153 Km. to miles.
4. 483 Dl. of wine to gallons.
5. 75.5 Kg. of beef to lb., Avoir.
6. 157.75 Ha. of land to acres.
7. 27.5 cm. of lace to inches.

Reduce

8. 87 yards of cloth to meters.
9. 187 cords of wood to sters.
10. 157.35 acres of land to ars.
11. 173 gallons of vinegar to Dl.
12. 3 ounces of gold to grams.
13. 187 bushels of oats to Hl.
14. 845 cu. yd. of earth to Cu. M.

251. Metric quantities are added, subtracted, multiplied, or divided like other decimals, first reducing them to the same denomination, if necessary, by Rule, **249.** The primary unit is usually the most convenient common denomination to adopt when there is no special reason for adopting another.

EXAMPLES FOR PRACTICE.

1. How many meters in 15.7 cm., 381.4 dm., 4.173 cm., 183.24 M., 4.1 dm., and 5.83 Dm. ?

2. A merchant bought 17.8 Dl. of wine, and sold at different times 1.52 L., 3.85 dl., and 53.4 cl. How many liters had he remaining ?

3. What should be paid for 24 Dl. of peaches at 40¢ per peck ?

4. What is the cost of 34.5 Kg. of sugar at 14¢ per kilo ?

5. Find the cost of 5.137 S. + 3.8 Ds. + 5.75 ds. of wood at $1.60 per S.

6. What is the value in U. S. money of 364.625 meters of cloth, invoiced at 8.40 francs per meter ?

7. A merchant imported 10000 Dl. of wine at a cost of 62 francs per Dl., and sold the same at $4.75 per gallon. What was his gain ?

8. At $5 per cord, what is the cost of a pile of wood which is 18.75 meters long, 1.5 meters wide, and 1.625 meters high ?

9. What is the total cost in U. S. money of the following : 52.75 M. of cloth bought at 3.20 fr. per M.; 28.375 M. at 4.75 fr. per M.; 91.85 dm. at 6.50 fr. per M.; and 3.25 Dm. at 8.70 fr. per. M. ?

10. Find cost in U. S. money of 517.32 Kg. + 875.75 Hg. + 81.65 Mg. + 837.25 Hg. + 8175.50 G. + 1.45 Ton. of sugar at .70 fr. per kilo.

11. An importer mixed 18 liters of wine costing 1.75 francs per liter with 15 liters costing 2.15 francs per liter, and sold the entire mixture at $1.75 per gallon. Did he gain or lose, and how much ?

SURFACE MEASUREMENTS.

261. A **Surface** is the outside of a solid or body. It can therefore contain only *two* of the three dimensions of a body, its depth or thickness not being considered.

A flat surface is called a plane surface, or, simply, a *plane*. To discover if any surface is a plane, place the straight edge of a ruler upon it in many directions. If all parts of the edge touch the surface, it is a plane.

262. A **Rectangular Surface** is a plane surface of four sides, each of which is straight, and perpendicular to its adjacent sides.

The opposite sides of a rectangular surface are equal to each other.

263. A **Square** is a rectangular surface having *all* of its sides equal in length.

A square inch is a rectangular surface, *each side* of which is one inch in length; a square foot is a rectangular surface each side of which is one foot; etc.

264. The **Area** of a rectangular surface is the number of square units which it contains.

Rem.—In obtaining areas, the most convenient unit of measure is a square, each side of which is a unit of the same denomination as the two given dimensions. Hence, to find the area of a rectangular surface 8 ft. long and 5 ft. wide, the measuring unit will be 1 sq. ft., since the denomination of the length and width is feet. Having selected the measuring unit, the area is found as follows:

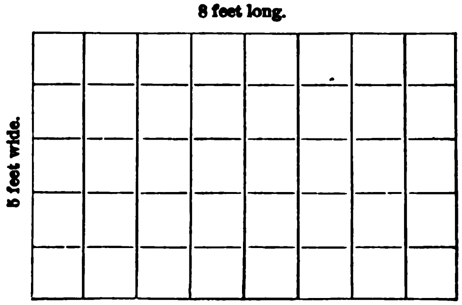


Figure 3.

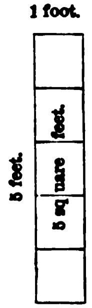


Fig. 2.

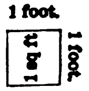


Fig. 1.

5 square feet × 8 = 40 square feet, area of Fig. 3.

Fig. 1 is the measuring square, which is 1 ft. long and 1 ft. wide, and therefore represents 1 sq. ft. In Fig. 2 are 5 such squares, and hence it represents 5 sq. ft. In Fig. 3 are 8 rows of measuring squares, each row containing as many measuring squares as Fig. 2, and it must therefore represent 8 times 5 sq. ft., or 40 sq. ft.

265. To find the area of a rectangular surface.

ILLUSTRATIVE EXAMPLE.

How many square yards are in a sidewalk 48 ft. long and 11 ft. 4 in. wide?

Explanation.—A sidewalk 48 ft. long and 1 ft. wide contains 48 sq. ft. Hence, a sidewalk 48 ft. long and 11⅓ ft. wide must contain 11⅓ times 48 sq. ft., or 544 sq. ft., which reduced to sq. yd. equal 60$\frac{4}{9}$.

Solution.

11 ft. 4 in. = 11⅓ ft.
48 sq. ft. × 11⅓ = 544 sq. ft.
544 ÷ 9 = 60$\frac{4}{9}$ sq. yd.

Rule.—*Multiply one dimension, considered as square units, by the other dimension, considered abstractly.*

NOTE.—If necessary, reduce both dimensions to the same denomination before applying the rule.

EXAMPLES FOR PRACTICE.

1. How many acres are contained in a rectangular field 847 yards long and 240 yards wide?

2. How many square feet are in the floor of a room 27 ft. 6¾ in. long and 13 ft. 7½ in. wide?

3. How many acres are in a rectangular piece of land 3 miles long and 175 rods wide?

4. How many square yards are in the walls of a room 24 ft. long, 18 ft. wide, and 12 ft. high?

5. How many square feet are in the surface of a box 4 ft. long, 3 ft. 9 in. wide, and 2 ft. high?

6. A street ¼ of a mile long was paved at a cost of 45 cents per square foot. What was the cost of paving if the street was 40 ft. wide?

7. How much will it cost to plaster the ceiling of a room 36 ft. long and 27 ft. wide, at 23 cents per square yard?

8. At $7.25 per 100 sq. ft., how much will it cost to tin the roof of a house 45 ft. long, and which measures 23 ft. from each eave to the ridge?

9. At $30 per acre, what is the cost of a rectangular piece of ground 40 chains long and 35 chains 75 links wide?

10. The length of a piece of land is 15 rd. 2 yd. 1 ft. 7 in., and its width 12 rd. 4 yd. What is its area?

266. To find either dimension of a rectangular surface.

ILLUSTRATIVE EXAMPLE.

If a rectangular field containing 30 acres is 40 rods wide, what is its length?

Explanation.—A field 40 rd. wide and 1 rd. long contains 40 sq. rd.; therefore to contain 30 acres or 4800 sq. rd., it must be as many times 1 rd. long as 4800 sq. rd. are times 40 sq. rd., or 120 rd. long.

Solution.

30 acres=4800 sq. rd.

4800÷40=120 rd., length.

Rule.—*Divide the area by either dimension; the quotient will be the other dimension.*

NOTE.—If necessary, reduce the area to square units and the given dimension to linear units of the same denomination, before applying the rule.

EXAMPLES FOR PRACTICE.

1. If a rectangular field of wheat contains 391 acres, and its length is 85 chains, what is its width?

2. A farmer raised 1575 bushels of wheat in a rectangular field which averaged 25 bushels to the acre. If the field was 112 rods long, what was its width?

3. A rectangular piece of land contains 11520 acres, and its length is 9 miles. What is its width?

4. If the area of a floor is 48 sq. yd. 5 sq. ft., and its breadth is 7 yd. 2 ft., what is its length?

5. A plasterer whose price was 25¢ per sq. yd., charged $8 for plastering the ceiling of a room 16 ft. wide. What was the length of the room?

6. If a rectangular field which contains 57 A. 155 P. is 175 rods long, what is its width?

7. What is the length of a blackboard which contains 9 sq. yd. 5 sq. ft. 36 sq. in., if its breadth is 3 ft. 9 in.?

267. To find the quantity of material to cover a given surface.

ILLUSTRATIVE EXAMPLE.

Making no allowance for waste, how many yards of carpet 30 in. wide are required to cover a floor 17 ft. 6 in. long and 16 ft. 6 in. wide?

Explanation.—1 yard of carpet of the given width can cover only 7½ sq. ft. of the floor; hence, to cover the entire surface of the floor, the length of the carpet must be	**Solution.** 17½ × 16½ = 288¾ sq. ft., area of floor. 3 × 2½ = 7½ sq. ft., area of 1 yd. of carpet. 288¾ ÷ 7½ = 38½ yd., length of carpet.

as many times 1 yard as the area of the floor (288¾ sq. ft.) is times the area of 1 yard of carpet (7½ sq. ft.), or 38½ yards.

Rule.—*Divide the area of the floor by the area of one yard of the given material. The quotient will be the number of yards of material.*

Note.—To find the number of stones, bricks, shingles, etc., to cover a given surface, *divide the area of the surface to be covered by the* EXPOSED *area of one unit of the given material.*

EXAMPLES FOR PRACTICE.

Allowing nothing for waste, how many yards of carpet are required

1. For a room 16 ft. long and 14 ft. wide; carpet 30 in. wide?
2. For a room 20 ft. 3 in. by 18 ft. 6 in.; carpet 1 yd. wide?
3. For a room 30 ft. 5 in. by 24 ft. 8 in.; carpet 2 ft. 4 in. wide?
4. For a room 50 ft. 7 in. by 42 ft. 3 in.; carpet 28 in. wide?
5. For a room 15 ft. 9 in. by 12 ft. 6 in.; carpet 20 in. wide?
6. What is the cost of carpeting a room 24 ft. 3 in. long and 18 ft. 8 in. wide; carpet 28 in. wide and $1.25 per yard?
7. How many bricks 8 in. long and 4 in. wide are necessary to cover a sidewalk 34 ft. 4 in. long and 14 ft. 3 in wide?
8. How many flagstones 20 in. long and 16 in. wide will pave a yard 57 ft. 11 in. long and 10 ft. 8 in. wide?
9. At $1.75 per yard for carpeting 34 in. wide, what will it cost to cover a room 19 ft. 10 in. long and 15 ft. 4 in. wide?
10. Making no allowance for waste, how many shingles 4 in. wide are required to cover the roof of a house 36 ft. long and measuring 20 ft. from each eave to the ridge, if only 5 inches of the length of the shingles are exposed to the weather?
11. How many sods 10 in. long and 8 in. wide are required to turf a plat of ground 28 ft. 4 in. long and 22 ft. 4 in. wide?
12. At 75 cents per yard, what is the cost of covering the floor of a dining-room with oil-cloth 1½ yards wide, if the length of the room is 27 ft. and its width 20 ft. 9 in.?

13. How many yards of carpet are required to cover the floor of a room 22 ft. 1 in. long and 18 ft. 9 in. wide, if the width of the carpet is 27 in., and the strips are laid crosswise?

Explanation. — Since the strips are to be laid crosswise (parallel to the less dimension), divide the length of the room (22 ft. 1 in.) by the width of the carpet (27 in.) to find the number of strips ($9\frac{22}{27}$), or, in practice, 10 strips, since the excess of the 10th strip (5 in.) is either turned under in laying, or cut off before laying.

Solution.

(22 ft. 1 in.) ÷ 27 in. = $9\frac{22}{27}$ strips, or practically 10 strips.

(18 ft. 9 in.) × 10 = 62 yd. 1 ft. 6 in.

As the carpet is to be laid crosswise, the length of each strip must be the same as the less dimension (18 ft. 9 in.). Hence, multiply the length of each strip (18 ft. 9 in.) by the number of strips (10) to find the number of yards to be purchased.

If the design of the carpet is such as to involve no loss in matching, how many yards must be purchased to cover the floor of a room

14. 18 ft. 6 in. by 15 ft. 3 in.; carpet 32 in. wide, laid lengthwise?

15. 31 ft. 8 in. by 25 ft. 2 in.; carpet 28 in. wide, laid crosswise?

16. I wish to carpet a room 28 ft. 6 in. long and 18 ft. 3 in. wide by laying the strips lengthwise. Which of three carpets respectively 28 in., 30 in., and 34 in. wide, will involve the least wastage in laying?

17. I wish to carpet a room 20 ft. long by 18 ft. wide with carpeting 2 ft. 6 in. wide. At $2.25 per yard, how much shall I save by laying the carpet crosswise instead of lengthwise?

MEASUREMENT OF VOLUMES.

268. A **Rectangular Volume** is one which is bounded by six rectangular surfaces (**262**).

All rectangular volumes have three dimensions: *length, breadth,* and *thickness.*

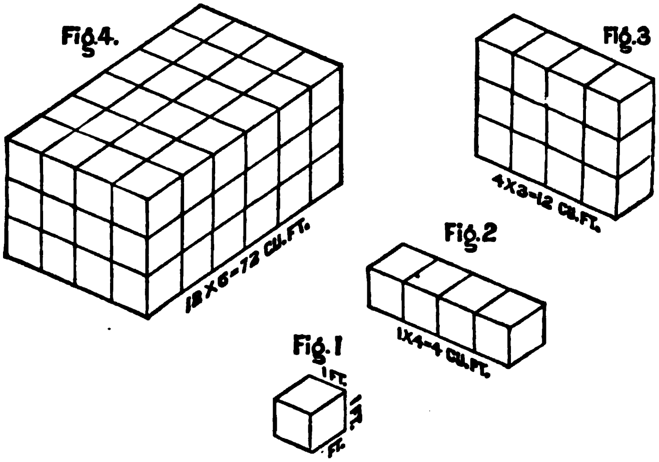
269. A **Cube** is a rectangular volume which is bounded by six equal squares, called *faces.* All the edges of a cube are of equal length.

A cubic inch is a cube each edge of which is one inch in length; a cubic foot is a cube each edge of which is one foot in length; etc.

270. The **Contents** or **Volume** of a rectangular solid is the number of cubic units which it contains.

Rem.—In obtaining volumes, the most convenient unit of measure is a cube each edge of which is a unit of the same denomination as the three given dimensions. Hence, to find the volume of a rectangular solid which is 6 ft. long, 4 ft.

wide, and 3 ft. deep, the measuring unit will be 1 cu. ft., since the denomination of the length, width, and depth is feet. After selecting the measuring unit, the required volume is found as follows:



1×1×1=1 cu. ft.

4×3×6=72 cu. ft., contents of Fig. 4.

Fig. 1 is the measuring cube, which is 1 ft. long, 1 ft. wide, and 1 ft. deep, and therefore represents 1 cu. ft. In Fig. 2, which represents the upper tier of Fig. 3, are 4 such cubes, and hence it represents 4 cu. ft. In Fig. 3, which represents an end section of Fig. 4, are 3 tiers equal to Fig. 2, and hence it represents 3 times 4, or 12 cu. ft. In Fig. 4 are 6 sections equal to Fig. 3, and hence it represents 6 times 12, or 72 cu. ft.

271. To find the contents of a rectangular volume.

ILLUSTRATIVE EXAMPLE.

What is the charge for freight upon a box 3 ft. 8 in. long, 3 ft. 6 in. wide, and 2 ft. 3 in. high, at 8¢ per cubic foot?

Solution.

3⅔ cu. ft. × 3½ × 2¼ = 28⅞ cu. ft.

8¢ × 28⅞ = $2.31. *Ans.*

Explanation.—The volume of the box is equal to the continued product of its three dimensions (Rem. **270**), or 28⅞ cu. ft. Hence, multiply the freight per cu. ft. (8¢) by the number of cu. ft. (28⅞).

Rule.—*Multiply one dimension considered as cubic units, by the product of the two remaining dimensions, considered abstractly.*

Note.—If necessary, reduce all three dimensions to a common denomination before applying the rule.

EXAMPLES FOR PRACTICE.

1. How many cubic feet of air are in a room 18 ft. long, 16 ft. wide, and 10 ft. 6 in. high?

2. How many cubic yards of stone are in a pile 15 ft. long, 12 ft. wide, and 9 ft. high?

3. What is the cost of digging a cellar 30 ft. long, 18 ft. wide, and 12 ft. deep, at 32 cents per cubic yard?

4. How much should be paid for a pile of wood 32 ft. long, 4 ft. wide, and 5 ft. high, at $6.50 per cord?

5. What is the weight of a squared piece of timber 28 ft. long, 15 in. thick, and 15 in. wide, at 45 pounds per cubic foot?

6. What is the cost of removing a rectangular embankment of earth 22 ft. 9 in. long, 8 ft. 6 in. wide, and 8 ft. high, at 54¢ per cubic yard?

7. What is the cost of a pile of wood 18 ft. long, 4 ft. wide, and 6 ft. high, at $6.80 per cord?

272. To find any dimension of a rectangular solid.

ILLUSTRATIVE EXAMPLE.

A rectangular volume 7 ft. long and 5 ft. wide contained 140 cu. ft. What was its height?

Explanation.—A rectangular volume 7 ft. long, 5 ft. wide, and 1 ft. high, contains 35 cu. ft. (**271**); hence a rectangular volume 7 ft. long and 5 ft. wide to contain 140 cu. ft. must be as many times 1 ft. high as 140 cu. ft. is times 35 cu. ft., or 4 ft.

Solution.

$$140 \div (7 \times 5) = 4 \text{ ft.}$$

Rule.—*Divide the solidity by the product of any two dimensions; the quotient will be the remaining dimension.*

Note.—If necessary, reduce the solidity to cubic units, and the two given dimensions to linear units of a common denomination, before applying the rule.

18

EXAMPLES FOR PRACTICE.

1. What must be the height of a pile of wood 32 ft. long and 6 ft. wide, to contain 9 cords?

2. A contractor agreed to dig a cellar 18 ft. long and 15 ft. wide at 65¢ per cu. yd., and received therefor $45.50. What was the depth of the cellar?

3. What must be the height of a wagon-body 10 ft. long and 4 ft. wide, that its contents may be 60 cu. ft.?

4. A man excavated a cellar 18 ft. 6 in. long and 15 ft. 3 in. wide, taking out 64 cu. yd. 11 cu. ft. 1332 cu. in. of earth. How deep was the cellar dug?

5. What is the height of a rectangular embankment 18 ft. 9 in. long, 5 ft. 1 in. wide, and containing 28 cu. yd. 6 cu. ft. 864 cu. in.?

6. I bought a pile of wood 24 ft. long and 4 ft. wide for $28.50. If the price per cord was $4.75, how high was the pile?

7. A room 30 ft. long and 25 ft. wide contains 7125 cu. ft. What is its height?

8. If 24 rectangular blocks of stone, each 9 ft. long, 2 ft. wide, and 1 ft. 6 in. thick, be purchased at $5.75 per cu. yd., what will be the total cost?

BOARD MEASURE.

273. **Board Measure** is used in measuring boards or planks, scantling, joists, and other sawed timber. The unit of board measure is the *Board Foot.*

274. A **Board Foot** is 1 ft. long, 1 ft. wide, and 1 in. thick; and hence is 1 sq. ft. of the *face* of a board 1 in. thick. A board foot is one-twelfth of a cubic foot.

Thus, a board 8 ft. long, 2 ft. wide, and 1 in. thick, contains 16 board feet. If the board had been 1½ in. thick, it would have been 1½ times 16, or 24 board feet; if 2 in. thick, 2 times 16, or 32 board feet, etc.

275. The **Average Width** of a regularly tapering board is found by taking *one-half the sum* of the width of its two ends, or by measuring the width of the board midway between its two ends.

Thus, if the width of one end of a tapering board is 12 in. and the width of the other end 8 in., the average width is one-half of 8 + 12, or 10 in.

276. **To find the number of board feet in a board.**

ILLUSTRATIVE EXAMPLE.

How many board feet in a board 20 ft. long, 9 in. wide, and 2 in. thick ?

Solution.—An inch board which is 1 ft. long and 9 in. wide contains ¾ of a board foot; and if 20 ft. long contains 20 times ¾ of a board foot, or 15 board feet. Hence, a 2-in. board must contain 2 times 15, or 30 board feet.

Rule.—I. *Multiply the length in feet by the width in feet. Or,*

II. *Multiply the length in feet by the width in inches, and divide the product by 12.*

NOTE.—If more than 1 in. thick, multiply the result obtained by the rule by the thickness in inches.

EXAMPLES FOR PRACTICE.

Find the number of board feet in a board which is

	Long.	*Wide.*	*Thick.*		*Long.*	*Wide.*	*Thick.*
1.	18 ft.	7 in.	1 in.	6.	20 ft.	10 in.	2½ in.
2.	20 ft.	8 in.	1 in.	7.	18 ft.	1 ft. 2 in.	3 in.
3.	16 ft.	10 in.	1½ in.	8.	16 ft.	1 ft. 3 in.	2 in.
4.	15 ft.	9 in.	2 in.	9.	15 ft.	1 ft. 6 in.	3 in.
5.	18 ft.	11 in.	2 in.	10.	12 ft.	2 ft. 1 in.	1½ in.

11. What is the cost of 15 joists, each 20 ft. long, 9 in. wide, and 3 in. thick, at $2.75 per hundred feet ?

12. What is the cost of seven 2-in. planks, each 18 ft. long, and their widths respectively 5 in., 7 in., 6 in., 9 in., 8 in., 11 in., and 10 in., at $2.50 per hundred feet ?

13. What is the cost of a lot of 1-in. boards 16 ft. long, and the united width of which is 58 ft. 6 in., at $23.50 per thousand feet ?

14. How many board feet in a regularly tapering 2-in. plank 16 ft. long, 11 in. wide at one end and 7 in. wide at the other (**275**) ?

15. What is the cost of 18 2-in. planks, each 16 ft. long, 8 in. wide at one end and 10 in. wide at the other, at $3.40 per hundred feet ?

16. What is the length of a 2-in. plank, which contains 42 board feet and is 18 in. wide ?

Solution.—42÷2=21, board feet if the plank were only 1 in. thick; (21 × 144) ÷18=168 in. or 14 ft., the required length.

17. What is the width of a 1-in. board if it is 18 ft. long and contains 21 board feet ?

18. What is the length of a 3-in. plank which contains 60 board feet and is 16 in. wide ?

19. What must be the width of a 3-in. plank which contains 33¾ board feet and is 9 feet long ?

20. What must be the length of a piece of sawed timber 14 in. wide and 11 in. thick to contain 19¼ cu. ft. ?

Solution.—19¼ × 12=231 board feet (**274**) ; 231÷11=21, board feet if only 1 in. thick ; (21 × 144)÷14=216 in. or 18 ft., length.

21. What is the length of a piece of sawed timber 15 in. wide and 10 in. thick, to contain 8⅓ cu. ft. ?

22. What is the width of a piece of sawed timber 13 ft. long by 8 in. thick and containing 6½ cu. ft. ?

23. What is the thickness of a piece of sawed timber 15 ft. long, 9 in. wide, and containing 7½ cu. ft. ?

Solution.—7½ × 12=90, board feet ; (15 × 9)÷12 = 11¼, board feet if only 1 in. thick (**276**) ; 90÷11¼=8, thickness in inches.

24. What is the thickness of a hewn log 16 ft. long by 13 in. wide and containing 13 cu. ft. ?

25. At $20 per hundred for posts, and $18 per thousand feet for boards, what is the cost of enclosing with a post and board fence a rectangular lot 32 rods long and 24 rods wide ; the posts to be placed 8 ft. apart, the boards to be 16 ft. long, 9 in. wide, and 1 in. thick, and the fence 4 boards high ?

STONE-WORK.

277. Stone-work is usually estimated by perches of 24¾ cu. ft. each, of which 22 cu. ft. is allowed for the stone, and the remaining 2¾ cu. ft. for mortar and filling.

Rem. 1.—In estimating the *quantity of stone* to be used, allowance should be made for doors, windows, and other openings ; but in estimating stone-*work*, no allowance is usually made for openings less than 3 feet wide.

Rem. 2.—The length of walls of stone is found by taking the *girth* or gross outside measurement. This practically allows double measurement of the corners, since the corners are more difficult to build.

278. To find the number of perches of masonry.

ILLUSTRATIVE EXAMPLE.

At $3.85 per perch, what is the cost of erecting the stone walls of a church which is 45 ft. long by 36 ft. wide, the walls being 24 ft. high and 2 ft. 3 in. thick, allowing for 6 windows, each 5 ft. wide by 12 ft. high, and for 2 doors, each 6 ft. wide and 9 ft. high ?

Solution.

$(45+36)\times 2=162$ ft., girth.
$162\times 24\times 2\frac{1}{4}=8748$ cu. ft., gross.
$5\times 12\times 2\frac{1}{4}\times 6=810$ cu. ft., windows.
$9\times 6\times 2\frac{1}{4}\times 2=243$ cu. ft., doors.
$8748-(810\times 243)=7695$ cu. ft., net.
$7695\div 24\frac{3}{4}=310\frac{10}{11}$ perches.
$\$3.85\times 310\frac{10}{11}=\$1197$, cost.

Explanation.—Twice the sum of the length (45 ft.) and the width (36 ft.), or 162 ft., gives the girth or outside measurement of the walls (Rem. 2, **277**). The continued product of the girth (162 ft.) × the height (24 ft.) × the thickness ($2\frac{1}{4}$ ft.), or 8748 cu. ft., gives the gross solidity (**271**), from which should be deducted the solidity of the 6 windows (810 cu. ft.) and of the two doors (243 cu. ft.) to obtain the net solidity (7695 cu. ft.). Divide 7695 by $24\frac{3}{4}$ to obtain the net solidity in perches ($310\frac{10}{11}$). Multiply the price per perch ($3.85) by the number of perches to find the cost.

Rule.—*Find the number of cubic feet of masonry by* **271**, *divide the result by 24.75, and the quotient will denote perches.*

EXAMPLES FOR PRACTICE.

1. What is the cost of the stone foundation of a house which is 30 ft. long and 20 ft. wide, the foundation being 6 ft. high and 2 ft. thick, at $7.30 per perch ?

2. I contracted with a mason to build the stone foundation of a house 36 ft. long by 30 ft. wide, at $3.25 per perch, and paid him $100.80 for the job. If the foundation was 2 ft. thick, what was its height ?

3. At $5.75 per perch, what is the cost of erecting the stone walls of a house 36 ft. long by 24 ft. wide, the walls being 30 ft. high and 2 ft. 6 in. thick, allowing for 8 windows, each 4 ft. wide and 6 ft. high, for 1 door 5 ft. wide and 8 ft. high, and 2 doors, each 4 ft. wide and 7 ft. high ?

4. A sea-wall 396 ft. long and 3 ft. thick cost $2268, at $5.25 per perch. What was the height of the wall ?

BRICK-WORK.

279. Brick-work is usually estimated by the thousand bricks.

Rem. 1.—In measuring brick-work, allowance should be make for doors, windows, or other openings; and for corners since, unlike stone-work, only the *net length* of the walls is taken.

Rem. 2.—The net length of the walls is obtained by deducting four times the thickness of the walls from the gross outside measurement.

Rem. 3.—The dimensions of an ordinary brick are 8 in. × 4 in. × 2 in; of Baltimore and Philadelphia bricks, $8\frac{1}{4}$ in. × $4\frac{1}{8}$ in. × $2\frac{3}{8}$ in.; of Maine bricks, $7\frac{1}{2}$ in. × $3\frac{3}{8}$ in. × $2\frac{3}{8}$ in.; of North River bricks, 8 in. × $3\frac{1}{2}$ in. × $2\frac{1}{4}$ in.; and of Milwaukee bricks, $8\frac{3}{4}$ in. × $4\frac{1}{8}$ in. × $2\frac{3}{8}$ in.

280. To find the number of bricks in a wall.

ILLUSTRATIVE EXAMPLE.

How many ordinary bricks are required for the walls of a house 60 ft. deep, 36 ft. high, and 24 ft. front, if 240 square feet of the surface of the walls are allowed for windows and doors and $\frac{1}{4}$ of an inch for mortar, the walls being 3 bricks thick?

Solution.

$8+\frac{1}{4}=8\frac{1}{4}$ in., length of brick and mortar.
$2+\frac{1}{4}=2\frac{1}{4}$ in., thickness of brick and mortar.
$8\frac{1}{4}\times 2\frac{1}{4}=18\frac{9}{16}$ sq. in., exposed surface per brick.
$144\div 18\frac{9}{16}=7\frac{25}{33}$, exposed bricks per sq. ft. of wall.
$7\frac{25}{33}\times 3=23\frac{9}{11}$, total number of bricks per sq. ft. of wall.
$(60+24)\times 2=168$ ft., outside girth of wall.
$(4\times 3)+\frac{2}{4}=12\frac{1}{2}$ in., thickness of walls, including 2 inner joints.
$12\frac{1}{2}\times 4=50$ in., or $4\frac{1}{6}$ ft., deduction for corners.
$168-4\frac{1}{6}=163\frac{5}{6}$ ft., net length of walls.
$(163\frac{5}{6}\times 36)-240=5658$ sq. ft., net surface of walls.
$5658\times 23\frac{9}{11}=131677\frac{2}{11}$ bricks, the answer.

Rule.—I. *Add $\frac{1}{4}$ of an inch to the length and thickness of the brick, and multiply the two results to find the exposed surface of 1 outside brick.*

II. *Divide 144 by the exposed surface of 1 brick, to find the number of outside bricks per square foot of wall.*

III. *The continued product of the number of outside bricks* *per square foot of wall × the number of bricks in the thickness of the walls × the number of square feet in the surface of the walls, will give the total number of bricks.*

EXAMPLES FOR PRACTICE.

1. The walls of a warehouse built of Baltimore bricks, are 3 bricks thick, 60 ft. high, 50 ft. front, and 200 ft. deep. How many bricks does the warehouse contain, if 675 square feet are allowed for doors and windows.

2. How many Milwaukee bricks are required for the walls of a building 40 ft. front, 75 ft. deep, 50 ft. high, and 3 bricks thick, allowing for 20 windows, each 6½ ft. high and 4 ft. wide, and for 4 doors, each 8 feet high and 3¾ feet wide?

3. How many North River bricks are required to build a house 40 ft. front, 75 ft. deep, 60 ft. high, 3 bricks thick, and with gables 12 ft. high at the front and rear, allowing 1250 sq. ft. for openings?

Rem.—The surface of a gable is found by multiplying the length of the wall upon which the gable is built by *one-half* the height of the gable above the wall.

MEASUREMENT OF CAPACITIES.

281. Measurement of Capacities, as here treated, will embrace the measurement of granaries, bins, boxes, cisterns, tanks, etc.

Rem. 1.—*Stricken Measure* means the measuring vessel even full, and stricken off by a rule or striker. Grains, seeds, berries, etc., are sold by stricken measure. The standard bushel when stricken contains 2150.42 cubic inches.

Rem. 2.—*Heaped Measure* means the contents of the measuring vessel heaped in the form of a cone. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles are sold by heaped measure. The standard bushel when heaped contains 2747.71+ cubic inches. It is customary to allow 5 stricken bushels for 4 heaped ones.

Rem. 3.—The gallon, dry measure (½ pk.), contains 268.6 cu. in., and the gallon, liquid measure, 231 cu. in.

282. To find the capacity of a bin in bushels.

ILLUSTRATIVE EXAMPLE.

1. A rectangular bin is 8 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels of rye will it hold?

Solution.—8 × 6 × 4 = 192 cu. ft.; 1728 × 192 = 331776, the contents of the bin in cubic inches. 331776 ÷ 2150.42 = 154.284+ stricken bushels.

Rule.—*Divide the contents of the bin in cubic inches by 2150.42 to find bushels stricken measure; or by 2747.71 to find bushels heaped measure.*

NOTE.—A sufficiently close approximation for practical purposes may be obtained by multiplying the contents of the bin in cubic feet by .8 to find stricken bushels, or by .63 to find heaped bushels.

EXAMPLES FOR PRACTICE.

1. How many bushels of wheat will exactly fill a rectangular bin 9 ft. 6 in. long, 4 ft. 8 in. wide, and 3 ft. 9 in. high?

2. A rectangular bin is 10 ft. long, 5 ft. wide, and 3 ft. 6 in. deep. How many bushels of turnips will it hold?

3. How many bushels of apples can be stored in a rectangular box 4 ft. long, 3 ft. wide, and 2 ft. 6 in. deep?

4. A wagon-body is 10 ft. long, 4 ft. wide, and 18 in. deep. How many bushels of shelled corn will it hold?

5. How many gallons of water will fill a rectangular cistern 8 ft. 6 in. long, 8 ft. 6 in. wide, and 7 ft. deep?

Capacity in cubic inches ÷ 231 = capacity in liquid gallons.

6. How many bushels of oats will fill a vessel which is capable of holding exactly 325 gallons of water?

7. How many hogsheads of water will fill a rectangular tank 25 ft. long, 18 ft. wide, and 16 ft. 9 in. deep?

8. How many gallons of water can be put into a vessel which will hold exactly 18 bushels of wheat?

9. What must be the width of a wagon-body 9 ft. long and 20 in. deep, to contain 40 bushels of potatoes?

Solution.—2747.71 × 40 = 109908.4 cu. in.; 109908.4 ÷ (108 × 20) = $50.88\frac{19}{54}$ in. = 4 ft. $2.88\frac{19}{54}$ in., required width.

10. What must be the depth of a rectangular cistern 8 ft. 6 in. long and 7 ft. 9 in. wide, to contain 1800 gallons of water?

11. What must be the length of a wagon-body 4 ft. wide and 18 in. deep, to hold 50 bushels of shelled corn?

12. A grocer bought a barrel of molasses containing 36 gallons at 45 cents per gallon, and retailed it at 50 cents per gallon, using by mistake a set of dry measures. What was his net loss?

13. Allowing 80 lb. to the bushel, what is the weight of coal which will exactly fill a rectangular bin 6 ft. long, 5 ft. wide, and 4 ft. deep?

14. Allowing 28 bu. to the ton of 2240 lb., how deep must be a rectangular bin 18 ft. long and 5 ft. wide, to hold 20160 lb. coal?

(In the following, apply Note, **282**, for obtaining approximate results.)

15. A rectangular crib the inside dimensions of which are 10 ft. in length, 8 ft. in width, and 6 ft. in height, is two-thirds full of potatoes. What is the value of the potatoes at 75 cents per bushel?

16. A merchant has a rectangular bin 12 ft. long, 5 ft. wide, and 4 ft. high, filled with shelled corn, which he retailed at 85 cents per bushel. How much did he receive for the corn?

17. A rectangular bin 12 ft. long and 10 ft. wide will hold exactly 378 bushels of apples. What is the depth of the bin?

When the crib or bin is flared at the sides, the average width is found by taking *one-half the sum of the top and bottom widths* (**275**).

Thus, the contents of a flaring crib 16 ft. long, 10 ft. high, 9 ft. wide at the bottom, and 13 ft. wide at the top, are $16 \times 10 \times \frac{1}{2}$ of $\overline{9+13}$, or 1760 cu. ft.

18. A bin is 8 ft. long, 5 ft. deep, 3 ft. wide at the bottom, and 5 ft. wide at the top. How many bushels of rye will it hold?

283. A cylindrical vessel is one whose sides are *uniformly* round and whose ends are equal and parallel circles.

Cylindrical Vessel.

Rem.—It can be proved by geometry that the contents of a cylindrical vessel are .7854 of the contents of a rectangular vessel which has two of its dimensions equal to the diameter of the cylindrical vessel.

19. A cylindrical vessel is 9 ft. deep and 3 ft. in diameter. How many gallons of water can it hold?

Solution.—$3 \times 3 \times 9 = 81$ cu. ft., contents of a rectangular cistern with two dimensions equal to the given diameter. $81 \times .7854 = 63.6174$ cu. ft., contents of cylindrical vessel. $(63.6174 \times 1728) \div 231 = 475.8912$ gallons.

20. How many gallons of water are in a well 5 ft. in diameter if the water is 7 ft. deep?

21. A cylindrical cistern is 8 ft. deep and 5 ft. in diameter. How many gallons of water will it hold when full?

22. A circular reservoir is 90 ft. in diameter and 15 ft. deep. How many gallons of water will it hold when full?

23. A cylindrical vessel 8 ft. high and 5 ft. in diameter is filled with potatoes. What is the value of the potatoes at 75¢ per bushel?

PERCENTAGE.

284. **Percentage**, as a general term, means any computation *by hundredths.*

The phrase *per cent.*, or its symbol (%), signifies *by the hundred.* Thus, 8% of $900 means $8 of every $100 in $900.

285. The **Base** is the quantity of which a number of hundredths is given or required.

286. The **Rate per cent.** denotes *how many* hundredths of the base are considered.

When the phrase *per cent.* is not added, Rate means *any* part of the base. Thus, ¾, .035, and .05 are all rates; the first being a common fractional rate in *fourths*, the second a decimal rate in *thousandths*, but only the last a decimal rate in *hundredths*, or a rate per cent.

287. The **Percentage** denotes the *value* of a certain number of hundredths of the base.

Rem. 1.—Problems in percentage differ in no respect from other problems involving decimal or common fractions. Hence, **129** and **130** are equally applicable to percentage. The only difference is a change in the name of the terms: the weight, measure, value, etc., of the whole quantity being now called the *Base;* the weight, measure, value, etc., of any part of the whole quantity being now called the *Percentage;* and any considered fractional part of the whole quantity being now called the *Rate.*

Rem. 2.—The Base, like its equivalent in common fractions (b, **129**), when not self-evident, may be identified by reference to the rate per cent. which will then be followed by *of*, *more than*, or *less than*, introducing the base (**132**).

Rem. 3.—After the base has been identified by Rem. 2, any concrete quantity which is more or less than the base, that is, any proper or improper fractional part of such base, may be regarded as a percentage.

Rem. 4.—To distinguish a percentage if formed by adding another percentage to the base, it is called the *Amount;* and if formed by subtracting another percentage from the base, it is called the *Difference.*

288. The Percentage corresponds with the product; and its two factors are the Base and the Rate of that percentage. Hence, if a percentage is required, multiply its two given or obtainable factors; and if a percentage is given, divide it by its given factor and the quotient will be its remaining factor.

General Direction: *Employ multiplication when the percentage is required, and division when the percentage is given.*

289. When the rate per cent. is equivalent to a more convenient common fractional rate, it is better to change it to a common fraction.

TABLE.

Rate %. Com. Frac.	*Rate %. Com. Frac.*	*Rate %. Com. Frac.*	*Rate %. Com. Frac.*
$2\frac{1}{2}\% = \frac{1}{40}$.	$10\% = \frac{1}{10}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$62\frac{1}{2}\% = \frac{5}{8}$.
$5\% = \frac{1}{20}$.	$12\frac{1}{2}\% = \frac{1}{8}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$66\frac{2}{3}\% = \frac{2}{3}$.
$6\frac{1}{4}\% = \frac{1}{16}$.	$16\frac{2}{3}\% = \frac{1}{6}$.	$40\% = \frac{2}{5}$.	$75\% = \frac{3}{4}$.
$8\frac{1}{3}\% = \frac{1}{12}$.	$20\% = \frac{1}{5}$.	$50\% = \frac{1}{2}$.	$80\% = \frac{4}{5}$.
$6\frac{2}{3}\% = \frac{1}{15}$.	$25\% = \frac{1}{4}$.	$60\% = \frac{3}{5}$.	$87\frac{1}{2}\% = \frac{7}{8}$.

290. The base and rate being given, to find the percentage.

ILLUSTRATIVE EXAMPLES.

1. What is 9% of $37.48?

Explanation.—*Of* follows the rate (9%) and precedes the base ($37.48). Hence, the base and rate (factors of the required percentage) are given; and the multiplication of its two factors will produce the required percentage.

Solution.

$37.48
.09
$3.3732

Ans., $3.37+

2. A drover bought 375 head of cattle and afterwards sold 60% of his purchase. How many cattle had he remaining?

Explanation. — *Of* follows the rate (60%) and precedes the base (his purchase). Hence, the purchase (375 cattle) is one factor of the required percentage.

Subtract the rate of the sale (60%) from the rate of the purchase (100%), to find the rate of the remaining cattle (40%), the other factor of the required percentage.

Multiply its two factors (375 × .40), obtaining 150 head of cattle as the required *percentage.*

Solution.

100%, rate of purchase.
60 " " " sale.
40 " " " remainder.
375 cattle × .40 = 150 cattle.

3. A man paid \$800 for a lot and 25% more than that sum for a house. What did he pay for both?

Explanation.—*More than* follows the rate (25%) and precedes the base (that sum, meaning \$800). Hence, \$800 is the base, or one factor of the required percentage.

Increase the rate of the lot (100%) by the rate of the house (25% more than the rate of the lot, or 125%), obtaining 225% or $\frac{9}{4}$ as the rate of both, the other factor of the required percentage.

Multiply its two factors (\$800 × $\frac{9}{4}$), obtaining \$1800 as the required percentage.

Solution.

100%, rate of the lot.
125 " " " " house.

225%, or $\frac{9}{4}$, rate of both.
\$800 × $\frac{9}{4}$ = \$1800, *Ans.*

Rule.—*Multiply the base by the rate % of any required percentage, and the product will be that percentage.*

NOTE.—All rates % denote hundredths of some quantity taken as the base; hence, as $\frac{100}{100}$ of any base equal the whole of that base, the rate % of all bases must be uniformly 100%; and the rate % of any required percentage must be so much of the rate of the base (such a % of the base) as equals that percentage.

EXAMPLES FOR PRACTICE.

1. Find 5% of \$3.75. 2. 45% of 786 pounds. 3. 34% of \$3127.85. 4. 33⅓% of 948 men. 5. 75% of 3184 feet. 6. ¼% of 364 miles. 7. 40% of £325 17s. 6d. 8. 72% of 839.75 marks. 9. 125% of \$7134.56.

10. What is the amount of 3488 ft. plus 12½% of itself? 11. Of 4371 bu. plus 83⅓% of itself? 12. Of \$575 plus 20% of itself?

13. What is the difference of \$5975 minus 10% of itself? 14. Of 856 francs minus 37½% of itself? 15. Of 3895 tons minus 80% of itself?

16. A gentleman is worth \$75000, 35% of which is invested in real estate. What is the value of his real estate?

17. A speculator invested \$35400 in stocks and lost 16⅔% of his investment? How much of his investment had he remaining?

18. A merchant bought goods for \$587.35, and sold them at an advance of 40% of what he paid for them. How much did he receive for the goods?

19. A grazier owned 580 sheep, but lost 15% of his flock in a snow storm. How many sheep had he remaining?

20. A merchant retired from business with a fortune of \$82400, invested 37½% of it in government bonds, 20% of it in manufacturing

stock, 15% of it in a farm, and deposited the remainder in bank. What was the amount of his deposit?

21. A farm contains 350 acres of land, 15% of it having been bought at $28 per acre, 25% at $35 per acre, 18% at $37 per acre, and the remainder at $40 per acre. What was the cost of the farm?

22. In a certain school 420 pupils are enrolled, 45% of the enrollment being boys. How many girls are enrolled?

23. A, B, and C are partners, A having invested $5200, B 25% more than A, and C 15% less than B. What is the capital of the firm?

24. A man deposited $18500 in bank, and afterwards withdrew ¾% of his deposit to buy a watch. What was the cost of the watch?

25. A merchant has on deposit $18750, 25% of which is coin, and the remainder in bank notes. How much in notes has he on deposit?

26. A farmer sold 530 bu. wheat at $1.18 per bushel, and 20% more oats than wheat at 70¢ per bushel. How much did he receive for both?

27. A grocer sells 28% more granulated sugar than loaf sugar. If his yearly sales of loaf sugar amount to 38275 pounds, what are his monthly sales of granulated sugar?

28. A merchant bought 780 yards of goods, sold 20% of the purchase at one sale, and 25% of the remainder at a second sale. How many yards remained unsold?

29. A music dealer paid $500 for each of 3 pianos, sold 2 of them at a profit of 30% and 40% of their respective cost, and the third at a loss of 25% of its cost. What was his net profit upon the three pianos?

291. The base and percentage being given, to find the rate.

ILLUSTRATIVE EXAMPLES.

1. What per cent. of $84 are $12.60?

Explanation.—*Of* follows the phrase per cent. and precedes $84. Hence, $84 is the base and one factor of the percentage. Since $12.60 is not the base, it must be a given percentage (Rem. 3, **287**).

Solution.

$12.60 ÷ $84 = .15 = 15%

Employing **288**, divide the given percentage ($12.60) by its given factor (the base, $84), and carry the quotient to hundredths, obtaining .15 or 15% as its other factor, or the required rate %.

2. A man's income is $875 per annum and his expenses $1050. What per cent. more than his income does he spend?

291

Explanation.—*More than* follows the phrase per cent. and precedes the base (his income). Hence, his income, $875, is the base and one factor of the percentage.

Solution.

$1050 —$875 = $175

175.00 ÷ 875 = .20 = 20%

Subtract his income ($875) from his expenses ($1050) to find *how much* more than his income are his expenses, obtaining $175 as the percentage of which it is required to find the rate.

Divide the percentage ($175) by its given factor (the base, $875), obtaining 20% as the remaining factor, or the required rate %.

Rule.—*Divide any percentage by its base, and the quotient carried to hundredths will be the rate % of that percentage.*

NOTE.—To obtain a quotient in hundredths, the dividend must contain exactly two decimal places more than the divisor.

EXAMPLES FOR PRACTICE.

1. What % of $75 are $25.25? 2. Are $455 of $875? 3. Of 2 years are 146 days? 4. Are 58 pounds of 725 pounds? 5. Are $800 of $500?

6. What % more than $750 are $800? 7. Are 660 gallons than 528 gallons? 8. Are $19.53 than $18.60? 9. Are 2848 pounds than 2136 pounds?

10. What % less than 750 bushels are 615 bushels? 11. Are $227.70 than $345? 12. Are 60 cents than $1.50? 13. Are 690 gallons than 828 gallons?

14. A collector charged $25.92 for collecting a bill of $324. What per cent. of the sum collected did he charge for his services?

15. A man's income is $840 per annum, and his expenses $560. What per cent. of his income does he save?

16. A and B engage in partnership, A investing $5976 and B $6474. What % of the capital of the firm did each invest?

17. A farm contained 42 acres in wheat, 75 acres in corn, 58 acres in oats, 65 acres in grass, 124 acres in woodland, and the remaining 11 acres in orchard. What % of the farm was in corn?

18. A grocer sold 175 pounds of coffee and had 1075 pounds remaining. What per cent. of his stock of coffee did he sell?

19. Last year a grocer sold 42580 pounds of sugar, and this year 68128 pounds. What % more did he sell this year than last year? What % less did he sell last year than this year?

20. If 7 gallons of water are added to 168 gallons of alcohol, what per cent. of the mixture is alcohol?

21. A merchant retired from business with a capital of $35825, invested $12180.50 of it in real estate and the remainder in U. S. bonds. What per cent. of his capital was invested in bonds?

22. A merchant commenced business with a capital of $8375, and at the expiration of the first year found that his capital was $11725. What was his per cent. of increase?

In problems involving increase or decrease, the quantity increased or diminished is the normal base. When bases are thus self-evident, Rem. 2, **287**, does not apply.

23. Immediately before a battle, the garrison of a fort numbered 775 men, and after the battle 713 men. What per cent. was lost?

24. Since 1880, a town has increased 5850 in population and now contains 38350 inhabitants. What is the % of increase since 1880?

25. The bread made from 720 pounds of flour weighs 972 pounds. What per cent. more than the flour does the bread weigh?

26. A tank contains 175 gallons of water, 50 gallons having previously leaked out. What % of what the tank originally held was lost by leakage?

27. A man's real estate is worth $7500 and his personal property $8200. If the former increase in value 18% and the latter 5%, what will be the per cent. of increase in the value of both?

28. An importer paid £243 15s. for an invoice of goods, and £73 2s. 6d. for freight, custom-house charges, etc. What per cent. of the invoice price were the charges?

292. The percentage and rate being given, to find the base.

ILLUSTRATIVE EXAMPLES.

1. An inventor sold 35% of his patent-right for $3065.30. At that rate, what was the estimated value of his whole patent-right?

Explanation.—*Of* follows the rate per cent. and precedes the base (his patent-right). Hence, as $3065.30 is the value of only a part of his patent-right, it is not the base and must be a percentage.

Solution.

$3065.30 ÷ .35 = $8758

Divide the given percentage ($3065.30, the value of the part sold) by its given factor (35%, the rate of the part sold), obtaining $8758 as its other factor (the base, or value of the whole patent-right).

292

2. A is worth $13021 or 45% more than B. How much is B worth?

More than follows the rate and precedes the base (B's worth). Hence, as $13021 is not B's worth, it must be a percentage.

Since $13021 is 45% more than the base (B's worth), the rate of $13021 must be 45% more than the rate of the base, or 145%. Hence, divide the given percentage ($13021, A's worth) by one of its factors (145%, the rate of A's worth), obtaining $8980 as its other factor (the base, or B's worth).

Solution.

100% × 45% = 145% = 1.45

$13021.00 ÷ 1.45 = $8980

Rule.—*Divide any percentage by its rate, and the quotient will be its base.*

NOTE.—The rate % of any percentage is as much greater or less than the rate of the base (100%), as such percentage is greater or less than the base.

EXAMPLES FOR PRACTICE.

1. Of what quantity are 360 lb. 45%? 2. Are 210 gal. 33⅓%? 3. Are 250 mi. 20%? 4. Are 365 T. 73%? 5. Are $1314.10 34%?

6. What quantity increased by 25% of itself equals $8915? 7. Increased by 31% of itself equals 19.65 francs? 8. Increased by 62% of itself equals $1333.26?

9. What quantity diminished by 25% of itself equals 225 T.? 10. Diminished by 40% of itself equals $4800? 11. Diminished by 28% of itself equals 2466 bu.?

12. A merchant sold 14% of his stock of prints. How many yards of prints had he in stock if the quantity sold was 1260 yards?

13. If 24% of a contract can be executed in 54 days, in what time can the whole contract be executed?

14. A farmer lost 12% of his flock of sheep and had 264 remaining. How many sheep did he lose?

15. A merchant withdrew $2058 from bank, which was 28% of his deposit. How much had he remaining in bank?

16. A's orchard contains 910 fruit trees, which is 4% more than the number in B's orchard. How many fruit trees in B's orchard?

17. A owns 15% of a ship; B, 25%; C, 28%; and D, the remainder. What is the value of A's share of the ship if D's is worth $17232?

18. A farmer bought 50 acres of land, which was 20% of what he previously owned. How many acres does he now own?

19. A merchant sold 35% of his stock and had $8027.50 worth of goods remaining. What was the value of the goods sold?

20. A dealer bought 325 bushels of corn and then had 13% more than before. How many bushels did he have after the purchase?

21. A man paid $25 for a suit of clothes, $22 for an overcoat, $7.66 for books, $120 for a watch, and had 18% of his money remaining. How much money had he remaining?

22. A grocer sold 20% of an invoice of sugar at one sale, 25% of the invoice at a second sale, 60% of what was left of the invoice at a third sale, and then had 473 pounds remaining. How many pounds did the invoice contain?

23. A's farm contains 375 acres, and 20% of his farm equals 15% of B's. How many acres are in B's farm?

24. A and B unite in buying a house, A investing $2964 or 38% of the purchase money. How much did B invest?

25. A debtor paid 35% of a creditor's claim, and still owed him $5538. What was the sum paid?

26. I sold a bill of merchandise, received $805 cash, and took the buyer's note for the remainder, which was 12½% of the bill. What was the face of the note?

27. A owned 40% of a mill and sold 25% of his share for $3575.25. At that rate, what was the value of B's share, who owned 45% of the mill?

28. A quantity of soap lost 6% of its weight by desiccation and then weighed 1739 pounds. What did it weigh at first?

293. REVIEW OF PERCENTAGE.

$$\left.\begin{array}{l}\textit{Base}\\ \textit{Rate}\end{array}\right\}\ \textit{are the two factors of a Percentage.}$$

$$\textit{Percentage}\ \left\{\begin{array}{l}\textit{if given, divide it by its given factor.}\\ \textit{if required, multiply its two given factors.}\end{array}\right.$$

1. The assets of a bank are $292312.50 and its liabilities $389750. What per cent. of its indebtedness can it pay?

2. In a city of 52340 inhabitants, 25% were Germans, 30% Irish, and 5% French. What number of each did the city contain?

3. A, B, and C are partners. A's investment is $19941, or 15% more than B's, and 25% less than C's. What is the capital of the firm?

4. A has $500, and 18% of his money equals 5% of B's. How much more money has B than A.?

5. A quantity of gunpowder is compounded of 6255 pounds saltpetre, 834 pounds sulphur, and 1251 pounds charcoal. What per cent. of each does the gunpowder contain?

6. A man paid $375 for a pair of horses and 18% more than that sum for a carriage. What was the total cost?

7. A farmer harvested 375 bushels of wheat from a field containing 25 acres. If the wheat had been sown at the rate of 1 bu. 3 pk. per acre, what was the per cent. of yield? The per cent. of increase?

8. A bankrupt owed $12840, and agreed to liquidate the debt in equal annual installments of 16⅔%. How much did he owe after paying the fourth installment?

9. A invested $32500 in business and at the end of the first year found that he had gained $5200, which he withdrew, and at the end of the second year that he had lost $2925 during that year. What was his per cent. of net gain for the two years?

10. A's taxes for 1886 amounted to $386, subject to a discount of 5% if paid before Nov. 1st of that year; to a discount of 3% if paid between Nov. 1st and Dec. 1st; and to an additional charge of 5% if paid on or after Jan. 1st, 1887. What was the net amount of his taxes if paid Oct. 30th, 1886? If paid Nov. 18th, 1886? If paid Dec. 24th, 1886? If paid Jan. 15th, 1887?

11. On Dec. 18, 1885, a merchant bought a bill of goods amounting to $736.80 on 4 months' credit, or 5% off if paid within 30 days of date of sale. What sum will pay the bill on Jan. 7, 1886?

12. If coffee loses 8% of its weight in roasting, how much green coffee will be required to produce 828 pounds when roasted?

13. A merchant lost $5000 of his capital and had $25000 remaining. What per cent. of his capital did he lose?

14. By purchasing improved machinery, a miller increased the capacity of his mill 25%, or 340 barrels of flour per month. What is the present capacity of the mill?

15. 18% of a man's wealth is in real estate, 24% in bank stock, 26% in railroad bonds, and the remainder in money. What is the value of his bank stock, if his money amounts to $10288?

16. A gentleman paid $1701 in settlement of an account, and afterwards discovered that he had paid 26% more than the correct sum. What was the amount of overcharge?

17. A man withdrew 30% of his bank deposit, and invested 12% of what he withdrew in purchasing an organ for $180. How much had he remaining in bank?

18. A bought a horse and buggy for $570, paying 28% more for the horse than for the buggy. What did he pay for each?

The cost of the buggy is the base (Rem. 2, **287**). As the given percentage ($570) is the cost of the buggy plus the cost of the horse, its rate must be the rate of cost of the buggy (100%) plus the rate of cost of the horse (128%), or 228%.

19. A lady spent $64.50 for jewelry and dress goods, paying 15% more for jewelry than for dress goods. How much did she pay for each?

20. In the manufacture of a certain quality of cloth, 1122 pounds of cotton and wool were mixed, the weight of the cotton being 30% of the weight of the wool. How many pounds of each were used?

21. A and B together have $1680, and A has 25% less money than B. How much money has A?

22. A firm exported 8494 barrels of flour in three shipments. Their second shipment was 25% greater than their first, and their third 32% less than their second. How many barrels were exported in each shipment?

23. A dealer purchased a quantity of oysters, fish, and clams, paying $166.75 for the entire quantity. The cost of the fish was 45% of that of the oysters, and the cost of the clams 25% of that of the oysters and fish together. What did he pay for each?

24. The sales of a firm were increased 25% during the second year, 20% the third year, and 16⅔% the fourth year. What was the amount of sales during the first year, if the fourth year's sales were $60248?

25. The charge for a certain class of "through freight" between two cities, 400 miles distant from each other, was 95 cents per 100 pounds, and the quantity transported 8600 pounds. The freight was divided *pro rata* between three railroad companies whose mileages were respectively 120, 180, and 100 miles. What per cent. of the total freight should each company have received? What sum?

26. The property of a testator was divided among his wife, son, and daughter. His wife received 35% of the property, and the remainder, $16250, was divided between the son and daughter, the daughter receiving 33⅓% less than the son. How much did each receive?

27. The salary of a clerk is $560 per annum, of which he pays $192 per annum for board, $85 for clothes, $59 for incidentals, and the remainder he deposits in bank. What per cent. of his salary does he deposit in bank?

28. A herder has charge of 1140 sheep and goats, the number of sheep being 40% greater than the number of goats. How many sheep were in charge of the herder?

29. The sum paid for two farms is $9310. If 80% of the sum paid for one farm equals 120% of the sum paid for the other, how much was paid for each?

30. The population of a certain city decreased 10% in 1882, and 8% in 1883. If the population on Jan. 1, 1884, was 26910, what was the population on Jan. 1, 1882?

31. In manufacturing a certain quality of blankets 2380 pounds of cotton and wool were used, the quantity of wool being 140% greater than that of the cotton. What was the weight of the wool?

32. The foreign-born inhabitants of a certain city were 30% of the native-born. What per cent. of the total population was each?

33. A has 15% less money than B. What per cent. more than A's money is B's?

34. If A has 25% more money than B, what % of A's money is B's?

35. I sold tea and sugar at a total gain of $7.54, the gain on sugar being 45% of the gain on tea. What was my gain on each?

36. A bought 75 barrels of flour, and B increased his stock of flour 28% by buying 16% less than A. How many barrels did B have after his purchase?

37. An army lost 4077 men in a certain battle, and after the battle numbered 41223 men. What per cent. of the army was lost?

38. Two railroads whose lengths were respectively 175 and 125 miles transported 7500 pounds of merchandise at the through rate of 45 cents per 100 pounds. What sum did each company receive?

39. A merchant found that his sales in 1881 were 20% greater than his sales in 1880; in 1882, 30% greater than in 1881; and in 1883, 25% greater than in 1882. If his sales in 1883 were $3600 greater than his sales in 1881, what were his sales in 1882?

40. Two R. R. companies transported 5318 lb. freight at the through rate of 35¢ per cwt. If one company received 45% of the through rate, how much should the other receive?

41. Considering each of the following columns as a separate question, if the terms marked √ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Base....	√	?	√	√	√	?	?	..	..	?	?	..	..
Rate of percentage	√	√	?	√	√	..	..	?	?	√	√	√	√
Percentage.......	?	√	√	..	..	√	√	√	√	..	..	?	?
Amount	..	..	..	?	..	√	..	√	..	√	..	√	..
Difference........	..	..	..	..	?	..	√	..	√	..	√	..	√

PROFIT AND LOSS.

294. **Profit** and **Loss** are commercial terms which denote the gain or loss in business transactions.

REM. 1.—The *gross cost* of merchandise is its prime cost plus all expenses incidental to the purchase, such as freight, commission, transit insurance, drayage, etc.; and its *net selling price* is the gross selling price minus all discounts allowed.

REM. 2.—In estimating the profit or loss on merchandise, its *gross cost* is taken as the *base* or standard of comparison. If its net selling price is greater than its gross cost, there is a profit; if less, a loss; and the amount of profit or loss will depend upon the difference between its gross cost and net selling price.

295. Computations in Profit and Loss are made in accordance with the principles of percentage, the cost being regarded as the base; and the profit, loss, and selling price as percentages. Hence, **288** may be modified into the following

General Direction: *If a profit, loss, or selling price is required, multiply its two given factors; and if a profit, loss, or selling price is given, divide it by its given factor and the quotient will be its remaining factor.*

REM. 1.—The two factors of a given or required profit, are the *cost* and the *rate of profit;* of a given or required loss are the *cost* and the *rate of loss;* of a given or required selling price are the *cost* and the *rate of selling price.*

REM. 2.—As the selling price equals the cost plus the profit or minus the loss, so the rate of selling price must equal the rate of cost (100%) plus the rate % of profit or minus the rate % of loss.

EXAMPLES FOR PRACTICE.

1. What is the gain on goods which cost $5876.40 and are sold at 15% profit? 2. Which cost $375.75 and are sold at 8% profit? 3. Which cost $735 and are sold at 26% profit? 4. Which cost $17.25 and are sold at 40% profit?

Cost × rate of profit = profit.

5. What is the loss on goods which cost $150 and are sold at 25% loss? 6. Which cost $5896.26 and sold at 33⅓% loss? 7. Which cost $175 and are sold at 20% loss? 8. Which cost $715.32 and are sold at 12½% loss?

Cost × rate of loss = loss.

9. At what price must goods be sold to gain 15%, if they cost $3175.34? 10. To gain 25%, if they cost $153.84? 11. To gain 10%, if they cost $918.10? 12. To gain 12½%, if they cost $1483.20?

Cost + (cost × rate of gain) = selling price.
Cost × (100% + rate % of gain) = selling price.

13. At what price must goods be sold to lose 12%, if they cost $43.50? 14. To lose 20%, if they cost $315? 15. To lose 6¼%, if they cost $1204.80?

Cost − (cost × rate of loss) = selling price.
Cost × (100% − rate % of loss) = selling price.

16. What is the % of gain on goods which cost $3745 and are sold at a profit of $749? 17. Which cost $5136 and are sold at a profit of $1712? 18. Which cost $8934 and are sold at a profit of $2233.50?

Gain ÷ cost = rate of gain.

19. What is the % of loss on goods which cost $7134 and are sold at a loss of $11 89? 20. Which cost $5824 and are sold at a loss of $1.82? 21. Which cost $7832 and are sold at a loss of $48.95?

Loss ÷ cost = rate of loss.

22. What is the % of gain or loss on goods which cost $5172 and are sold for $5301.30? 23. Which cost $3072 and are sold for $2560? 24. Which cost $850 and are sold for $765? 25. Which cost $8292 and are sold for $11056? 26. Which cost $24 and are sold for $23.97?

(Selling price − cost) ÷ cost = rate of gain.
(Cost − selling price) ÷ cost = rate of loss.

27. What is the cost of goods which are sold at $1346.40 profit, if

the rate of gain is 15%? **28.** Which are sold at $75 profit, if the rate of gain is 20%? **29.** Which are sold at $33 profit, if the rate of gain is 4%?

Gain ÷ rate of gain = cost.

30. What is the cost of goods which are sold at a loss of $129, if the rate of loss is 33⅓%? **31.** Which are sold at a loss of $51.75, if the rate of loss is ¼%? **32.** Which are sold at a loss of $885, if the rate of loss is 15%?

Loss ÷ rate of loss = cost.

33. What is the cost, if goods are sold for $406.25, or at a gain of 25%? **34.** If sold for $73.85, or at a gain of 16⅔%? **35.** If sold for $81.42, or at a loss of 40%? **36.** If sold for $7583.40, or at a gain of 10%? **37.** If sold for $38.24, or at a loss of 20%?

Selling price ÷ 100% + rate of gain = cost.
Selling price ÷ 100% − rate of loss = cost.

38. A farmer bought 35 acres of land for $1750 and sold it at 20% profit. What was his gain per acre?

39. I bought a house for $5600, paid $400 additional for repairs, and then sold it at 12% profit. What was my gain?

40. What is the per cent. of profit on a piano which cost $350 and is sold for $420?

41. A bill of goods was sold for $13 less than cost, by which a loss of 5% was sustained. What was the cost of the goods?

42. What is the loss on 153 tons of iron bought at $45 per ton and sold at 20% below cost?

43. What is the rate per cent. of profit on merchandise which cost $573 and is sold for $647.49?

44. A dealer sold a quantity of lumber for $3172.50 and lost 37½%. What did he pay for the lumber?

45. A merchant bought 80 bushels of oats at 56 cents per bushel, paid $1.40 for sundry expenses, and sold out at 15% profit. How much did he receive for the oats?

46. A stationer bought 9 reams of paper at $2.40 per ream, and retailed it at 1 cent per sheet. What was his per cent. of gain?

47. A quantity of wheat which cost $1.14 per bushel was sold at a loss of 20%. How many bushels were sold if the total loss was $30.78?

48. A set of jewelry was sold for $126.30. What was the cost if the seller gained 25%?

49. A speculator bought three houses at $1800 each, and sold them at a profit respectively of 8%, 12%, and 15%. What did he receive for the three houses ?

50. A grain dealer bought 735 bu. of wheat at $1.14 per bushel, and sold the entire quantity for $977.55. What was his per cent. of gain ?

51. A grocer bought 174 gallons of molasses at 55 cents per gallon, paid 87 cents drayage to get it to his store, and retailed it at a profit of 33⅓%. What was his selling price per gallon ?

52. A drover bought 15 horses at $125 per head, sold 2 of them at $127.75 per head, 8 at $140 per head, and the remainder at $150 per head. If his expenses in taking them to market amounted to $5 per head, what was his per cent. of gain ?

53. A man sold a house for $9775 and gained $1275. What was his rate per cent. of profit ?

54. A merchant lost $9.96 by selling an article for $73.04. What was his rate per cent. of loss ?

55. A music dealer sold a piano for $413 and gained 18%. What was his profit ?

56. A merchant sold a quantity of rye at 15% profit and gained $26.70. What did he receive for the rye ?

57. I bought two houses for $11700, paying 25% more for one than for the other. I sold the cheaper house at 20% profit and the higher priced at 16⅔% profit. What was my total gain ?

58. Oats purchased at 45⅝ cents per bushel were sold for 54¾ cents per bushel. What was the rate per cent. of gain ?

59. A grocer sold a quantity of sugar for $330 and thereby lost 12%. How much did he lose ?

60. A gentleman gained 24% by selling land for $195.60 more than he paid for it. What did he obtain for the land ?

61. A jeweler purchased a pound of gold at $16.58⅓ per ounce, added 2 ounces, Troy, of alloy at 50 cents per ounce, and converted the whole into rings which he sold at $1 per pennyweight. What was his per cent. of gain ?

62. What did I receive for 3872 pounds of sugar bought at 5¾ cents per pound, and sold at 16⅔% profit ?

63. A grocer bought 536 gallons of vinegar for $150.08, sold 175 gallons at 30 cents per gallon, 124 gallons at 35 cents per gallon, and the remainder for $91.70. What was his rate per cent. of gain ?

64. A farmer bought two horses at $175 each, sold one at a profit of 20%, and the other at a loss of 5%. What was his net gain?

65. A music dealer sold a violin for $35.70, and lost $15.30. What was his rate per cent. of loss?

66. A bought a farm from B for $4550, which was 30% less than B paid for it, and sold it for 10% more than B paid for it. What were A's gain and B's loss?

67. A man sold two lots for $1800 each, gaining 25% on one lot and losing 25% on the other. Did he gain or lose, and how much?

68. A grocer sold a quantity of butter for $22.85, and gained $4.57. What was his rate per cent. of gain?

69. A manufacturer gained 35% on one kind of wares and 44% on another. If the cost of manufacturing each kind was the same, and the gain was $360 more on the higher-priced wares, what were the total sales?

70. A merchant sold a bill of goods for $89.30 and thereby lost 6%. How much did he lose?

71. A sold a carriage to B and lost 20%; B sold it to C at a loss of 5%; C expended $50 in repairs and sold it for $442.40, gaining 40%. How much more did C obtain for the carriage than A?

72. A house was sold for $5616 at a gain of 8%. What was the gain?

73. If a house increase in value 20% per annum for three successive years and is then worth $6912, what was its value at the beginning of the first year?

74. A dealer sells goods, wholesale, at 10% profit; and, retail, at 20% advance on wholesale rates. How much greater is his profit on goods sold at $1.98 retail than if sold at wholesale rates?

75. What is the per cent. of loss on goods sold at $\frac{5}{8}$ of their cost?

76. What is the per cent. of gain if $\frac{4}{5}$ of a barrel of sugar be sold at $\frac{7}{8}$ of the cost of a whole barrel?

77. If a manufacturer sells $\frac{5}{6}$ of an invoice of hardware at $\frac{3}{4}$ of the cost of the entire invoice, what is his loss per cent.?

78. A grocer sold 50% of an invoice of coffee at 40% gain, and the remainder at 20% gain. What was his per cent. of gain on the whole?

79. A merchant sold 40% of a barrel of molasses at 10% gain, and the remainder at cost. What was his gain per cent. on the whole?

80. An importer invested an equal sum in Oolong and Gunpowder teas, gaining 20% on one kind and losing 5% on the other. What was his per cent. of gain on the total investment?

81. A merchant sold 30% of an invoice of grain at 20% profit, and the remainder at 10% loss. If his net loss was $3.15, what was the total amount of sales?

82. A dealer sold 20% of an invoice of tea at a loss of 25%, and the remainder at a gain of 12½%. What was the cost of the tea if the total sales amounted to $840?

83. I bought a row of six houses and afterwards sold five of them for what I paid for the six. What was my per cent. of gain?

84. A bill of goods cost $520, and freight 5% additional. If 15% of the goods were sold at 20% profit, 60% of the remainder at 33⅓% profit, and what was left of the two preceding sales at 25% profit, what was the total gain?

85. A lost 40% of an invoice of oysters, and sold the remainder at a profit of 75%. What per cent. of the total cost did he gain or lose?

86. If a man lose 25% of a lot of apples, what must be his % of gain on the remainder to net a gain of 25% on the cost of the whole?

87. A fruit dealer bought 530 crates of peaches at 95 cents per crate, sold 150 crates at a gain of 20%, 75 crates at a gain of 12%, 50 crates at cost, 200 crates at a loss of 5%, 25 crates at a loss of 50%, and threw away the remainder as worthless. What was his net gain or loss?

88. I bought a quantity of cloth at $2 per yard. Allowing 5% for bad debts, what must I ask per yard to net a profit of 20%.

89. A cabinet-maker sold a set of furniture for $18.75 more than cost, and gained 30%. What would have been his rate per cent. of gain or loss if he had sold the furniture for $87.50?

90. A merchant sold 20% of an invoice at 25% profit. At what per cent. loss must he sell the remainder to net cost on the whole?

91. A merchant sold 40% of his stock of goods at 25% gain, and 50% of the remainder at 10% gain. What was his total gain, if the cost of the goods remaining unsold was $3150?

92. An importer bought 240 yards of English broadcloth at 12s. 6d. per yard, paid 40% additional for freight, customs dues, etc., sold 30% of the importation at 25% profit, and the remainder at 20% profit. What were the total sales in U. S. money, estimating $4.85 to the £?

93. A man sold a house at 20% profit, and with the proceeds purchased another which he sold at 30% profit, realizing a total gain of $3136 on both. What did he pay for each?

94. A clothier bought 50 yards of broadcloth at $3.60 per yard and 76 yards of cassimere at $2.50 per yard. He sold the cassimere at a loss of 10%. What should he ask per yard for the broadcloth to net a profit of 20% on the cost of both?

95. A grain dealer bought 3500 bushels of wheat at $1.30 per bushel, 25% more oats than wheat at 53 cents per bushel, and 40% less rye than oats at 95 cents per bushel. He sold the wheat at 10% profit, the oats at 12½%, and the rye at 15%. What was the total gain?

96. The profits of a wholesale house in 1880 were 20% of its capital; in 1881, 25%; in 1882, 16⅔%, and in 1883 it suffered a loss of 4%. At the close of each year the firm reinvested the entire capital and gain in the business, and on Jan 1, 1884, the capital of the firm was $75264. What was the capital of the firm on Jan. 1, 1881?

97. A grain dealer sold a quantity of rye and wheat for $1320, gaining 33⅓% on the rye and 12½% on the wheat. He received 20% more for the wheat than for the rye. What was his total gain?

98. A grocer purchased a quantity of vinegar of which 20% was lost by leakage. Expecting to obtain cost for the entire quantity, he sold the remainder at 20% advance on its cost. What was his per cent. of gain or loss, if any?

99. A bookseller bought a number of "Orton and Sadler's Lightning Calculators." Being damaged by transportation, he sold them for $72, thereby losing 25% of the selling price. What was his per cent. of loss?

100. A grocer invested equal sums in Rio and Java coffee, and gained $5.70 more on one kind than on the other. His gain on Java coffee was 40% and on Rio 25%. How many pounds of each did he buy, if the cost of the Rio coffee was 12½ cents per pound, and of the Java 20 cents per pound?

101. The retail price of a set of furniture was $131.25 or 25% in advance of retailer's cost. By selling to the retailer the jobber realized a gain of 20%, and by selling to the jobber the manufacturer made a profit of 16⅔%. What did the furniture cost the manufacturer?

102. A merchant bought a lot of notions, 25% of the cost of which he paid in cash, and on the balance due, amounting to $741, he was allowed a credit of 60 days. If he sold the notions at a uniform profit of 12½%, what did he receive for them?

103. A man bought a house for $5600, paid $900 for improvements, and then sold it at a profit of 12%, receiving 30% of the selling price in cash, and accepting a note secured by mortgage for the balance. What was the face of the note?

104. A dealer sold a quantity of wheat at 20% gain, and with the proceeds of the sale purchased corn which he sold at a loss of 10%, receiving $729 for the corn. How much did he pay for the wheat?

105. How much should be asked for coffee which costs 18 cents per pound that a gain of 5% may be realized, allowing for a loss of 10% in roasting?

106. A stationer sold an invoice of books at 25% profit; and with the sum received therefor, he purchased ink which he sold at 20% gain. His gain on both was $75. What did he receive for the ink?

107. By selling a lot for $89.75 above cost, I realized a profit of 5%. I sold another lot which cost the same for $1848.85. What was my gain per cent. on the second sale?

108. A fruit dealer bought an invoice of oranges for $735, sold 80% of the invoice at 75% of the entire cost, and the remainder at 20% profit. What was his net loss?

109. The profit on a sale of butter was 20%, and the sum received for the butter was invested in eggs which were sold at 33⅓% profit. What was the gain on the butter if the gain on the eggs was $48?

110. A dairyman sold a quantity of milk, butter, and cheese, receiving $31.14 for all. He gained 20% on the milk, 15% on the butter, and 25% on the cheese. If an equal sum was paid for each, how much did the dairyman receive for the butter?

111. Considering each of the following columns as a separate question, if the terms marked √ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cost....................	√	√	?	?	?	..	√	√	√	..	√
Rate of Profit.............	√	√	√	√	..	√	..	..	?	?	?
Profit....................	?	..	√	..	√	?	√	?	..	√	√
Selling Price at a Profit....	..	?	..	√	√	√	?	√	√	√	..

	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Cost....................	√	√	?	?	?	..	..	√	√	..	√
Rate of Loss.............	√	√	√	√	..	√	√	..	?	?	?
Loss....................	?	..	√	..	√	?	√	?	..	√	√
Selling Price at a Loss.....	..	?	..	√	√	√	?	√	√	√	..

TRADE DISCOUNTS.

296. Trade Discounts are allowances or abatements made by manufacturers and dealers upon their catalogue or list prices.

Rem. 1.—It is customary in some branches of business for manufacturers or dealers to list or catalogue their goods or wares at a fixed price for each article, and allow an abatement or discount on all orders of a certain amount, a second discount on larger orders, a third discount on still larger orders, etc. In some cases, trade discounts are designed as a fee or compensation to agents or other middlemen for handling a line of goods, without regard to the quantities involved.

Rem. 2.—Since the price of an article will vary with the supply and demand, instead of publishing a new catalogue or list price with each fluctuation, manufacturers or jobbers simply *change the rate* of discount with each variation of price.

Rem. 3.—Merchandise is frequently sold at "time prices," upon which certain rates of discount are allowed if paid at an earlier date. Thus, an invoice of goods may be sold for $325, payable in 60 days, and subject to a discount of 1% if paid in 30 days, or 2% if paid in cash or within 10 days. Even when no discount *is offered* upon credit sales, sellers are generally willing to deduct the legal interest for the remainder of the time, if "time payments" are made before maturity.

297. Computations in Trade Discount are made in accordance with the principles of percentage, the gross selling price (that is, the list or marked price) is the base; and the discount and net selling price are percentages. Hence,

General Direction: *If a discount or net selling price is required, multiply its two given factors; and if a discount or net selling price is given, divide it by its given factor and the quotient will be its remaining factor.*

Rem. 1.—The two factors of a given or required discount are the gross selling price and the rate of discount; of a given or required net selling price are the gross selling price and the rate of net selling price.

Rem. 2.—If there is more than one discount, the factors of the second discount are the proceeds of the first discount × rate of second discount; the factors of any subsequent discount are the *proceeds of the preceding discount* × rate of that subsequent discount.

Rem. 3.—As the net selling price is the gross selling price minus the discount, so the rate of net selling price must be the rate of gross selling price (100%) minus the rate % of discount.

298. To find the net selling price.

ILLUSTRATIVE EXAMPLE.

What is the net price of hardware, invoiced at $460, and subject to a discount of 20%, 10%, and 5%?

Explanation.—Find the first discount ($\frac{1}{5}$ of $460), and subtract it ($92) from the gross price ($460), obtaining $368 as the proceeds of the first discount.

Next find the second discount ($\frac{1}{10}$ of $368) and subtract it ($36.80) from the base of the second discount ($368), obtaining $331.20 as the proceeds of the second discount.

Next find the third discount ($\frac{1}{20}$ of $331.20) and subtract it ($16.56) from the base of the third discount ($331.20), obtaining $314.64 as the proceeds of the third discount, or the required net price.

Solution.

$460,	gross selling price.
92,	20% or $\frac{1}{5}$ off.
$368,	proceeds of 1st disct.
36.80,	10% or $\frac{1}{10}$ off.
$331.20,	proceeds of 2d disct.
16.56,	5% or $\frac{1}{20}$ off.
$314.64,	net selling price.

EXAMPLES FOR PRACTICE.

Find the net selling price.

	Invoice price.	*Discount off.*		*Invoice price.*	*Discount off.*
1.	$375	20%, 10%, 10%.	4.	$3145.75	20%, 10%, 2%.
2.	$537.50	10%, 10%, 5%.	5.	$1873.25	20%, 10%, 5%.
3.	$825.30	2%, 1%, 1%.	6.	$535.15	30%, 20%, 10%.

7. What is the net cost to the purchaser of hardware invoiced at $815, and subject to a discount of 20%, 10%, and 5%?

8. The gross amount of a bill of goods is $750.35, and the rates of discount are 10%, 10%, and 5%. What is the net cost to the purchaser?

9. An invoice of crockery amounting to $1575.50 was sold Jan. 18, 1883, payable in 90 days, and subject to a discount of 20% and 10%, with an additional discount of 1% if paid in 60 days, 2% if paid in 30 days, or 3% if paid within 10 days of date of purchase. How much will pay the bill on Feb. 20, 1883?

10. A and B offer the same quality of hats at the same list price, but A offers a discount of 10% and 5%, and B a single discount of 15%. Of whom will it be more advantageous to buy, and how much will be saved in an order the gross amount of which is $185.50?

11. A's list price of a certain quality of pocket-knives is $9 per dozen, 20% and 5% off; and B's list price of the same quality of knives

is $9.50 per dozen, 25% and 10% off. How much will be saved by ordering 15 dozen knives from B?

12. The gross amount of a bill of tinware is $275.30, of which $75 is sold at a discount of 25% and 10%; $150 at 20% and 5%; and the remainder at 10% and 5%. What is the net amount of the bill?

299. To reduce a discount series to a single discount.

ILLUSTRATIVE EXAMPLE.

What single discount on the gross price is equivalent to a discount series thereon of 25%, 20%, and 10%?

Explanation.—Since the gross price is the base, its rate is 100%.

Deduct the rate of first discount (25%) from the rate of the gross price (100%), obtaining 75% as the rate of 1st proceeds.

Then reduce the rate of second discount (20%) which is expressed on the 1st proceeds (Rem. 2, **297**) to an equivalent rate expressed on the gross price (20% of 75% of the gross price = 15% of the gross price) and subtract, obtaining 60% as the rate of 2d proceeds.

Solution.

100% = gross price.
25% = 1st disct.
75% = 1st proceeds.
[$\frac{1}{5}$ of 75] 15% = 2d disct.
60% = 2d proceeds.
[$\frac{1}{10}$ of 60] 6% = 3d disct.
54% = final proceeds.
100% − 54% = 46%, *Ans.*

Continuing, it is found that 54% of the gross price is the rate of final proceeds. Hence, 100% of the gross price minus 54% of the gross price, or 46% of the gross price equals the required rate of single discount.

NOTE.—The net result of a series of discounts is the same regardless of the order of their arrangement. Thus, 25%, 20%, and 10% off an invoice is the same as 20%, 25%, and 10%; or 10%, 20%, and 25% off the same invoice; but the net result will be different even if the *sum* of each discount series is the same, if the rates themselves differ. Thus, 20% and 10% off an invoice will not produce the same result as 25% and 5% off.

Reduce the following discount series to equivalent single discounts:

1. 10% and 5%.
2. 20% " 10%.
3. 15% " 10%.
4. 30%, 10%, and 5%.
5. 20%, 10%, " 5%.
6. 10%, 5%, " 3%.

What is the difference between a discount series

7. Of 10%, 5%, and 2%, and a direct discount of 17%?
8. Of 30% and 10%, and a single discount of 40%?
9. Of 30%, 10%, and 5%, and a single discount of 55%?

300. ADDITIONAL PROBLEMS IN TRADE DISCOUNT.

1. What must be asked for goods that a discount of 20% and 10% may be allowed, and net $2.88 to the seller?

Net price ($2.88) ÷ rate of net price (72%) = gross price ($4).

At what price must goods be marked

2. To net $2.72, after allowing a discount of 15%?
3. To net $12, after allowing a discount of 10% and 10%?
4. To net $3.60, after allowing a discount of 25%, 20%, and 10%?
5. To net $30, after allowing a discount of 10%, 10%, and 10%?
6. To net $14.58, after allowing a discount of 40%, 10%, and 10%?
7. At what price must goods be marked to net 10% profit after allowing a discount of 10% and 5%, if they cost $4.75?

Cost ($4.75) + profit ($.475) = net selling price ($5.225).
Net price ($5.225) ÷ rate of net price (85½%) = marked price ($6.11 +).

At what price must goods be marked to net

8. 25% profit after allowing 25%, 20%, and 10% off; cost $216?
9. 20% profit after allowing 10%, 5%, and 3% off, if they cost $96?
10. 12½% profit after allowing 20%, 10%, and 2% off; cost $171.20?
11. 10% loss after allowing 10% and 5% off, if they cost $15.30?
12. 20% loss after allowing 25%, 10%, and 5% off; cost $15.75?
13. If goods which cost $120 are marked $150, what per cent. of discount must be allowed on marked price to net cost?

Discount ($150 — $120 = $30) ÷ marked price ($150) = rate of discount (20%).

What per cent. of discount must be allowed on marked price

14. To net cost, if the goods cost $5 and are marked $6?
15. To net cost, if the goods cost $3.24 and are marked $4.05?
16. To net cost, if the goods cost $8.50 and are marked $12.75?
17. To net 20% profit, if the goods cost $7.50 and are marked $12?
18. To net 10% loss, if the goods cost $8 and are marked $9?
19. A dealer was allowed 25% discount on an invoice of furniture, paid an amount equal to the discount by giving his check for $35, and gave his note for the remainder. What was the face of the note?
20. The retail price of sewing-machines is $45 each, upon which agents are allowed a discount of 33⅓%. What is their % of gain?
21. What must be a jobber's list price of axes which cost $7.50 per dozen, that a discount of 20%, 10%, and 5% may be offered on the list price and net a profit of 10% to the jobber?

22. In a closing-out sale, a jobber desires to sell dishes costing $5 per dozen at 10% loss, and offer a discount of 30% and 20% on the marked price. At what price per dozen should the dishes be marked?

23. The net cost of 240 yards of dress goods is $126.36. What was the gross price per yard, if purchased subject to a discount of 10%, 10%, and 2½%?

24. What must be the marked price of clothing which cost $18 per suit that a discount of 10% may be offered on the marked price and yet net cost to the seller?

25. If I mark augers at 60% profit, what rate of discount can I allow on the marked price and net cost?

26. If goods are sold at their marked price the rate of profit will be 20%. What will be the rate of profit if the same goods are sold at 10% discount on their marked price?

27. Merchandise was bought at a certain gross price with 20% and 10% off, and sold at the same gross price with 10% and 5% off. What was the per cent. of profit?

28. If goods are marked at a profit of 25%, what per cent. of discount should be allowed on the marked price to net cost?

29. A merchant bought a bill of goods upon which the total discount was $318, the discount-series being 20%, 20%, and 10%. What was the net price of the goods?

30. If a jobber's prices are based upon sales on 60 days' credit and amounting to less than $100 gross, with a discount of 10% on all sales from $100 to $500, a second discount of 10% on all sales from $500 to $1000, a third discount of 5% on all sales over $1000, and a further discount of 2% for cash payment, what is the net cash price of a bill of goods amounting to $1865 gross? Amounting to $400 gross? What is the net price of a bill amounting to $800 gross and sold on 60 days' credit?

31. Considering each of the following columns as a separate question, if the terms marked ✓ are given how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Gross Price.....	✓	✓	?	?	?	..	..	✓	✓	✓	..
Rate of Discount	✓	✓	✓	✓	..	?	✓	?	..	?	✓
Discount........	?	..	✓	..	✓	✓	✓	✓	?	..	?
Net Price.......	..	?	..	✓	✓	✓	?	..	✓	✓	✓

32.

STATIONERY.

All accounts to be settled by *cash* or *note* on the first of every month.

New York, *May* 13, 1886.

Messrs. Theron B. Parks & Co.,

Chicago Ills.,

Bought of Daniel Slote & Co.

Terms: 4 mo., or 5% 30 da.

Payable with exchange on New York.

Forwarded for your account and risk by Penna. R. R.

Quantity.	*Items.*	*Total Quires.*	*Price.*	*Extension List Prices.*		*Extension Net Prices.*	
6	1.17 1.89 doz. Blank Books ea. 884 732 . .	749	Net			$**	**
3	1.17 1.68 2.34 1.98 " " " " 730 731 742 744	812	"			**	**
2	" " " 210 . . .		24.00	**	**		
½	" " " 1620 . . .		31.68	**	**		
3	3.00 4.25 " " " ea. 2267 2268 . .			**	**		
½	" " " 2272 . . .		4.75		**		
3½	" " " 908 . . .		9.00	**	**		
2½	" " " 905 . . .		5.50	**	**		
3	15.00 5.50 7.20 7.60 " " " ea. 912 1350 905 1351			***	**		
50	lbs. Pads, Card size, . . . 8¢		Net			*	**
100	" " Cabinet size, . . 8½		"			*	**
100	" ' Panel " . - 8¾		"			*	**
				***	**		
	60%			***	**		
				**	**		
	5%			*	**	**	**
						$***	**
	3 Cases and Cartage,					4	75
						$***	**

What sum will pay the above bill on July 1, 1886?

The student should be required to copy the preceding and following bills, make the necessary extensions, and "foot" the columns.

300

33. HARDWARE.

All accounts remaining unsettled 90 days subject to sight draft.

BALTIMORE, *June* 15, 1886.

MR. FAIRMAN A. SADLER,

Bought of S. G. B. COOK & Co.

Terms: 60 days, net; 30 days, 3% off; 10 days, 5% off.

1	Doz. Table Knives 4008	$10	00		
	25% & 10%			*	**
	3.00 4.50				
½	" Butcher " ea. 8-in. 10-in.			*	**
1	" Sticking " G. B. 6-in.			2	10
	3.00 5.00 4.50				
½	" Pocket " ea. 261 310 318 . .			*	**
	3.65				
3	" Brandford Locks 01800	**	**		
	2.50				
3	" " Knobs 202	*	**		
		**	**		
	60% & 10%			*	**
1	" Kipp's A. E. Farrier's Hammers . .	6	00		
	30%			*	**
	36.00				
½	" Ives Braces 132	*	**		
	60% & 5%			*	**
	2.50 6.00				
½	" Oneida Traps ea. $1 $1½ . . .	*	**		
	9.00				
2	" " " $2	**	**		
	14.50				
½	" " " $4	*	**		
		**	**		
	60%, 10% & 5%			*	**
	1.85 3.35				
1	" Cotton Hooks ea. $1 $4	*	**		
	25%			*	**
	6.00				
1½	" Bolts 481 6-in.	*	**		
1	" " " 8-in.	7	50		
	5.00 6.50				
1	" " 490 ea. 6-in. 8-in.	**	**		
		**	**		
	60%, 10% & 10%			*	**
	Pkg. and Drayage,				75
				**	**

What sum will pay the above bill on June 20, 1886?

300

COMMISSION AND BROKERAGE.

301. **Commission** or **Brokerage** is the sum charged by an agent for transacting business for his principal; as, for buying or selling property, for collecting or investing money, etc.

Rem. 1.—Commission or brokerage is usually estimated at a per cent. of the gross proceeds of a sale or of the prime cost of a purchase; but on some kinds of merchandise it is often computed at a certain price per unit; as, grain per bushel, flour per barrel, cotton per bale, etc.

Rem. 2.—If sales are made on credit, a certain per cent. of the sum credited is usually charged by the agent for guaranteeing its final payment. This additional charge is called *guaranty*.

Rem. 3.—The agent who buys or sells merchandise for another is called a *Commission Merchant* or *Broker*. A *commission merchant* is usually placed in possession of the goods bought or sold, and acts in his own name; a *broker* does not take possession, but simply effects contracts to buy or sell in the name of his principal.

Rem. 4.—The term *broker* is also applied to those whose business it is to buy and sell stocks, bonds, bills of exchange, promissory notes, specie, etc., etc.

Rem. 5.—A *Consignment* is a shipment of merchandise by one party to another; the party who ships the merchandise being called the *Consignor* or *Shipper*, and the party to whom the merchandise is shipped being called the Consignee.

302. The **Gross Proceeds** of a sale or collection is the total amount received by the agent for his principal *before* deducting the commission or other charges.

Rem. 1.—After deducting the commission and all other charges from the gross proceeds, the remainder is called the *net proceeds*.

Rem. 2.—An *account sales* is a written statement rendered by an agent to his principal, specifying the goods sold and prices obtained, detailing the several charges or expenses, and exhibiting the net proceeds or balance due the principal.

303. The **Prime Cost** of a purchase is the net sum paid by an agent, and does not include his commission or other charges.

Rem. 1.—After adding the commission and other charges to the prime cost, the sum is called the *gross cost*.

Rem. 2.—An *account purchase* is a written statement rendered by an agent to his principal, specifying the property bought and prices paid, detailing the various charges or expenses, and exhibiting the gross cost of the purchase.

304. Computations in Commission and Brokerage are made in accordance with the principles of percentage, the gross proceeds (if a sale) or the prime cost (if a purchase) being regarded as the base; and all other terms than gross proceeds or prime cost (such as commission, guaranty, net proceeds, gross cost, etc.) being regarded as percentages. Hence,

General Direction: *If a commission, guaranty, net proceeds or gross cost is required, multiply its two given factors; and if a commission, guaranty, net proceeds, or gross cost is given, divide it by its given factor and the quotient will be its remaining factor.*

REM. 1—The two factors of a given or required commission are the prime cost (if a purchase) or the gross proceeds (if a sale) and the rate of commission; of a given or required net proceeds are the gross proceeds of the sale and rate of net proceeds; of a given or required gross cost are the prime cost of the purchase and the rate of gross cost.

REM 2.—Since the net proceeds is the gross proceeds minus all the expenses, the rate of net proceeds must be the rate of gross proceeds (100%) minus the rate of all the expenses; and since the gross cost is the prime cost plus all the expenses, the rate of gross cost is the rate of prime cost (100%) plus the rate of all the expenses.

REM. 3.—All expenses which have no rates, as freight, drayage, storage, etc., should be added to a given net proceeds or subtracted from a given gross cost to obtain a result which has a rate for one of its factors.

EXAMPLES FOR PRACTICE.

1. What is the commission if the gross proceeds of a sale are $180 and the rate of commission for selling is 2½%?

Gross proceeds ($180) × rate of commission (2½%)=commission ($4.50).

What is the commission

2. If the gross proceeds are $53.50, and the rate of commission 2%?
3. If the prime cost is $12360, and the rate of commission $\frac{9}{32}$%?
4. If the gross proceeds are $1240, and the rate of commission ⅝%?
5. If the gross proceeds are $236, and the rate of commission 1¼%?
6. If the prime cost is $7500, and the rate of commission ¾%?

What are the net proceeds of the sale if

7. The gross proceeds are $750, the rate of commission 3%, and the other expenses $5.25?

8. The gross proceeds are $1275, the rate of commission 4%, and the other expenses $18.30?

9. The gross proceeds are $18000, the rate of brokerage ⅜%, and the other expenses $175.30?

10. The gross proceeds are $700, the rate of commission 2½%, and the other expenses $12.75 ?

What is the gross cost of the purchase if

11. The prime cost is $15.50, the rate of commission 4%, and the other expenses $1.30 ?

12. The prime cost is $7320, the rate of commission 2¼%, and the other expenses $150.25 ?

13. The prime cost is $2480, the rate of brokerage $\frac{5}{16}$%, and the other expenses $25 ?

14. The prime cost is $73.20, the rate of commission 2¾%, and the other expenses $5.25 ?

15. What is the rate per cent. of commission if the prime cost of merchandise is $480, and the commission for buying $4.20 ?

Commission ($4.20) ÷ prime cost ($480) = rate of commission (⅞%).

What is the rate per cent. of commission if

16. The gross proceeds are $960 and the commission $15.60 ?
17. The prime cost is $3264 and the commission for buying $3.06 ?
18. The gross proceeds are $3200 and the commission for selling $6?
19. The prime cost is $460 and the commission for buying $23 ?
20. The prime cost of the purchase is $275 and the gross cost $286 ?
21. The gross proceeds are $105 and the net proceeds $102.90 ?
22. The gross proceeds are $380 and the net proceeds $379.05 ?
23. The prime cost is $124 and the gross cost $128.34 ?

24. What are the gross proceeds of the sale if the commission is $132 and the rate of commission 4% ?

Commission ($132) ÷ rate of commission (4%) = gross proceeds ($3300).

What are the gross proceeds or prime cost if

25. The commission for selling is $50.30 and the rate 2% ?
26. The commission for buying is $75.30 and the rate 5% ?
27. The commission for selling is $36.60 and the rate 1⅓% ?
28. The brokerage for buying is $29.25 and the rate ¾% ?
29. The net proceeds are $736.96 and the rate of commission 2% ?

Net proceeds ($736.96) ÷ rate of net proceeds (98%) = gross proceeds ($752).

30. The net proceeds are $77.90 and the rate of commission 5% ?
31. The net proceeds are $344.75 and the rate of commission 1½%?
32. The net proceeds are $495.36 and the rate of commission 3¼%?
33. The gross cost is $138.69 and the rate of commission 3½% ?

Gross cost ($138.69) ÷ rate of gross cost (103½%) = prime cost ($134).

What are the gross proceeds or prime cost if

34. The gross cost is $185.40 and the rate of commission 3% ?

35. The gross cost is $564.20 and the rate of brokerage ¾% ?

36. The gross cost is $1606 and the rate of brokerage ⅜% ?

37. The net proceeds are $51.90, the rate of commission 5%, and the other charges $1.30 ?

($51.90 + $1.30) ÷ .95 = $56, gross proceeds of sale.

38. The net proceeds are $145.50, the rate of commission 2%, and the other charges $2.50 ?

39. The net proceeds are $704.20, the rate of commission 1¾%, and the other charges $3.20 ?

40. The net proceeds are $774.25, the rate of commission 2⅛%, and the other charges $5.75 ?

41. The gross cost is $668.60, the rate of commission 4%, and the other charges $8.20 ?

($668.60 − $8.20) ÷ 1.04 = $635, prime cost of purchase.

42. The gross cost is $33.50, the rate of commission 3⅛%, and the other charges $.50 ?

43. The gross cost is $141.56, the rate of commission 2¼%, and the other charges $2.50 ?

44. The gross cost is $256.15, the rate of commission 1⅞%, and the other charges $3.50 ?

45. A commission merchant sold a consignment of cotton for $736 and charged 1½% commission. What was his commission ?

46. A grain broker charged $20 for effecting the sale of 16000 bu. corn at 80¢ per bushel. What was his rate per cent. of brokerage ?

47. An agent charged $13.40 for selling a consignment of merchandise. What were the gross proceeds of the sale if his rate of commission for selling was 2½% ?

48. A commission merchant received $544.35 to invest in wool after deducting all expenses. How much did he pay for the wool, if his rate of commission for buying was 2%, and the sum of the remaining charges $3.75 ?

49. A fruit buyer received an order to purchase 324 barrels of apples. The purchase was made at $2.25 per barrel, and in settlement the buyer drew on his principal for $750.87. What was his rate % of commission ?

50. An attorney collected 30% of a note the face of which was $75,

and charged 10% commission for collecting. What were the net proceeds due the principal?

51. An agent invested $7500 in cotton, charged 1⅛% commission for buying, $125 for freight prepaid, $15 for drayage, and drew on his principal for the gross cost. What was the face of the draft?

52. If an agent's commission was $21.30 for selling 284 yards of cloth at $3 per yard, what was his rate per cent. of commission?

53. A commission merchant remitted to his principal $158 as the net proceeds of a consignment of eggs. What were the gross proceeds if the rate of commission was 3%, of guaranty 2%, and the sum of the other charges $6.54?

54. An agent sold 42 barrels of potatoes at $3.25 cash per barrel, and 58 barrels at $3.50 per barrel, payable in 30 days. His charge for selling was 3%, for guaranty 2%, and for cooperage $3. What were the net proceeds due the principal?

55. A commission merchant remitted $1323 as the net proceeds of a consignment, after deducting $27 as commission. What was his rate per cent. of commission?

56. I paid ¼% brokerage to a grain broker for effecting the sale of a quantity of wheat at $1.15 per bushel. How many bushels of wheat were sold, if the total brokerage was $63.48?

57. How much did I realize on a house and lot sold on commission at 3%, the total charges, including $25 for advertising, being $130?

58. What is the rate per cent. of commission if the gross cost of a purchase is $759.60, the commission $11.10, and the sum of the other charges $8.50?

59. An agent purchased 7200 bushels of wheat at $1.34 per bushel, charged $\frac{3}{16}$% commission for buying, $108 for storage, and $120 for cartage. What was the total cost to the principal?

60. The gross proceeds of a sale were $512, the net proceeds $503.10, and the sum of the charges other than commission $2.50. What was the rate per cent. of commission?

61. I remitted to my agent $508.95 to invest in potatoes after deducting all expenses. He paid $46 for barrels, $38.60 for drayage, and charged 2½% commission for buying. How many barrels of potatoes did he buy at $2.25 per barrel?

62. A produce commission merchant received a consignment of 15 cases, each containing 36 dozen eggs, and 3 firkins of butter whose total net weight was 160 pounds. He sold the eggs at 18 cents per dozen and the butter at 18¾ cents per pound, and remitted to the consignor $115.84 as the net proceeds of the consignment. What was the rate per cent. of commission, if the charges other than commission amounted to $5 ?

63. A collector received a claim of $540, of which he succeeded in collecting $380. What sum did he return to his principal if his rate of commission for collecting was 5% ?

64. A commission merchant received $4456.40 to invest in wheat after deducting his commission of 2%, and drayage of 50 cents per load of 36 bushels. How much wheat did he purchase at $1.20 per bushel ?

Gross cost of all the bushels ÷ gross cost of 1 bushel = number of bushels.

65. The gross cost of a purchase is $1221, which includes the commission $6, and other charges amounting to $15. What is the rate per cent. of commission ?

66. What is the brokerage for effecting the sale of 7000 bushels of wheat at $1.25 per bushel if the rate of brokerage is ⅛¢ per bushel ?

67. A commission merchant sold 80 bales of cotton, averaging 400 pounds per bale, at 12½ cents per pound, charged 2½% for selling, 3% for guaranty, and $24.36 for other expenses. He invested the proceeds in coffee at 14 cents a pound, and charged 2% commission for buying. How many pounds of coffee did he buy ?

68. A commission merchant sold 350 barrels of flour at $7.50 per barrel, paid 50 cents per load of 10 barrels for drayage, and $3 for cooperage. He purchased for the consignor 1570 bushels of wheat at $1.20 per bushel, paid 1½ cents per bushel for storage, and 1 cent per bushel for drayage. What is the balance due the consignor if the rate of commission was 2% for selling and the same for buying ?

69. A commission merchant received a consignment of 82 bags wheat, net weight 9870 pounds, and 43 bags corn, net weight 2422 pounds, which he sold at $1.30 per bushel of 60 pounds for the wheat, and 75 cents per bushel of 56 pounds for the corn. His charges were 3% commission for selling, 25 cents per 100 pounds for freight, and $3 for cartage. What were the net proceeds due the principal ?

70. An agent expended $864.55 in purchasing a quantity of tobacco, which included his commission $25.05, his charge for drayage $3.50, and for cooperage $1. What was his rate per cent. of commission ?

71. A commission merchant remitted to his principal $310.16 as the net proceeds of a consignment. His charges were $2.50 for drayage and 3½% for commission. What was his commission?

72. An agent received $300 to invest in apples after deducting all expenses. How many barrels can he buy at $3.50 per barrel, his charges being 4% commission for buying, 2½ cents per barrel for drayage, 12 cents per barrel for freight, $2.15 for cooperage, and $3.50 for advertising? What was the unexpended balance of the remittance?

It is customary in the commission business to find the number of *whole* barrels, by first deducting from the remittance all charges which are not estimated per barrel, and dividing the remainder by the prime cost per barrel plus the commission and remaining charges per barrel. If the division is not exact, treat the remainder as an unexpended balance to be returned to the principal or placed to his credit.

73. An agent received $96 to invest in onions after deducting his commission of 5% and drayage of 25 cents per load of ten barrels. How many barrels of onions did he buy at $1.60 per barrel and what was the unexpended balance of the remittance?

After finding the number of whole barrels as in Ex. 72, divide by 10 to find the number of loads, considering the fraction of a load as a full load. Deduct from the unexpended balance the increased drayage per barrel on the fraction of a load.

74. A commission merchant received $815 to invest in flour after deducting all charges. If the market price of the grade of flour ordered was $6.75 per barrel, the commission 3%, and the drayage 50 cents per load of ten barrels, how many barrels of flour were purchased, and what was the unexpended balance?

75. An agent charged $46.75 for selling a consignment of oats. What was the sum returned to the consignor if the agent's rate of commission was 2½%?

76. I received $1386.90 to invest in wheat after deducting my commission of 3½%. How much did I invest in wheat, and what was the sum retained as commission?

77. A commission merchant sold a quantity of coal oil, and remitted the net proceeds after deducting his commission of 3½% for selling, 2% for guaranty, and $5.30 for other expenses. What were the gross proceeds of the sale if the sum remitted was $298.30?

78. I paid $21.60 to a commission merchant for selling goods at 3% commission, and his other charges amounted to $18. What were the net proceeds of the sale?

79. A commission merchant received from a speculator $3806.25

to invest in corn after deducting his commission of 1½%, and to store the corn subject to the speculator's order. The anticipated rise in corn taking place, the commission merchant was ordered to sell. He did so, obtaining 80 cents per bushel; and after deducting his commission of 1⅛% for selling and $75 for storage, paid the speculator $3865 as the balance due him. What did the commission merchant pay per bushel for the corn?

80. A commission merchant sold a consignment of flour at 2% commission and invested the net proceeds in bacon at 5% commission. What did he receive for the flour and pay for the bacon, if his total commission amounted to $150?

81. The net proceeds of a consignment were $685.03, the commission $14.37, drayage $8.75, advertising $5.40, storage $3.60, and sundry other expenses $1.35. What was the rate of commission?

82. An agent sold a consignment of coffee at 13 cents per pound, charged 5% commission for selling, invested $1348.62 of the proceeds in corn, including 4% commission, and remitted $380.38 as the balance due his principal. How many pounds of coffee were sold? What was the total commission?

83. I ordered my agent in St. Louis to buy flour which I afterwards sold at 20% profit and gained $1.56 per barrel. If my agent's rate of commission was 4%, and his total commission $23.40, how many barrels did he buy?

84. At what price shall an agent be ordered to buy potatoes at 2% commission that, after paying 7 cents per barrel for transportation, they can be sold at $1.76 per barrel and net 10% profit?

85. I remitted $578 to my agent in Chicago with instructions to invest in wheat after deducting his commission of 2%, and consign the purchase to my agent in New Orleans. What was my net gain, if my agent in New Orleans sold the wheat for $625, charged 3% for selling, and $12.35 for sundry other expenses?

86. A commission merchant received a consignment of 420 barrels of flour, on which he advanced to the consignor $200 cash, paid $42 freight, $5 cooperage, $12.60 storage, and $10.50 cartage. He sold 80 barrels at $7.25 per barrel, 150 barrels at $7.75 per barrel, 50 barrels at $8.25 per barrel, and the remainder at $8.50 per barrel. What was the balance due the consignor, the commission for selling being 2%?

87. A merchant in St. Louis sent 5000 bushels of corn and 15000 *pounds* of bacon to his agent in Galveston, and instructed him to sell the consignment and invest $6000 of the proceeds in wool. The agent paid $525 for freight, $72 for drayage, and $3 for cooperage, sold the corn at $1.15 per bushel, and the bacon at 14 cents per pound, charging a commission of 1⅛% on the bacon and 1 cent per bushel on the corn. The commission for buying the wool was 2¼%. What was the balance due the merchant in St. Louis ?

88. I consigned 2420 pounds of leather to a firm in Philadelphia, which they sold at 25 cents per pound. I received $588.17 as the net proceeds of the consignment. What was the rate per cent. of commission if the charges other than commission amounted to $4.73 ?

89. What is the cost of 17$^{\underline{14}}$ bushels of clover seed at $7.25 per bushel, commission 2⅛% additional ?

90. A broker effected a purchase of sugar at $\frac{3}{16}$% brokerage, receiving $75 for his services. He afterwards effected the sale of the same sugar at a net profit to his principal of $2710, charging ⅛% brokerage for selling. What did the broker obtain for the sugar ?

91. An agent received a consignment of 86000 pounds of wheat, which he sold at 95 cents per bushel of 60 pounds. He paid 18 cents per cental for freight, 40 cents per load of 30 bushels for drayage, and charged 1⅛% commission for selling. What were the net proceeds due the consignor ?

92. ACCOUNT SALES.

BALTIMORE, *Dec.* 15, 1886.

SOLD *for account of* J. T. JANNEY

By S. M. GORDON & Co.

1886.							
Dec.	8	75 bbls. Queen City Flour . .	$7.25	***	**		
"	"	50 " Snow Flake . . .	7.30	***	**		
"	9	130 " Patapsco	7.00	***	**		
"	10	200 " Howard St. . . .	6.25	****	**		
"	15	80 " Western Family, . .	5.25	***	**	****	**
		Charges:					
"	6	Freight, 535 bbls. @ 15c.		**	**		
"	"	Drayage, 535 bbls. @ 5c.		**	**		
"	10	Storage, 535 bbls. @ 8c.		**	**		
"	15	Inspection, 535 bbls. @ 1c. . . .		*	**		
		Commission, 3%		**	**	***	**
		Net Proceeds to your Credit . . .				****	**

93.

ACCOUNT PURCHASE.

St. Louis, Mo., *March* 12, 1886.

Bought *for account and risk of* S. Meems & Co.

By Sutro & Williams.

15	bbls. Early Rose Potatoes . . . $3.25	**	**		
8	" Goodrich " 2.75	**	**		
4	" Burbank " 2.63	**	**		
85	bu. Seed Corn 1.25	**	**		
75	" " Oats 75	**	**		
83	" " Wheat 1.45	**	**		
72	" Buckwheat 1.15	**	**		
5	" Clover Seed 7.85	**	**	***	**
	Charges:				
	Drayage	1	50		
	Commission, 2%	*	**	*	**
	Debited your account			***	**

94. Considering each of the following columns as a separate question, if the terms marked √ are given how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Gross proceeds....	√	√	√	?	?	..	?	..	?	?	√	√	√	..	..
Rate of commission	√	√	√	√	..	?	..	?	√	√	..	?	?	√	√
Commission.......	?	..	..	√	√	√	√	√	..	..	?	..	..	√	√
Net proceeds......	..	?	?	..	√	√	√	√	√	√	√	√	√	?	?
Storage..........	..	..	√	..	..	..	√	√	..	√	..	..	√	..	√
Drayage..........	..	..	√	..	..	..	√	√	..	√	..	..	√	..	√
Freight..........	..	..	√	..	..	..	√	√	..	√	..	..	√	..	√

	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
Prime cost..	√	√	√	?	√	?	..	?	..	?	?	√	√	..	..
Rate of com.	√	√	√	√	?	..	?	..	?	√	√	?	?	√	√
Commission.	?	..	..	√	√	√	√	√	√	..	..	..	..	√	√
Gross cost...	..	?	?	..	..	√	√	√	√	√	√	√	√	?	?
Storage.....	..	..	√	..	..	..	..	√	√	..	√	..	√	..	√
Drayage....	..	..	√	..	..	..	..	√	√	..	√	..	√	..	√
Freight.....	..	..	√	..	..	..	..	√	√	..	√	..	√	..	√

INTEREST.

305. **Interest** is a charge for the use of money.

306. The **Principal** is the sum for the use of which interest is charged.

307. The **Rate of Interest** is the per cent. of the principal which is charged for its use one year.

If A borrows $1000 from B, and agrees to pay him 6% per annum, the sum borrowed ($1000) is the *principal;* the per cent. of the principal which is charged *per year* for its use (6%) is the *rate of interest;* and the percentage of $1000 which is paid for its use is the *interest*, as 6% of $1000 = $60, or one year's interest; 2 times 6% of $1000 = $120, or two years' interest; ½ of 6% of $1000 = $30, or 6 months' interest; etc.

308. The **Time** is the period during which the principal bears interest.

309. **Simple Interest** is the sum charged for the use of the original principal, or of so much of the original principal as may at any time remain unpaid.

For definition of annual interest and compound interest, see **330** and **331**.

310. **Ordinary Interest** is interest computed on the basis of 360 days to a year.

By reason of its great convenience, this method of computing interest is generally employed by business men; and, in a number of States, its use has been sanctioned by law.

311. **Accurate Interest** is interest computed on the basis of 365 days to a common year or 366 days to a leap year.

312. The **Amount** is the *sum* of the principal and its accrued interest.

313. **Legal Interest** is interest which is computed at the rate % per annum established by law.

1. When no particular rate is specified in a contract, the legal rate is understood. In several States, when *agreed to* by both borrower and lender, more than the legal rate is allowed, ranging from 7% to 12%; and in a few States, the higher rate is unlimited.

2. To charge a higher rate of interest than is allowed by law is called *usury*, for which penalties are imposed by the laws of nearly all the States.

3. Debts of all kinds draw interest from the time they become due; but not before unless so specified.

314. The legal and maximum rates of interest allowed by law in the different States and Territories of the United States are as follows:

TABLE OF INTEREST RATES.

States and Territories.	*Rate per annum.*		*States and Territories.*	*Rate per annum.*	
	Legal.	*Maximum.*		*Legal.*	*Maximum.*
Alabama	8%	8%	Mississippi	6%	10%
Alaska	. . .	. . .	Missouri	6%	10%
Arizona Ter.	10%	any %	Montana Ter.	10%	any %
Arkansas	6%	10%	Nebraska	7%	10%
California	7%	any %	Nevada	7%	any %
Colorado	10%	any %	New Hampshire	6%	6%
Connecticut	6%	any %	New Jersey	6%	6%
Dakota Ter.	7%	12%	New Mexico Ter.	6%	any %
Delaware	6%	6%	New York	6%	6%
District of Columbia	6%	10%	North Carolina	6%	8%
Florida	8%	any %	Ohio	6%	8%
Georgia	7%	8%	Oregon	8%	10%
Idaho Ter.	10%	24%	Pennsylvania	6%	6%
Illinois	6%	8%	Rhode Island	6%	any %
Indiana	6%	8%	South Carolina	7%	7%
Indian Ter.	6%	any %	Tennessee	6%	6%
Iowa	6%	10%	Texas	8%	12%
Kansas	7%	12%	Utah	10%	any %
Kentucky	6%	6%	Vermont	6%	6%
Louisiana	5%	8%	Virginia	6%	6%
Maine	6%	any %	Washington Ter.	10%	any %
Maryland	6%	6%	West Virginia	6%	. . .
Massachusetts	6%	any %	Wisconsin	7%	10%
Michigan	7%	10%	Wyoming	12%	any %
Minnesota	7%	10%			

315. Computations in Interest may be made in accordance with the principles of percentage, the principal being regarded as the base; and the interest and amount as percentages. Hence,

General Direction: *If the interest or amount is required, multiply its two given factors; and if the interest or amount is given, divide it by its given factor and the quotient will be its other factor.*

Rem. 1.—The two factors of a given or required interest are the *principal* and the *rate of interest for the given time*; and of a given or required amount are the *principal* and the *rate of amount for the given time.*

Rem. 2.—The rate of interest for the given time has the same relation to the rate per annum as the given time has to one year. Hence, to find the rate of ordinary interest for *less* than one year, divide the rate per annum into 360, and the result into the given time in days regarded as hundredths. Thus, at 8% per annum, the rate for 95 days will be .95 ÷ (360 ÷ 8) or .02$\frac{1}{9}$.

Rem. 3.—Since the amount equals the principal plus the interest for the given time, the rate of amount equals the rate of the principal (100%) plus the rate of interest for the given time.

SIX PER CENT. METHOD.

316. If the rate is 6% for one year, the equivalent rate for $\frac{1}{6}$ of a year, that is, for 2 months or 60 days, will be $\frac{1}{6}$ of 6%, or 1%; and if the rate for 60 days is 1%, the equivalent rate for $\frac{1}{10}$ of 60 days, or 6 days, will be $\frac{1}{10}$ of 1%, or $\frac{1}{10}$%. Hence,

The ordinary interest of $1 at 6% per annum

For	1 year,	12 months,	or 360 days,	is	.06	of the principal,	or 6 cents.
"	$\frac{1}{6}$ "	2 "	" 60 "	"	.01	" " "	" 1 cent.
"	$\frac{1}{12}$ "	1 "	" 30 "	"	.005	" " "	" 5 mills.
"	$\frac{1}{60}$ "	$\frac{1}{5}$ "	" 6 "	"	.001	" " "	" 1 mill.
"	$\frac{1}{360}$ "	$\frac{1}{30}$ "	" 1 "	"	.000$\frac{1}{6}$	" " "	" $\frac{1}{6}$ mill.

317. To find the interest at 6% per annum when the time is expressed in days. [illegible]

ILLUSTRATIVE EXAMPLE.

What is the interest of $751.45 for 23 days at 6% per annum?

Explanation.—At 6% per annum, the rate for any number of days is *one-sixth* as many thousandths of the principal as there are days (**316**). Hence, if the principal ($751.45) is multiplied by as many thousandths as there are days (.023), or 6 times the correct rate of interest, the resulting product will be 6 times the correct interest. Therefore, divide the product by 6 to find the correct interest.

Solution.

$$\begin{array}{r} \$751.45 \\ \underline{.023} \\ 225435 \\ \underline{150290} \\ 6)\$17.28335 \\ \hline \$2.88+ \end{array}$$

Rule.—*Multiply the principal by the number of days, point off from the right of the product 3 more decimal places than are contained in the principal, and divide the result by 6.*

NOTE.—When it is seen that the time in days is exactly divisible by 6, the correct interest may be obtained in one operation by using its correct factors (Rem. 1, **315**), *multiplying the principal by one-sixth as many thousandths as there are days.*

EXAMPLES FOR PRACTICE.

Find the interest at 6% per annum.

Principal.	*Time.*	*Principal.*	*Time.*	*Principal.*	*Time.*
1. $348	38 days.	7. $5187.24 .	141 days.	13. $357.19 .	95 days.
2. $846	219 "	8. $7152 . . .	59 "	14. $1238 . . .	18 "
3. $250.20 . .	55 "	9. $128.35 .	53 "	15. $3187.35 .	96 "
4. $918.30 . .	62 "	10. $5341.75 .	36 "	16. $329.32 .	215 "
5. $182.10 . .	45 "	11. $6275 . . .	48 "	17. $1568.75 .	39 "
6. $8531.40 .	31 "	12. $135.75 .	77 "	18. $718.25 .	312 "

(In the following examples, find the exact time in days by **219.**)

What is the interest at 6% per annum

19. Of $738.25 from Oct. 25, 1884, to Jan. 20, 1885?
20. Of $8271.43 from May 12, 1883, to March 5, 1884?
21. Of $1538.65 from Feb. 18, 1884, to Nov. 18, 1884?
22. Of $350.50 from Sept. 30, 1884, to Dec. 31, 1884?
23. Of $837.15 from Jan. 13, 1884, to July 6, 1884?
24. Of $1213.16 from March 5, 1884, to Jan. 3, 1885?
25. Of $628.45 from Nov. 13, 1883, to March 8, 1884?
26. Of $75.80 from May 25, 1884, to Jan. 31, 1885?
27. Of $950.38 from Apr. 18, 1886, to Oct. 17, 1886?
28. Of $32.12 from June 9, 1885, to Aug. 13, 1885?
29. Of $723 from May 4, 1886, to July 3, 1886?
30. Of $172.35 from Nov. 18, 1884, to Feb. 22, 1885?
31. Of $5280 from Aug. 14, 1883, to Apr. 3, 1884?
32. Of $1538.25 from May 7, 1885, to July 12, 1885?
33. Of $7125.40 from Oct. 15, 1885, to May 3, 1886?
34. Of $917.25 from Apr. 20, 1886, to Dec. 12, 1886?
35. Of $250.70 from Jan. 25, 1885, to Nov. 15, 1885?
36. Of $1215.30 from Mar. 7, 1886, to Aug. 3, 1886?
37. Of $720.19 from Nov. 15, 1886, to Feb. 4, 1887?

317

318. To find the interest at 6% by the "Banker's Method," when the time is less than 60 days?

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $650 for 36 days at 6% per annum?

Explanation.—One thousandth of the principal is the interest for 6 days (**316**). Hence, using a perpendicular line as a separatrix, cut off 3 figures from the right of the principal, obtaining $.65 as the interest for 6 days; and 6 times this result or $3.90 will be the interest for 6 times 6 days or 36 days.

Solution.

$	650, int. for 6 da.
$3	90, " " 36 "

2. What is the interest of $928.50 for 45 days at 6% per annum?

Explanation.—Using a perpendicular separatrix, cut off three figures to find the interest for 6 days, obtaining $.9285. Then 7 times this interest for 6 days, or $6.4995, will be the interest for 42 days; and one-half this interest for 6 days, or $.4643, will be the interest for 3 days; and the sum of the interest for 42 days + the interest for 3 days will be the interest for 45 days.

Solution.

$	928.50, int. for 6 da.
6	4995, " " 42 "
	4643, " " 3 "
$6	96+, " " 45 "

3. What is the interest of $8000 for 22 days at 6% per annum?

Explanation.—If $8 is the interest for 6 days, 4 times $8 or $32 is the interest for 24 days (2 days more than the given time). Subtract the interest for 2 days (⅓ of $8, or $2.667) from the interest for 24 days ($32) to find the interest for 24—2, or 22 days.

Solution.

8	000, int. for 6 da.
32	" " 24 "
2	667, " " 2 "
$29	333, " " 22 "

Rule.—I. *Draw a perpendicular line three places to the left of the decimal point of the principal. The result will be the interest of the principal for 6 days, the figures to the left of the perpendicular line being dollars.*

II. *Take as many times the interest for 6 days as the given days are times 6 days, or such a part of the interest for 6 days as the given days are a part of 6 days.*

NOTE 1.—Consider the interest for 1 day as ⅙ of the interest for 6 days; for 2 days as ⅓ of the interest for 6 days; for 3 days as ½ of the interest for 6 days; for 4 days, deduct ⅓ of the interest for 6 days from the interest for 6 days; for 5 days, deduct ⅙ of the interest for 6 days from the interest for 6 days.

318

NOTE 2.—In applying the above rule, if the time is less than 60 days, a judicious subtraction, as in Ill. Ex. 3, will always prevent more than *two* statements.

EXAMPLES FOR PRACTICE.

Using the above Rule, and proving results by 317, find the interest.

Principal.	*Time.*	*Principal.*	*Time.*	*Principal.*	*Time.*
1. $937.58..	40 days.	6. $532.50..	19 days.	11. $418.30..	49 days.
2. $1250.75.	52 "	7. $1286.30.	25 "	12. $125.90..	53 "
3. $391.40..	47 "	8. $9850....	55 "	13. $90.75...	11 "
4. $862.65..	38 "	9. $3275.25.	32 "	14. $8.35....	43 "
5. $258.....	27 "	10. $891.50..	29 "	15. $7.60....	34 "

319. To find the interest at 6% by the "Banker's Method" for any number of days.

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $780 for 75 days at 6%?

Explanation.—If 6% is the rate for 1 yr. or 360 da., ⅙ of 6%, or 1% must be the corresponding rate for ⅙ of 360 da., or 60 da. Hence, using a perpendicular separatrix, cut off two figures to find the interest for 60 da., and ¼ of the interest for 60 days will be the interest for 15 da. Add the two results to find the interest for 60 + 15, or 75 days.

Solution.

$7	80,	int.	for	60	da.
1	95,	"	"	15	"
$9	75,	"	"	75	"

2. What is the interest of $538.75 for 318 days at 6%?

Explanation.—If $5.3875 is the interest for 60 da., 5 times that sum or $26.9375 is the interest for 5 times 60 da., or 300 da.; and, using the 6-day rule, .3 times $5.3875, or $1.61625 is the interest for $\frac{3}{10}$ of 60 da., or 18 da.

REM.—To multiply by any number of tenths, proceed as if multiplying by an integer, and place the orders of the product *one place to the right* of the orders multiplied.

Solution.

$5	38.75,	int.	for	60	da.
26	9375,	"	"	300	"
1	61625	"	"	18	"
$28	55 +	"	"	318	"

3. What is the interest of $950 for 112 days at 6%?

Explanation.—Add the interest for 60 da. ($9.50); the interest for $\frac{8}{10}$ of 60 da. or 48 da. ($7.60); and the interest for $\frac{1}{12}$ of 48 da. or 4 da. ($\frac{1}{12}$ of $7.60 = $.63 +), obtaining $17.73 + as the required interest for 112 da.

Solution.

$9	50,	int.	for	60	da.
7	60,	"	"	48	"
	63, +	"	"	4	"
$17	73 +	"	"	112	"

Rule.—I. *Draw a perpendicular line two places to the left of the decimal point of the principal. The result will be the interest of the principal for 60 days or 2 months.*

II. *Take as many times the interest for 60 days as the given days are times 60 days; or such a part of the interest for 60 days as the given days are a part of 60 days.*

Note.—By applying the above rule for the largest multiple of 60 days contained in the given time, and then using rule, **318**, for the remaining days, as was done in Ill. Ex. 2 and 3, the maximum number of statements in any example will be *three*.

EXAMPLES FOR PRACTICE.

Using the above Rule, and proving results by Rule, 317, find the interest.

	Principal.	*Time.*		*Principal.*	*Time.*		*Principal.*	*Time.*
1.	$750	137 da.	7.	$7136.20	250 da.	13.	$4500	192 da.
2.	$175.50	173 "	8.	$7500.50	117 "	14.	$825.30	144 "
3.	$5282	121 "	9.	$596.20	136 "	15.	$612.20	156 "
4.	$723	161 "	10.	$317.40	112 "	16.	$718.25	312 "
5.	$1835.80	174 "	11.	$4181.25	85 "	17.	$875.75	189 "
6.	$812.80	311 "	12.	$950	76 "	18.	$1532.38	320 "

At 6% per annum, what is the interest

19. Of $875.10 from June 1, 1885, to Aug. 3, 1885?
20. Of $74.50 from Jan. 3, 1884, to May 4, 1884?
21. Of $512.75 from Dec. 15, 1885, to Jan. 18, 1886?
22. Of $5826.45 from Aug. 15, 1885, to June 3, 1886?
23. Of $925.38 from May 13, 1885, to Dec. 24, 1885?
24. Of $1917.25 from Jan. 25, 1884, to Aug. 8, 1884?
25. Of $5863.50 from Feb. 28, 1886, to Oct. 18, 1886?
26. Of $928.80 from June 2, 1884, to Aug. 19, 1885 (1 yr. 78 da.)?
27. Of $759.40 from May 5, 1885, to Sept. 19, 1886?
28. Of $2359.65 from Jan. 12, 1884, to May 15, 1885?
29. Of $8211.75 from Mar. 7, 1882, to Aug. 31, 1883?
30. Of $506.15 from Dec. 15, 1884, to Mar. 8, 1886?
31. Of $1275.25 from Feb. 15, 1885, to Sept. 8, 1885?
32. Of $5918.70 from Oct. 21, 1885, to Feb. 23, 1886?
33. Of $2612.50 from Jan. 12, 1886, to Aug. 7, 1886?
34. Of $4105.09 from May 30, 1885, to Nov. 18, 1886?

319

320. To find the interest at 6% for years, months and days.

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $875 for 5 months and 12 days at 6% ?

Solution.

$8	75, int. for 2 mo.
17	50, " " 4 mo.
6	125, " " 1 mo. 12 da. (42 da.)
$23	63—, " " 5 mo. 12 da.

Proof.

5 mo. 12 da. = 162 da.

($875 × .162) ÷ 6 = $23.625 (**317**).

Explanation.—If 6% is the rate for 1 yr. or 12 mo., 1/6 of 6%, or 1%, will be the corresponding rate for 1/6 of 12 mo., or 2 mo.

Hence, cut off two figures from the right of the principal, obtaining $8.75 as the interest for 2 mo.; and twice this result, or $17.50, will be the interest for twice 2 mo., or 4 mo.; and .7 times the interest for 2 mo. (60 da.), or $6.125, will be the interest for .7 of 60 da., or 42 da. (See Rem. to Ill. Ex. 2, **319**.) Add the two results, obtaining $23.63— as the required interest for 5 mo. 12 da.

Proof.—An ordinary interest year is supposed to be divided into 12 months of 30 days each. Hence, to prove the first solution, multiply the months (5) by 30 and add in the odd days (12), obtaining 162 days as the equivalent time in days. Then apply Rule, **317**.

2. What is the interest of $1248 for 1 yr. 4 mo. 5 da. at 6% ?

Solution.

$12	48, int. for 2 mo. or 60 da.
99	84, " " 8 times 2, or 16 mo.
1	04, " " 1/12 of 60, or 5 da.
$100	88, int. for 1 yr. 4 mo. 5 da.

Proof.

1 yr. 4 mo. 5 da. = 16 mo. 5 da.

(16 × 30) + 5 = 485, time in days.

($1285 × .485) ÷ 6 = $100.88 (**317**).

Explanation.—Draw a perpendicular separatrix as before, obtaining $12.48 as the interest for 2 mo. Hence, 8 times $12.48, or $99.84, must be the interest for 8 times 2 mo., or 1 yr. 4 mo.; and 1/12 of the interest for 2 mo. (60 da.), or $1.04, will be the interest for 5 da. Add the interest for 1 yr. 4 mo. ($99.84) to the interest for 5 da. ($1.04) to find the interest for 1 yr. 4 mo. 5 da.

Proof.—Reduce the time to days, considering the year as containing 360 da., and each month as containing 30 da.; then find the interest by Rule, **317**.

Rule.—I. *Draw a perpendicular separatrix two places to the left of the decimal point of the principal. The result will indicate the interest for 2 months or 60 days.*

320

II. *Take as many times the above result as the given months are times 2 months; and apply* **318** *or* **319** *for any time less than 2 months.*

Proofs.—1. *Reduce the time to days by multiplying the years by 360, the months by 30, and adding in the odd days. Then apply Rule,* **317**.

2. Or, *multiply the principal by the rate for the given time, which is one-half as many hundredths as there are months plus one-sixth as many thousandths as there are days.*

Note.—The second method of proof is convenient when the odd days are exactly divisible by 6.

Examples for Practice.

Find the interest at 6% by the above rule and prove results.

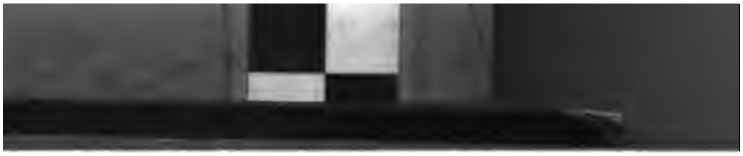
	Principal.	*Time.*		*Principal.*	*Time.*
1.	$7500	3 mo. 15 da.	**11.**	$325.45 . .	1 yr. 6 mo. 18 da.
2.	$8125.42 . .	9 " 18 "	**12.**	$896.30 . .	2 " 3 " 6 "
3.	$918.30 . . .	11 " 25 "	**13.**	$7125.30 .	1 " 7 " 15 "
4.	$4126.50 . .	8 " 7 "	**14.**	$85.15 . . .	1 " 2 " 19 "
5.	$816.10 . . .	5 " 3 "	**15.**	$815.20 . .	2 " 3 " 11 "
6.	$915.20 . . .	7 " 11 "	**16.**	$240.35 . .	1 " 8 " 6 "
7.	$8175.25 . .	5 " 13 "	**17.**	$8734.12 .	3 " 4 " 24 "
8.	$4150.35 . .	10 " 18 "	**18.**	$342.15 . .	4 " 9 " 15 "
9.	$523.05 . . .	5 " 16 "	**19.**	$1875.65 .	2 " 5 " 9 "
10.	$8275.75 . .	1 " 20 "	**20.**	$517.40 . .	3 " 11 " 25 "

(In the following examples, find the time by compound subtraction, **218**.)

What is the interest at 6% per annum

21. Of $478.32 from May 3, 1882, to July 6, 1884?
22. Of $1375.70 from Oct. 8, 1879, to Jan. 30, 1885?
23. Of $51.30 from Feb. 13, 1881, to July 9, 1883?
24. Of $1286.58 from Sept. 25, 1880, to Mar. 3, 1883?
25. Of $5.36 from Nov. 10, 1878, to May 8, 1885?
26. Of $138.50 from Jan. 18, 1884, to Jan. 15, 1885?
27. Of $8270.75 from Mar. 12, 1881, to Feb. 28, 1885?
28. Of $415.60 from Aug. 8, 1882, to May 19, 1884?
29. Of $12.80 from July 30, 1880, to Aug. 31, 1883?
30. Of $280.70 from May 1, 1878, to Apr. 12, 1884?

320



What is the interest at 6% per annum

31. Of $1508.25 from Mar. 15, 1884, to Sept. 9, 1886?
32. Of $395.42 from Aug. 19, 1880, to July 3, 1882?
33. Of $531.73 from June 23, 1882, to May 12, 1884?
34. Of $9217.15 from Apr. 5, 1883, to Oct. 17, 1885?
35. Of $3519.20 from Feb. 15, 1885, to July 25, 1886?

321. To find the interest at any rate per annum by the 6% rule.

ILLUSTRATIVE EXAMPLE.

What is the interest of $825.40 for 7 mo. 15 da. at 4%?

Explanation.—First find the interest at 6%, as shown in **320**, obtaining $30.953.

Since the given rate (4%) is ⅓ less than the assumed rate (6%), the interest at 4% is ⅓ less than the interest at 6%. Hence, diminish $30.958 by ⅓ of itself ($10.318), obtaining $20.64— as the required interest at 4%.

Solution.

$8	25.40,	int. for 2 mo. at 6%.
24	762	" " 6 " " "
4	127	" " 1 " " "
2	064 —	" " 15 da. " "
$30	953 —	" 7 mo. 15 da. " 6%
10	318 —	" " " " 2%
$20	635	" " " " 4%.

Proof.—Reduce the time to days and find the interest at 6% by Rule, **317**. 4% is ⅔ of 6%, therefore take ⅔ of the interest at 6% to find the interest at 4%.

Proof.

7 mo. 15 da. = 225 da.

($825.40 × .225) ÷ 6 = $30.953, int. at 6%

($30.953 × 4) ÷ 6 = $20.635, int. at 4%.

Rule.—*Find the interest at 6% for the given time, and increase or diminish the result by such a part of itself as the given rate per annum is greater or less than 6%.*

Proof.—*If necessary, reduce the time to days; and find the interest at 6% by Rule,* **317.** *Multiply the interest at 6% by the given rate per annum, and divide the product by 6.*

NOTE 1.—The interest at 6% being found, to obtain the interest at 1%, divide by 6; at 1½%, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract ⅓ of the interest from itself; at 4½%, subtract ¼ of the interest from itself; at 5%, subtract ⅙ of the interest from itself; at 5½%, subtract $\frac{1}{12}$ of the interest from itself; at 6½%, add $\frac{1}{12}$ of the interest to itself; at 7%, add ⅙ of the interest to itself; at 7½%, add ¼ of the interest to itself; at 8%, add ⅓ of the interest to itself; at 9%, add ½ of the interest to itself; *and at any per cent., multiply the interest at 6% by the given rate per annum, and divide the product by* 6.

321

EXAMPLES FOR PRACTICE.

Find the interest by the above rule, and prove results.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	mo.	da.
✓ 1.	$105	8%	157 da.	✓ 6.	$895.80	7%		3	5
2.	$450	4%	79 "	7.	$375.20	10%		2	15
3.	$575.30	3%	85 "	8.	$4105.30	8%		10	24
4.	$350.90	5%	48 "	9.	$9500	9%	1	9	12
✓ 5.	$7500.	7%	139 "	✓ 10.	$182.40	4%	1	3	18

Finding the exact time in days, what is the interest

11. Of $375.45 from May 9, 1886, to July 7, 1886, at 8%?
12. Of $1240.35 from Sept. 15, 1885, to Feb. 5, 1886, at 5%?
13. Of $526.62 from June 8, 1884, to Dec. 30, 1884, at 7%?
14. Of $857.25 from Feb. 12, 1884, to Nov. 9, 1884, at 4%?
15. Of $7285.15 from July 4, 1886, to Jan. 3, 1887, at 4½%?
16. Of $612.75 from Jan. 1, 1886, to Mar. 5, 1886, at 9%?

Finding the time by compound subtraction, what is the interest

17. Of $2536.75 from May 19, 1879, to July 15, 1884, at 3%?
18. Of $746.20 from Apr. 5, 1882, to Jan. 9, 1886, at 7%?
19. Of $428.50 from Oct. 25, 1878, to May 21, 1885, at 12%?
20. Of $815.30 from July 15, 1883, to Oct. 19, 1883, at 7%?

322. Direct method of finding the interest at any rate per annum.

ILLUSTRATIVE EXAMPLES.

1 What is the interest of $157 for 84 days at 5% per annum?

Explanation.—If 5% of the principal is charged for its use one year (360 da.), $\frac{1}{5}$ of 5% or 1% of the principal should be charged for its use $\frac{1}{5}$ of 360 da., or 72 da. Hence, draw a perpendicular line as previously explained, obtaining $1.57 as the interest for 72 da.; take $\frac{1}{6}$ of $1.57, or $.26, as the interest for $\frac{1}{6}$ of 72 da., or 12 da. Add the two results.

Solution.

$1	57,	int. for 72 da.
	26 +	" " 12 "
$1	83 +	" " 84 "

Proof.

$(1.57 \times 84) \div 72 = \$1.83+$

Proof.—Cut off two figures from the right of the principal, obtaining $1.57 as the interest for 72 da. Hence, for 84 da., the interest must be $\frac{84}{72}$ of $1.57, which by cancellation can be simplified to $\frac{7}{6}$ of $1.57, or $1.83+.

322

2. What is the interest of $385.20 for 108 days at 8%?

Explanation.—If 8% is the rate for 1 yr. (360 da.), ⅛ of 8%, or 1%, must be the corresponding rate for ⅛ of 360 da., or 45 da. Therefore, draw a perpendicular line two places to the left of the decimal point, obtaining $3.852 as the interest for 45 da.; take twice $3.852, or $7.704, as the interest for twice 45, or 90 da.; ⅓ of $3.852, or $1.284, as the interest for ⅓ of 45 da., or 15 da.; take ⅕ of of $1.284, or $.2568 as the interest for ⅕ of 15 da., or 3 da. Add the three results.

Solution.

$3	8520,	int. for 45 da.
7	704,	int. for 90 da.
1	284,	" " 15 "
	2568,	" " 3 "
$9	24+	" " 108 "

Proof.

($3.852 × 108) ÷ 45 = $9.24+

Proof.—If, as already explained, the interest for 45 da. is $3.852, the interest for 108 da. is 108/45 of $3.852, which by reducing fraction to lowest terms, is simplified to 12/5 of $3.852, or $9.24+.

3. What is the interest of $750 for 1 yr. 5 mo. 18 da. at 9%?

Explanation.—If 9% of the principal is the rate for 1 yr. or 360 da., 1% of the principal must be the rate for ⅑ of a year, or 40 da. (= 1 mo. 10 da.). Hence, multiply the interest for ⅑ yr. ($7.50) by 9, obtaining $67.50 as the interest for 9/9 yr. or a whole year; multiply the interest for ⅑ yr. (40 da. or 1 mo. 10 da.) by 4, obtaining $30 as the interest for 4 times 1 mo. 10 da., or 5 mo. 10 da.; take ⅕ of the interest for 40 da. ($7.50), obtaining $1.50 as the interest for ⅕ of 40 da., or 8 da. Add the three results.

Solution.

$7	50,	int. for ⅑ yr. or 40 da.
67	50,	" " 9/9 yr. or 1 yr.
30	00,	" " 5 mo. 10 da.
1	50,	" " 8 da.
$99	00,	" " 1 yr. 5 mo. 18 da.

Proof.

1 ordinary interest year = 360 da.
5 " " months = 150 da.
18
1 yr. 5 mo. 18 da. = 528 da.
($7.50 × 528) ÷ 40 = $99.

Proof.—Reduce the time to days, obtaining 528 days. If the interest for 40 da. is $7.50, the interest for 528 da. must be 528/40, or 66/5 of $7.50.

Rule.—I. *Draw a perpendicular line two places to the left of the decimal point of the principal, and the result will be the interest of the principal for as many days as the given rate is contained times in 360.*

II. *Take as many times the above result as the given period is times the period required to produce such result, or such a part of the above result as the given period is part of the period required to produce such result.*

NOTE 1.—Cutting off two integral places from the right of the principal will give the interest

At 1% per an., for 1 year, or 360 da.	At 6% per an., for 2 mo., or 60 da.
" 2% " " " 6 mo., or 180 da.	" 8% " " " 1 mo. 15 da., or 45 da.
" 3% " " " 4 mo., or 120 da.	" 9% " " " 1 mo. 10 da., or 40 da.
" 4% " " " 3 mo., or 90 da.	"10% " " " 1 mo. 6 da., or 36 da.
"4½% " " " 2 mo. 20 da., or 80 da.	"12% " " " 1 mo., or 30 da.
" 5% " " " 2 mo. 12 da., or 72 da.	

Proof.—Multiply 1% of the principal by the given time in days, and divide the product by the number of days required to produce interest equal to 1% of the principal.

NOTE 2.—To find how many days are required to produce interest equal to 1% of the principal, mentally divide 360 by the given rate per annum.

EXAMPLES FOR PRACTICE.

Find the interest by the above rule, and prove results.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	mo.	da.
1.	$126	5%	39 da.	9.	$317.95	4%		5	15
2.	$720	10%	51 "	10.	$758	4½%		8	17
3.	$1350	4%	79 "	11.	$375	3%		11	2
4.	$1800.50	8%	140 "	12.	$8736.28	10%		7	21
5.	$475.08	9%	65 "	13.	$3275.30	8%		4	8
6.	$1950.15	3%	118 "	14.	$528.75	5%	1	3	18
7.	$2758.25	4½%	315 "	15.	$719.80	9%	1	7	15
8.	$1275.50	5%	189 "	16.	$2125.25	12%	2	5	24

Finding the time in exact days, what is the interest

17. Of $750 from Mar. 19, 1886, to Aug. 16, 1886, at 10%?
18. Of $238.40 from Oct. 15, 1885, to Mar. 18, 1886, at 5%?
19. Of $7128.15 from Nov. 13, 1884, to Feb. 2, 1885, at 8%?
20. Of $1875.40 from Apr. 15, 1885, to Sept. 12, 1885, at 4½%?
21. Of $8175 from Aug. 13, 1883, to Jan. 5, 1884, at 4%?
22. Of $5320.25 from May 15, 1885, to Jan. 15, 1886, at 9%?

Finding the time by compound subtraction, what is the interest

23. Of $575.10 from May 4, 1884, to Feb. 2, 1885, at 5%?
24. Of $348.52 from Jan. 21, 1886, to Sept. 15, 1886, at 8%?
25. Of 9275.40 from July 21, 1880, to May 16, 1882, at 4%?
26. Of $5325.30 from Sept. 30, 1882, to Aug 14, 1884, at 3%?
27. Of $375.25 from Mar. 28, 1883, to Dec. 19, 1884, at 10%?
28. Of $2384.20 from Apr. 3, 1884, to July 13, 1885, at 4½%?

323. To find the amount.

ILLUSTRATIVE EXAMPLE.

On Jan. 5, 1884, I gave a note of $713.40, drawing interest at 4% per annum. What was the amount due on May 28, 1884?

Rule.—*Find the interest of the principal by the most convenient of the preceding rules; and add the principal thereto.*

Solution.

From Jan. 5 to May 28=144 da.

7	1340,	int. for 90 da.
3	567,	" " 45 "
	713+,	" " 9 "
713	40,	given principal.
$724	81+,	principal and interest.

Proof.

($7.134 × 144) ÷ 90 = $11.41, int.

$713.40 + $11.41 = $724.81, amt.

EXAMPLES FOR PRACTICE.

Find the amount, and prove result.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	mo.	da.
1.	$890.40	5%	29 da.	7.	$515.50	9%		1	19
2.	$315.30	4%	72 "	8.	$734.40	8%		7	28
3.	$3750.30	7%	33 "	9.	$600	5%		8	15
4.	$840	6%	168 "	10.	$325.80	7%		1	2
5.	$25	8%	92 "	11.	$815.15	4%	1	5	8
6.	$415.75	3%	75 "	12.	$5120.90	6%	2	11	12

324. To find the accurate interest (311).

ILLUSTRATIVE EXAMPLE.

What is the accurate interest of $325 for 24 days at 6%?

Explanation.—First find the interest for 1 yr., obtaining $19.50. Since the accurate number of days in a common year is 365, the accurate interest for 24 days must be $\frac{24}{365}$ of $19.50, or $1.28+.

Solution.

$325

.06

19.50, int. for 1 yr.

×24

365) $468.00 ($1.28+

Rule.—*Multiply the principal by the rate per annum to find the interest for one year.*

Multiply the interest for one year by the number of days and divide the product by 365.

Note 1.—If the time is greater than one year, multiply the interest for one year by the number of years, and add the product to the interest for the odd days in the fractional part of year as obtained by the above rule.

Note 2.—If it is seen that the ordinary interest at the same rate and for the same time will not much exceed $10, first find the ordinary interest, and mentally deduct $\frac{1}{73}$ of itself to obtain the accurate interest.

Note 3.—If the rate is 6% or less, it will be more convenient to move the decimal point of the principal *one place to the left*, multiply the result by *twice* the rate % per annum, then by the number of days, and then divide by 73. Thus, in the Ill. Ex., ($32.5 × .12 × 24)÷73=$1.28+.

Note 4.—Notes 2 and 3 may be used as proof methods to verify results obtained by the rule.

EXAMPLES FOR PRACTICE.

Find the accurate interest, and prove results.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	da.
1.	$187.30	5%	15 da.	7.	$8135.25	3%		93
2.	$1837.75	4%	115 "	8.	$782.50	5%		113
3.	$853.25	6%	18 "	9.	$4150.35	7%		25
4.	$75.50	8%	93 "	10.	$913.25	3½%	1	320
5.	$375.40	3½%	146 "	11.	$2150.75	7%	5	271
6.	$982.75	4½%	75 "	12.	$538.50	6%	2	118

What is the accurate interest

13. Of $758.20 from Mar. 13, 1886, to Aug. 28, 1886, at 6%?
14. Of $1290.75 from Jan. 10, 1885, to Oct. 7, 1885, at 7%?
15. Of $75.30 from May 13, 1880, to Jan. 15, 1885, at 3½%?
16. Of $945.38 from Aug. 5, 1879, to May 12, 1883, at 8%?
17. Of $3200 from Oct. 12, 1882, to May 31, 1885, at 4%?
18. Of $625.80 from Nov. 28, 1881, to Sept. 16, 1883, at 5%?
19. Of £235 17s. 6d. from May 2, 1885, to Sept. 25, 1885, at 5%?

Solution.—Reduce 17s. 6d. to a decimal of a £ by **215**, and annex to the integral £'s, obtaining £235.875. Then find the accurate interest at 5% for 146 days, as in the preceding examples, obtaining £4.7175. Reduce the decimal part of a £ (.7175) to integers of the lower denominations by **211**, obtaining 14s. 4.2d.; making the required interest £4 14s. 4.2d.

324

NOTE 5.—In reducing the lower denominations to a decimal of a £, extend the reduction to 5 decimal places and omit the terminal common fraction.

What is the accurate interest

20. Of £430 10s. from June 3, 1885, to Aug. 5, 1885, at 4%?
21. Of £280 15s. 6d. from May 4, 1886, to July 13, 1886, at 5%?
22. Of £790 8s. 5d. from Feb. 18, 1885, to May 6, 1885, at 3%?
23. Of 3219.40 marks from May 3, 1882, to Aug. 18, 1883, at 5%?

INTEREST PROBLEMS.

325. To find the rate per annum, when the principal, interest, and time are given.

ILLUSTRATIVE EXAMPLE.

At what rate will $630 produce $6.72 interest in 48 days?

Explanation.—The interest of $630 for 48 da. at 1% is $.84.

Divide the given interest ($6.72) by the interest at 1% ($.84), and it is seen that the given interest is 8 times the interest at 1%. Hence, the rate of the given interest must be 8 times 1%, or 8%.

Solution.

$ | 630, int. for 6 da. at 6%.
6)5 | 04, " " 48 " " "
| 84, " " " " " 1%.

$7.72 ÷ $.84 = 8 times (1%, or 8%).

Proof:—Int. of $630 for 48 da. at 8% = $6.72.

Rule.—*Find the interest of the given principal for the given time at 1% per annum; and divide the result into the given interest.*

NOTE.—If the amount is given, diminish it by the principal to find the interest.

EXAMPLES FOR PRACTICE.

Proving all results, at what rate per annum will

1. $648 yield $7.29 interest in 81 days?
2. $3500 produce $105 interest in 4 mo. 15 da.?
3. $8730 amount to $8974.44 in 8 mo. 12 da.?
4. $296 in 45 days produce $3.70 interest?
5. $500 in 9 mo. 6 da. yield $23 interest?
6. $3800 amount to $3874.10 in 78 days?
7. $810 yield $17.64 interest from May 3, '86, to Nov. 15, '86?
8. $750 amount to $776.50 from Mar., 1, '85, to Aug. 7, '85?
9. $380 amount to $396.91 from Apr. 7, '86, to Oct. 2, '86?

326. To find the time, when the principal, interest, and rate are given.

ILLUSTRATIVE EXAMPLE.

At 4% per annum, in what time will \$315 produce \$20.79 interest ?

Explanation.—Divide the given interest (\$20.79) by the interest of the principal for 1 year at the given rate (\$12.60), obtaining 1 yr. and a remainder of 819, that is, $\frac{819}{1260}$ of a year. Reduce $\frac{819}{1260}$ of a yr. to months by multiplying the numerator 819 by 12, and dividing the product (9828) by the denominator (1260), obtaining 7 mo. and a remainder of 1008, that is, $\frac{1008}{1260}$ of a mo. Reduce $\frac{1008}{1260}$ of a mo. to days by multiplying the numerator by 30, and dividing the product (30240) by the denominator (1260), obtaining 24 da.

Solution.

\$315 × .04 = \$12.60
\$12.60)\$20.79(1 yr.
1260
8.19
12
12.60)98.28(7 mo.
88.20
10.08
30
12.60)302.40(24 da.
2520
5040
5040

Proof:—Int. of \$315 at 4% for 1 yr. 7 mo. 24 da. = \$20.79.

Rule.—I. *Find the interest of the principal at the given rate for 1 unit of the highest apparent denomination of time; and divide the result into the given interest to find the number of units of that denomination of time.*

II. *If there is a remainder, multiply it by the next lower denomination of time, and divide the product by the first divisor.*

NOTE.—If the amount is given instead of the interest, apply Note, **325.**

EXAMPLES FOR PRACTICE.

Proving all results, in what time will

1. \$395 produce \$82.16 interest, if loaned at 6% per annum ?
2. \$1940 amount to \$2273.68, if loaned at 8% per annum ?
3. \$936 yield \$69.68 interest, if loaned at 8% per annum ?
4. \$765 amount to \$765.51, if loaned at 4% per annum ?
5. \$856 produce \$2.14 interest, if loaned at 5% per annum ?
6. \$7200 amount to \$7239.20, if loaned at 7% per annum ?
7. Any principal double itself at 3% ? At 4% ? At 5% ? At 6% ?
8. Any principal treble itself at 8% ? Quadruple itself at 7% ?

327. To find the principal, when the interest, time, and rate are given.

ILLUSTRATIVE EXAMPLE.

What sum loaned from Apr. 20, 1883, to Dec. 19, 1883, at 8% per annum, will produce $8.10 interest?

Solution.

May 15 to Dec. 19=243 da.
243÷45=5$\frac{4}{10}$ times 1¢=$.054.
$.054) $8.100 (150 times $1, or $150.

Proof.

Interest of $150 at 8% for 243 da.=$8.10.

Explanation.—8% of $1, or 8 cents, is the interest of $1 of principal for 1 yr. or 360 da. If 8 cents is the interest of $1 for 360 da., ⅛ of 8 cents, or 1 cent, is the interest of $1 for ⅛ of 360 da., or 45 da.

If 1 cent is the interest of $1 for 45 days, the interest of $1 for 243 da. will be as many times 1 cent as 45 is contained times in 243, or $.054.

If $1 of principal will produce $.054 interest, it will require as many dollars of principal to produce $8.10 interest as $.054 is contained times in $8.10, or $150.

Rule.—*Divide the given interest by the interest of $1 for the given time and at the given rate.*

NOTE.—To find the interest of $1, divide the rate per annum into 360, and the result into the given time in days, regarded as cents. If the time is given in years, months, and days, reduce to days by Proof I, **320**.

EXAMPLES FOR PRACTICE.

What principal will yield

1. $21.85 interest in 69 days at 5%?
2. $8.82 interest in 84 days at 6%?
3. $22.50 interest in 135 days at 4%?
4. $30.94 interest in 6 mo. 28 da. at 9%?
5. $40.60 interest in 1 yr. 5 mo. 12 da. at 8%?
6. $67.20 interest in 2 yr. 9mo. 18 da. at 4%?
7. $19.60 interest in 1 yr. 2 mo. 21 da. at 5%?
8. What sum loaned from Sept. 18, 1883, to Mar. 4, 1884, at 6% per annum, will produce $24.22 interest (find exact time in days)?
9. A lender received $182 interest on a sum loaned at 8%, on May 30, 1882, and paid Jan. 19, 1883. What was the sum loaned (find the time by comp. subt.)?

328. To find the principal, when the amount, time, and rate are given.

ILLUSTRATIVE EXAMPLE.

What sum of money loaned from Jan. 8, 1883, to May 12, 1883, at 9% per annum, will amount to $3134.24 ?

Explanation.—Find the interest of $1 at 9% for 124 da. as explained in **327**, obtaining $.031; annex the result to $1, forming the amount of $1.

If $1 of principal in 124 da. at 9% will amount to $1.031, it will require as many times $1 of principal to amount to $3134.24 as the given amount ($3134.24) is times the amount of $1 ($1.031), or $3040.

Solution.

Jan. 8 to May 12 = 124 da.

124 ÷ 40 = $.031, int. of $1.

$1 + $.031 = $1.031, amt. of $1.

$1.031) $3134.240 (3040 times $1.

Proof:—Amt. of $3040 at 9% for 124 da. = $3134.24.

Rule.—*Divide the given amount by the amount of $1 for the given time and at the given rate.*

NOTE.—To obtain the amount of $1, find the interest of $1 by Note, **327**; and annex the result to $1.

EXAMPLES FOR PRACTICE.

What principal will amount to

1. $176.13 in 135 days, if loaned at 8% ?
2. $419.80 in 297 days, if loaned at 6% ?
3. $677.10 in 120 days, if loaned at 5% ?
4. $7420.74 in 3 mo. 19 da., if loaned at 4% ?
5. $9051.84 in 1 mo. 11 da., if loaned at 9% ?
6. $1302.75 in 1 yr. 10 mo. 25 da., if loaned at 4½% ?
7. What principal loaned Sept. 5, 1885, at 6% per annum will amount to $4513.60 on Mar. 2, 1886 (find exact time in days) ?
8. On Mar. 18, 1885, I borrowed a sum of money at 5% per annum, and on Aug. 3, 1885, I paid $488.80 in full of principal and interest. What was the sum borrowed (find time by comp. subt.) ?
9. A merchant sold goods on 2 months' credit. The bill was paid 5 mo. 12 da. after the date of purchase, with interest at 6% per annum, by remitting a check for $762.75. What sum was allowed for interest ?

REVIEW OF INTEREST.

329. General Direction.—1. *If interest is required, multiply the principal by the rate of interest for the given time.*

The rate of interest for the given time is found by dividing the rate per annum into 360, and the result into the given time in days regarded as hundredths.

2. *If interest is given, divide it by the interest of 1 unit of the required term.*

REVIEW EXAMPLES.

1. On May 16, 1886, I borrowed $7500, and on Aug. 5, 1886, I paid $7635 in full of principal and ordinary interest computed for the exact time in days. What rate of interest did I pay?

2. If a man borrow $3450 in Maryland at the legal rate and loan it in California at the legal rate, what will be his gain in 11 mo. 9 da.?

3. A note of $525.30, bearing accurate interest at 5%, was given Jan. 19, 1881. What sum will discharge the note on July 15, 1883, no previous payments having been made?

4. A note of $582.75, at 5%, dated May 2, 1884, was paid Feb. 14, 1885. What was the amount of the payment (exact time in days)?

5. A note of $850 amounted to $855.10 on July 9, 1884. When was the note drawn, if interest was computed for the exact time in days at 4% per annum?

6. What sum loaned at 9% per annum on April 13, 1885, will amount to $977.16 on June 11, 1885, (time obtained by comp. subt.)?

7. On a sum borrowed at 6% per annum and loaned at 8% per annum, I realized a gain of $31.20 in 3 mo. 18 da. What amount did I owe the lender at the expiration of that time?

8. On Mar. 3, 1885, a merchant bought goods for $780.16 on 60 days' credit, but did not pay for them until June 15, 1885. What was the amount then due, computing interest at 8% for the exact time in days?

9. On June 17, 1884, I loaned $812 at 5% per annum, and when the debt became due I received $818.09 in full of principal and ordinary interest computed for exact days. At what time was the debt paid?

10. How much must I pay for the use of $8350, borrowed at 5% per annum on Jan. 18, 1885, and returned Dec. 12, 1885 (find the time by comp. subt.) ?

11. A partner was allowed 6% per annum on all sums invested with his firm, and charged 6% per annum on all sums withdrawn. At the beginning of the year he had $8000 invested; on May 13, he invested $1500 additional; and on Aug. 9, he withdrew $800. What was the interest balance to his credit at the close of the year (find the exact time in days) ?

12. A contractor borrowed a sum of money for 3 mo. 18 da. at 6%. Not having sufficient funds to meet the whole obligation at maturity, he gave his check for $3054, and agreed to continue the remainder at 8% per annum until paid. The sum paid by check was 60% of the debt and accrued interest. The remainder was paid 1 mo. 24 da. later, with interest. What was the amount of the second payment ?

13. On July 16, 1884, I gave my note for $380 with interest at 6% per annum, and when the note became due I paid $409.64 in full of principal and ordinary interest computed for time obtained by compound subtraction. When was the note paid ?

14. A house which cost $5400 rents for $30 per month. What is the net rate per annum of interest received on the investment, if the average annual expenses are $144 ?

15. On Dec. 15, 1883, a manufacturer borrowed a sum of money at 6%, and on Mar. 3, 1884, he paid $9118.50 in full of principal and ordinary interest computed for time obtained by compound subtraction. What sum did he borrow ?

16. I bought a house and lot on speculation for $15325; and 5 mo. 11 da. from date of purchase, I sold the property for $16200. If money was worth 8% per annum, how much more did the transaction yield than by lending the purchase money at interest ?

17. An attorney collected a claim of $450 with ordinary interest thereon for the exact time from Aug. 29, 1885, to Dec. 14, 1885, at 7%. What were the net proceeds due the creditor, if the attorney's rate of commission was 10% ?

18. In what time will the interest at 8% be three-fifths of the principal ?

19. On August 15, 1881, I borrowed $7350, and on Mar. 7, 1883, I paid $7810.60 in full of principal and ordinary interest, computing the time for the fraction of a year in exact days. At what rate per annum was the sum loaned?

20. What is the difference between the accurate interest and the ordinary interest of $8500 from May 12, 1885, to Oct. 19, 1885, at 8% per annum, computing the exact time in days in both cases?

21. A sum of money loaned at 5% yielded $184.80 interest in 1 yr. 11 mo. 3 da. In what time would the same sum have produced $10.88 interest at 6% per annum?

22. A note of $584, dated Apr. 18, 1886, was paid July 2, 1886. What was the rate of interest charged, if the amount paid was $594.95?

23. A house which cost $8400 is rented for $700 per annum. What is the net annual rate % of interest on the investment, the average annual expenses being $125 for taxes, $21 for insurance, $75 for ground rent, and $59 for repairs?

24. What annual rate of interest on the net sales is equivalent to a discount of 5% on the gross sales, if the term of credit is 60 days?

25. What trade discount will equal 8% per annum interest on the net sales, if the term of credit is 90 days?

26. Considering each of the following columns as a separate question, if the terms marked ✓ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Principal........	✓	✓	✓	✓	?	?	✓	?	✓	..	✓	✓	..
Rate per annum .	✓	✓	✓	?	✓	✓	..	..	..	✓	?	✓	✓
Time............	✓	✓	?	✓	✓	✓	..	..	..	✓	✓	?	✓
Interest........	?	..	✓	✓	✓	..	✓	✓	?	✓	..	..	?
Amount.........	..	?	..	..	..	✓	?	✓	✓	?	✓	✓	✓

ANNUAL INTEREST.

330. Annual Interest is interest which is contracted to be paid at the end of each interest year.

REM. 1.—When notes or other contracts do not specify the time at which interest shall be paid, the law presumes that it is not due or collectible until the principal matures.

REM. 2.—If interest is contracted to be paid at specified periods, and falls in arrears, a few States sanction the collection of simple interest on the deferred interest payments from the time at which they were due. Where interest upon deferred interest payments cannot be legally collected, the lender can demand from the borrower a non-interest-bearing note for the principal and additional non-interest-bearing notes for the interest payments, one maturing at the end of each interest period. Since these notes, if not paid at maturity, will thereafter draw legal interest, the borrower is thus practically compelled to pay interest upon interest.

ILLUSTRATIVE EXAMPLE.

What is the amount of the following note at maturity, no payments having been previously made?

$800. Rutland, Vt., May 4, 1882.

Four years after date, I promise to pay J. S. Sinclair, or order, for value received, Eight Hundred Dollars, with interest at 6%, payable annually. J. E. Herrell.

Solution.

Face of note at interest from May 4, 1882, is	$800.00
Interest of $800 for 4 years at 6% ($800 × .06 × 4) is	192.00
Annual interest of $800 ($800 × .06) = $48.	
Int. of $48 for 3 + 2 + 1 (= 6 yr.) at 6% ($48 × .06 × 6) is . .	17.28
Amount due at maturity of note	$1009.28

Explanation.—If the interest had been paid when due, the total interest payment would have been four years' interest of $800 at 6%, or $192; but, since the first annual interest payment ($48) was retained 3 years after it was due; the second annual interest payment, 2 years; and the third, 1 year; which is equivalent to *one* annual interest payment ($48) retained 3 + 2 + 1, or 6 years after it was due, increase the previous interest by the simple interest on the deferred interest payments (= 6 years' interest of $48 at 6%), amounting to $17.28, etc.

Rem. 3.—The total time during which the interest in the above example was held after it became due may be mentally obtained as follows:

Number of interest periods minus 1 = 4 − 1, or 3.
Half the number of interest periods = ½ of 4, or 2.
Total time = 3 × 2, or 6 years.

That is, multiply the number of interest periods minus 1 by half the number of interest periods; or, if more convenient, the number of interest periods by half that number minus 1.

Rule.—*Find the simple interest of the principal to the date of settlement, and increase the result by the interest of each deferred interest payment from its due date to the date of its payment. The result will be the total interest; to which add the principal to find the amount.* **330**

NOTE.—If the time between the date of the note and the date of settlement contains a fraction of a period, multiply such fraction of a period by the number of whole interest periods, and add the product to the result obtained by applying Rem. 3 to the whole periods.

EXAMPLES FOR PRACTICE.

Find the amount due at maturity.

	Principal.	*Rate per annum.*	*Time.*	*Interest payable.*
1.	$9500	6%	5 years	annually.
2.	$350	8%	3 "	"
3.	$4500	5%	4 "	"
4.	$8000	9%	5 " 3 mo.	"
5.	$5000	10%	3 "	semi-annually.
6.	$7600	7%	2 "	"
7.	$9250	6%	3 "	"
8.	$7300	8%	4 " 1 mo.	"
9.	$420	4%	1 "	quarterly.
10.	$3800	6%	2 " 3 mo	"
11.	$175	5%	1 " 2 "	"
12.	$825	6%	2 " 11 "	"

13. No interest having been previously paid, what is the amount of a note of $2400, at 6%, interest payable annually, dated Jan. 1, 1879, and paid Feb. 1, 1884?

14. A merchant bought a warehouse for $8000, for which sum he gave his note without interest, payable in 2 years from date, and 8 separate non-interest-bearing notes for the quarterly interest at 8% per annum, payable respectively in 3 mo., 6 mo., 9 mo., 12 mo., 15 mo., 18 mo., 21 mo., and 24 mo. from date. If nothing was paid until the maturity of the note for $8000, what was the amount then due?

15. What amount will be due Aug. 15, 1884, on a note of $3500, drawing interest at 5% per annum, payable semi-annually, and dated Feb. 13, 1882, no previous payments having been made?

16. A note of $1532.60, dated Jan. 15, 1880, and drawing interest at 6% per annum, payable quarterly, was paid in full Mar. 13, 1883. What was the sum paid, no previous payments having been made?

17. What sum will be due Jan. 18, 1885, on a note of $3500, dated May 8, 1881, and drawing interest at 5% per annum, payable semi-annually, if the first four interest payments were paid when due, and no subsequent payments made?

COMPOUND INTEREST.

331. Compound Interest is interest which for the first interest period is computed upon the principal alone; and for subsequent periods, upon the principal increased by its accrued interest.

Rem.—Interest may be compounded annually, semi-annually, quarterly, or at still shorter intervals, if previously agreed to by the parties interested. In most States, the collection of compound interest cannot be legally enforced; but if the debtor consents to pay it, to receive compound interest does not constitute technical usury.

ILLUSTRATIVE EXAMPLE.

What is the compound interest of $4000 for 3 years at 6% per annum?

Explanation.—Find the interest of the original principal ($4000) for one year at 6%, obtaining $240, to which add the principal to find the sum due at the close of the first year, which will constitute a new principal for the second year ($4240). Next find the interest of this new principal ($4240) for the second year at 6%, obtaining $254.40, to which add the second year's principal ($4240) to find the sum due at the close of the second year, which will constitute a new principal for the third year. Proceed in this manner until the amount due at the close of the third year ($4764.06) is obtained, from which subtract the original principal ($4000) to find the compound interest.

Solution.

$4000, original principal.
.06, rate per annum.
$240.00, interest for 1st yr.
4000
$4240, new principal, 2d yr.
.06
$254.40, interest for 2d yr.
4240
$4494.40, new principal, 3d yr.
.06
$269.6640, interest for 3d yr.
4494.40
$4764.064, amt. at close of 3d yr.
4000, original principal.
$764.06, comp. interest for 3 yr.

Rule.—I. *Find the interest of the principal for the first interest period, add it to the principal, and the sum will be the second principal.*

II. *Find the interest of the second principal for the second period of time, add it to the second principal, and the sum will be the third principal.*

III. *Proceed in this manner as many different times as there are periods in the given time. The last result will be the amount due.*

IV. *Subtract the original principal from the amount due, and the remainder will be the compound interest.*

NOTE.—If the given time contains a fraction of an interest period, find by the rule the amount of the principal to the end of the last *complete* period; then compute the interest of this amount for the fraction of a period, and add it thereto.

EXAMPLES FOR PRACTICE.

Find the compound interest.

	Principal.	*Rate per annum.*	*Time.*	*Interest payable.*
1.	$8500	5%	3 yr.	annually.
2.	$6200	8%	2 "	semi-annually.
3.	$3500	4%	4 "	annually.
4.	$7200	6%	3 "	semi-annually.
5.	$5300	5%	3 " 5 mo.	semi-annually.
6.	$9000	7%	2 "	quarterly.
7.	$8200	8%	1 " 8 mo.	quarterly.
8.	$725	6%	1 " 7 "	quarterly.
9.	$1575	10%	2 " 5 " 18 da.	quarterly.
10.	$1836	4%	1 " 2 " 12 "	quarterly.

[In the following examples, find the time by comp. subtraction.]

11. A young man deposited $150 in a savings-bank, and received 5% per annum interest, payable quarterly, and compounded if not withdrawn. What amount was to his credit at the close of the second year?

12. What is the amount due Feb. 15, 1885, upon a note of $375.15, dated May 7, 1881, and drawing interest at 6% per annum, compounded semi-annually, no payment having been previously made?

13. What sum will, on May 19, 1884, discharge a note of $2750, dated Aug. 15, 1882, and drawing interest at 8% per annum, compounded quarterly, no previous payments having been made?

14. A note of $3500, dated May 5, 1880, drawing interest at 6% per annum, compounded semi-annually if not paid, had the first 5 interest payments paid when due. What was the amount due upon the note Dec. 25, 1884, if no subsequent payments were made?

332. The labor of computing compound interest may be abridged by the use of the following

COMPOUND INTEREST TABLE,

Showing the amount of $1 at compound interest for any number of years from 1 year to 55 years inclusive.

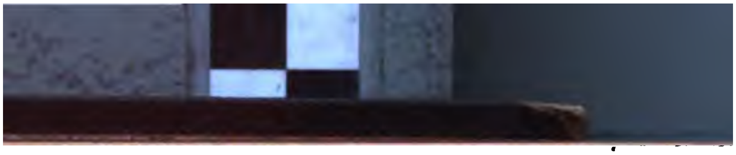
Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	2
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	11
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	12
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4460	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345	38
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50
51	1.6610 781	2.1368 211	2.7454 1979	3.5230 3644	4.5154 2320	5.7803 9930	7.3909 5068	51
52	1.6776 889	2.1688 734	2.8003 2819	3.6111 1235	4.6508 8590	5.9827 1327	7.6865 8871	52
53	1.6944 658	2.2014 065	2.8563 3475	3.7013 9016	4.7904 1247	6.1921 0824	7.9940 5226	53
54	1.7114 105	2.2344 276	2.9134 6144	3.7939 2491	4.9341 2485	6.4088 3202	8.3138 1435	54
55	1.7285 246	2.2679 439	2.9717 3067	3.8887 7303	5.0821 4859	6.6331 4114	8.6463 6692	55

NOTE 1.—To find the amount to which any given principal, at compound interest, will increase at any rate per annum, and for any number of years given in the preceding tables:

Multiply the given principal by the amount of $1 of principal for the given time and rate per annum, as shown in the Table.

NOTE 2.—To find the compound interest which any given principal will produce at any rate per annum, and for any number of years given in the preceding tables:

Subtract $1 from the amount of $1 for the given time and rate per annum, as shown in the table; the result will be the compound interest of $1 of principal. Multiply the compound interest of $1 of principal by the given principal.



COMPOUND INTEREST TABLE,

Showing the amount of $1 at compound interest for any number of years, from 1 year to 55 years inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 278	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992	42
43	6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 096	60.2400 692	43
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338	48
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529	50
51	9.4391 0490	12.0407 698	19.5253 635	31.5190 168	50.6537 415	81.0496 969	129.1299 382	51
52	9.8638 6463	12.6428 083	20.6968 853	33.7253 486	54.7060 408	88.3441 696	142.0429 320	52
53	10.3077 3853	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 252	53
54	10.7715 8677	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477	54
55	11.2563 0817	14.6356 309	24.6503 216	41.3150 015	68.9138 561	114.4082 616	189.0591 425	55

NOTE 3.—The amount of $1 for more than 55 years is equal to the product of the amount of $1 for any two or more numbers of years in the Table whose sum is equal to the given time. Thus, the amount of $1 at compound interest for 98 years at 8% per annum, is equal to the product of the amount of $1, at 8%, for 50 years ($46.9016125) multiplied by the amount of $1, at 8%, for 48 years ($40.2105731), that is, $46.9016125 × 40.2105731, or $1885.9407179+.

NOTE 4.—When the interest is to be compounded semi-annually, find, from the Table, the amount or interest at *one-half* the rate per annum for twice the number of years. Thus, the compound interest of $1 for 10 years at 5% per annum, *payable semi-annually*, is the same as the compound interest of $1 for 20 years at 2½% per annum, *payable annually*; and the compound interest of any sum for 8 years at 4% per annum, *compounded quarterly*, is the same as the compound interest of the same sum for 32 years, at 1% per annum, *compounded annually*.

333. Application of the compound interest tables.

EXAMPLES FOR PRACTICE.

1. What is the interest of $8000 for 28 years, if compounded annually at 6% per annum?

Solution.

$5.1116867, compound amount of $1 of principal for 28 years at 6%.
1.   , the principal, above compounded, which is to be deducted.
$4.1116867, compound interest of $1 of principal for 28 years at 6%.
8000, given principal.
$32893.4936000, comp. int. of $8000 of principal for 28 years at 6%.

Find from the Table the

2. Interest of $9600 for 17 years if compounded annually at 7%.
3. Amount of $7500 for 51 years if compounded annually at 5%.
4. Interest of $3200 for 36 yr. 3 mo. if compounded yearly at 6%.
5. Interest of $875 for 15 years if compounded half-yearly at 4%.
6. Amount of $425 for 23 years if compounded half-yearly at 8%.
7. Interest of $8200 for 9 yr. 5 mo., compounded half-yearly at 6%.
8. Interest of $9300 for 8 years if compounded quarterly at 4%.
9. Amount of $1500 for 12 years if compounded quarterly at 6%?
10. Amount of $5700 for 14 yr. 4 mo. if compounded quarterly at 6%.
11. If I deposit in a savings-bank $175, and the interest thereon is compounded quarterly at 4% per annum, how much should I receive at the end of 18 years, if nothing had been previously withdrawn?
12. For the benefit of his son, who is then 8 years old, a father invests $5000 at 6% per annum, interest payable semi-annually, and, on the same terms, promptly invests the interest as collected. How much should the son receive when he becomes 21 years old?
13. At what rate per annum will $750 amount to $2549.6747 in 21 years?

Solution.

Amt. of principal in 21 years.		Principal.		Amt. of $1 of prin. in 21 years.
$2549.6747	÷	$750	=	3.3995636.

Then, glance down the column of years until the given number of years (21) is reached, and thence to the right until the obtained amount of $1 ($3.3995636) is reached, finding it in the 6% column. Hence, the required rate is 6%.

Find from the Table the rate per annum

14. At which $5000 will amount to $20580.678 if compounded annually for 29 years.

15. At which $800 will amount to $1520.23416 if compounded half-yearly for 13 years.

16. At which $2780 will amount to $3744.2569 if compounded quarterly for 5 years.

17. At which $750 will produce $22807.065 interest, if compounded annually for 40 years.

18. At which $3500 will amount to $8007.74688, if compounded semi-annually for 14 years.

19. At which $50 will yield $14.1216 interest in 6 yr. 3 mo. if compounded quarterly.

20. In what time will $5000 amount to $6100.95 if compounded quarterly at 4% per annum ?

Solution.

Amt. of principal for required time.		Principal.		Amt. of $1 for required time.
$6100.95	÷	$5000	=	$1.22019.

Since the given rate is 4% per annum, or 1% per quarter, glance down the 1% column until the obtained amount of $1 ($1.22019) is reached, which will be found in the amounts for 20 years, or, in this instance, 20 quarters. Hence, the required time is 20 quarters or 5 years.

Find from the Table the time in which

21. $750 will amount to $1578.639 if compounded annually at 7%.

22. $100 will amount to $175.350605 if compounded half-yearly at 6%.

23. $300 will amount to $411.83571 if compounded quarterly at 8%.

24. A gentleman at the age of 30 years invested $1500 at 8% per annum, compounded annually. How old was he when the compound interest amounted to $6654.8106, no interest having been previously collected ?

25. What principal will amount to $9899.658 if compounded annually for 14 years at 5% per annum ?

Solution.

Amt. of required principal for 14 years at 5%.		Amt. of $1 for same time and rate.		Required principal.
$9899.658	÷	$1.9799316	=	$5000.

Find from the Table the principal which will

26. Amount to $2626.674 in 34 yr., compounded annually at 5%.

27. Amount to $11439.6384 in 14 yr., compounded half-yearly at 6%.

28. Yield $46.357 interest in 10 yr., compounded annually at 8%.

29. Yield $859.0056 interest in 6 yr., compounded quarterly at 6%.

30. What sum should be invested by a father for the benefit of his daughter who is now 4 years old, that, compounded semi-annually at 6% per annum, will amount to $10000 when she is 18 years old?

31. What sum, invested at 4% per annum, and the interest compounded quarterly for 12 years, will produce $325.40 interest?

32. What principal will amount to $9899.658, if compounded annually for 14 years at 5% per annum?

33. Considering each of the following columns as a separate question, if the terms marked ✓ are given, how may the term marked ? be found, with and without the use of the Compound Interest Table, and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Principal..........	✓	✓	?	?	✓	✓	✓	✓	..
Rate per annum....	✓	✓	✓	✓	?	?	✓	✓	✓
Time..............	✓	✓	✓	✓	✓	✓	?	?	✓
Compound interest.	?	..	✓	..	✓	..	✓	..	?
Compound amount.	..	?	..	✓	..	✓	..	✓	✓

PRESENT WORTH AND TRUE DISCOUNT.

334. The **Present Worth** of a debt due at a future time is its value *now*. Hence, it is a sum which, if put at interest to-day, will amount to the debt at the time that debt is due.

Rem.—To find the present worth of a debt is an application of **328,** the debt corresponding to the *amount;* the present worth, to the *principal;* the rate per annum which money is worth, to the *rate;* and the intervening time to the maturity of the debt, to the *time.*

335. Discount is an allowance for the payment of money before it is legally due.

336. True Discount is the difference between a debt due at a future time and its present worth.

337. To find the present worth of a debt.

ILLUSTRATIVE EXAMPLE.

I owe $3816.67, which is due in 4 mo. 3 da. without interest. What is the present worth of this debt, if the current rate of interest is 6%? What is the true discount if paid to-day?

Solution.

$1.0205, amt. of $1 for 4 mo. 3 da. at 6%.
$3816.6700 ÷ $1.0205 = 3740 times $1.
$3816.67 − $3740 = $76.67, true disc't.

Explanation. — $1 placed at interest to-day at the current rate (6%) will amount to $1.0205 in 4 mo. 3 da. Hence, $1 paid to-day is worth as much as $1.0205 paid in 4 mo. 3 da. from to-day.

If the present worth of $1.0205 of my debt is $1, the present worth of $3816.67, or the whole of my debt, must be as many times $1 as $1.0205 is contained times in $3816.67, that is, 3740 times $1, or $3740.

Rule.—*Divide the debt by the amount of $1 for the intervening time at the prevailing rate of interest to find the present worth. Subtract the present worth from the debt to find the true discount.*

NOTE 1.—To find the amount of $1, prefix $1 to the result obtained by Note, 327.

NOTE 2.—To find the present worth of a debt drawing interest, first find what will be the amount of the debt at its due date, and then apply the rule.

EXAMPLES FOR PRACTICE.

Find the present worth and true discount

1. Of $834.08 if due in 18 days and money is worth 5%.
2. Of $8320.01 if due in 73 days and money is worth 6%.
3. Of $306.20 if due in 93 days and money is worth 8%.
4. Of $305.55 if due in 7 mo. 15 da. and money is worth 8%.
5. Of $276.30 if due in 5 mo. 18 da. and money is worth 5%.
6. I bought 900 barrels of flour at $8 per barrel, one-half of which was to be paid in 6 months, and the remainder in 9 months. I paid the present worth of the bill at the date of purchase, the current rate of interest being 6%. How much less than the sum due at maturity did I pay for the flour?
7. A merchant offers a lot of goods for $587.25 and 90 days' credit. What will be an equivalent cash offer, money being worth 5%?

8. I can buy flour at $8.10 per barrel and 4 months' credit, at $7.95 and 2 months' credit, or at $7.86 for cash. What is the cost of 350 barrels bought on the most advantageous of these terms, money being worth 8% per annum?

9. A merchant paid the present worth of a debt of $803.60, 18 days before it was due. What was the true discount allowed, money then being worth 9%?

10. The invoice price of merchandise purchased on 60 days' time was $840, on which I was allowed a trade discount of 20% and 10%. I paid the present worth of the debt one week after the date of purchase, the rate of interest then current being 8%. How much less than the invoice price did I pay for the merchandise?

11. A merchant sold a house and lot for $8500, one-fourth to be paid in cash, and the remainder in 3 equal installments, due respectively in 1, 2, and 3 years, without grace, with interest at 6% from date of purchase. Five months after the sale, the merchant became embarrassed and sold his notes for the unpaid installments to a broker at such a price that the broker realized 8% interest on his investment. What did the merchant receive from the broker?

12. A bill of $5300, payable in 4 months, was settled 3 months after the date of purchase by paying its present worth. What was the true discount at 6% per annum?

13. If a debt of $3200, payable in 6 months with interest at 6% per annum, be given, what will be its present worth three months afterwards, money then being worth 8% per annum?

14. On Jan. 18, 1884, I bought a quantity of merchandise for $8000, on 3 months' credit. On Feb. 1, 1884, I made a payment of $2500; on Mar. 4, 1884, I made a second payment of $1800; and on Mar. 15, 1884, I paid the remainder of the debt at its present worth. How much did I pay Mar. 15, 1884, if the current rate of interest was 8%?

15. If, in each of the following columns, the terms marked √ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Present Worth……	√	√	?	?	..	..	√	√	√	√	√	√	?
Rate of Disc't per Annum……	..	..	..	√	√	√	?	√	√	√	√	?	√
Time……	..	..	..	√	√	√	√	?	√	√	?	√	√
True Discount……	√	?	√	..	?	√	..	..	?	..	√	√	√
Future Worth……	?	√	√	√	√	?	√	√	..	?	..	..	..

COMMERCIAL PAPER.

338. Commercial Paper embraces notes, checks, drafts, bills of exchange, letters of credit, etc., etc.

339. A **Note** (often called a promissory note) is a written promise by one party to pay to another a specified sum of money on demand, or at a designated time.

340. The **Maker** of a note is the party who makes the promise over his own signature; the **Payee** of a note is the party to whom, or to whose order, the note is made payable; the **Holder** of a note is the party who owns it; and the **Face** of a note is the sum of money (not including the interest which it may accrue) which the maker promises to pay the payee or holder of the note.

341. A **Negotiable Note** is one which is made payable to the bearer, or to the order of the payee. A negotiable note can be sold or transferred.

Rem. 1.—When a note is made payable to *bearer*, any person who is the lawful holder of the note is the payee.

Rem. 2.—When a note is made payable only to the party named therein, it is said to be *non-negotiable*.

342. An **Indorsement** is matter written on the back of a note or other commercial paper.

Rem. 1.—An indorsement is usually the signature of the payee, guarantor, or lawful holder, upon the back of a negotiable note or other negotiable paper; and its purpose is to enable the payee or lawful holder to sell or transfer his title or ownership to another.

Rem. 2.—The person who signs his name upon the back of a note is called an *indorser*, and the person in whose favor the indorsement has been made is called the *indorsee*.

Rem. 3.—One effect of an indorsement is to make the indorser liable to the indorsee or any subsequent holder of the note, for its payment at maturity, if the maker of the note or prior indorsers fail to pay it at maturity.

Rem. 4.—An **indorsement in blank** is simply the signature of the indorser written upon the back of the negotiable paper, by which it is made payable without further indorsement to *any* person who may subsequently become its holder. An **indorsement in full** is one in which the indorser states over his signature to *whose order* the negotiable paper is payable. A **conditional indorsement** is one which is subject to certain specified conditions, without the performance of which the indorser cannot be held liable. A **restrictive indorsement** is one which restricts the payment of the negotiable paper to some *particular person* so that he cannot transfer it, even if he wished. A **qualified indorsement** is one in which the indorser relieves himself from responsibility for payment by writing over his signature the words "*without recourse.*" The purpose of a qualified indorsement is to enable the indorser to sell or transfer his title to negotiable paper without subjecting himself to liability for its payment if protested.

SPECIMEN FORMS OF INDORSEMENT.

(*In Blank.*)	(*In Full.*)	(*Conditional.*)	(*Restrictive.*)	(*Qualified.*)
W. S. Hill.	Pay to R. S. Jones, or order. W. S. Hill.	Pay to R. S. Jones, or order, unless, before maturity, I give you notice not to pay. W. S. Hill.	Pay to R. S. Jones only. W. S. Hill.	Pay to R. S. Jones, or order, without recourse. W. S. Hill.

343. A **Protest** of a negotiable note is a formal statement by a notary public, at the request of the holder, that said note was presented to the maker thereof for payment, and by him refused.

REM. 1.—In case of non-payment of a note by the maker, each indorser can be held liable only after a formal protest has been made, and proper notice given to him. Each indorser may, in turn, be held liable to every subsequent indorsee, and may look for indemnity to every antecedent indorser.

REM. 2.—A note, when due, must be presented at the place at which it is made payable. When no particular place is specified, it is payable at the maker's place of business *during business hours;* or if his place of business is not known, at his place of residence *during family hours.*

344. **Days of Grace** are the *three days* beyond the time specified in the note, which are usually allowed for its payment unless the note contains the words "without grace."

345. The **Maturity** of a note is the time at which the note becomes legally due, and includes the "days of grace."

REM. 1.—Demand for the payment of a note should be made at maturity; and upon failure of the maker to pay the note, it should at once be protested, notice of which protest must be sent to the indorsers on the day of maturity or the next, otherwise they cannot be held responsible.

REM. 2.—If a note mature on Sunday or any legal holiday, it must be paid on the day previous; and if a legal holiday occur on Monday, notes maturing on that day must be paid on the preceding Saturday.

Rem. 3.—It is lawful to compute interest or discount for *all* 3 of the days of grace, though the debtor may lose the benefit of the last day of grace because it happens to fall on a Sunday or a legal holiday.

Rem. 4.—Calendar months are understood if the time is expressed in months. By a calendar month is meant the length of time from any date of one month to the same date of the succeeding month, whether the included days of the calendar month be 28, 29, 30, or 31.

Rem. 5.—If, in estimating the time of a note by calendar months, it should be found to fall due on the 29th, 30th, or 31st of a month which contains less than that number of days, the note would be nominally due on the last day of such month, and legally due on the 3d day of the succeeding month.

346. To find the date of maturity when the time is expressed in days.

ILLUSTRATIVE EXAMPLE.

A promissory note dated Sept. 9, 1885, was made payable 60 days after date. What was the date of legal maturity?

Explanation.—Subtract the number of days which the note still has to run in Sept. (30−9=21) from the full time the note has to run (60 da. + 3 da. grace=63), obtaining 42 days as the time which the note has to run in the months succeeding September. Subtract the full number of days in the next succeeding month to Sept. (Oct. = 31 da.), obtaining 11 da. which the note has to run in the next succeeding month (Nov.).

Solution.

60 + 3 = 63 days, time to run.
21 " remaining in Sept.
42 " after September.
31 " in October.
Nov. 11, 1885, maturity.

Mental Process.—Reduce the time to uniform calendar months of 30 da. each (63 da. = 2 mo. 3 da.); add the result to the date of the note (Sept. 9 + 2 mo. 3 da. = Nov. 12); make correction by deducting 1 day for the included 31-day month of Oct. (Nov. 12 − 1 = Nov. 11).

EXAMPLES FOR PRACTICE.

Find the date of legal maturity.

	Date of note.	*Nominal time.*		*Date of note.*	*Nominal time.*
1.	Feb. 18, 1883 . .	60 days.	6.	Jan. 19, 1884 . . .	90 days.
2.	Mar. 31, 1884 . .	30 "	7.	Apr. 4, 1886 . . .	60 "
3.	Aug. 5, 1886 . .	90 "	8.	June 25, 1885 . .	4 months.
4.	Dec. 31, 1885 . .	50 "	9.	May 13, 1886 . . .	3 "
5.	Sept. 30, 1885 . .	40 "	10.	Oct. 31, 1885 . . .	2 "

347. To find the number of days to legal maturity.

ILLUSTRATIVE EXAMPLE.

A note dated Mar. 12, 1886, was made payable 3 months after date. What was the number of days to legal maturity?

Explanation.—Mar. 12 + 3 mo. = June 12, nominal date of maturity. Add 3 days for grace, obtaining June 15 as the legal date of maturity. Find the number of days between Mar. 12 and June 15 by Rule, **219**, obtaining 95.

Solution.

Mar. 12 + 3 mo. 3 da. = June 15.
Days remaining in March = 19.
" in April = 30.
" " May = 31.
" included in June = 15.
" from Mar. 12 to June 15 = 95.

Mental Process.—Multiply the number of calendar months by 30 (3 × 30 = 90); to the product add 3 days for grace (90 + 3 = 93); and increase the result 1 day for each of the included 31-day months of Mar. and May (93 + 2 = 95).

EXAMPLES FOR PRACTICE.

Find the number of days to legal maturity.

	Date of note.	*Nominal time.*		*Date of note.*	*Nominal time.*
1.	Sept. 12, 1883	4 months.	6.	July 4, 1885	5 months.
2.	Aug. 30, 1885	2 "	7.	Feb. 15, 1884	2 "
3.	Jan. 8, 1884	3 "	8.	Oct. 12, 1883	5 "
4.	Dec. 15, 1883	4 "	9.	May 8, 1884	3 "
5.	Apr. 3, 1886	3 "	10.	July 2, 1885	4 "

348. To find the number of days between two dates.

ILLUSTRATIVE EXAMPLE.

A promissory note dated May 15, 1885, was nominally due Aug. 2, 1885. What was the number of days to legal maturity?

Explanation.—Add 3 days of grace to the result obtained by applying Rule, **219**.

Mental Process.—May 15 to Aug. 15 = 3 mo. or 90 da.; therefore to Aug. 2, it will be 13 days less than 90, or 77 da.; to which add 1 day for each of the 31-day months of May and July, obtaining 79; and 3 days for grace, obtaining 82.

Solution.

Days remaining in May, 16
" in June, 30
" in July, 31
" included in Aug., 2
" of grace, 3
" note has to run, 82

EXAMPLES FOR PRACTICE.

Find the number of days to legal maturity.

	Date of note.	*Nominally due.*		*Date of note.*	*Nominally due.*
1.	May 3, 1886,	Sept. 15, 1886.	6.	Jan. 8, 1884,	Apr. 15, 1884.
2.	Aug. 19, 1884,	Dec. 5, 1884.	7.	Dec. 31, 1883,	Mar. 3, 1884.
3.	July 8, 1885,	Oct. 12, 1885.	8.	Feb. 12, 1886,	July 8, 1886.
4.	Apr. 5, 1886,	July 8, 1886.	9.	Sept. 8, 1885,	Dec. 12, 1885.
5.	Sept. 9, 1883,	Jan. 12, 1884.	10.	May 30, 1886,	Sept. 28, 1886.

BANKS AND BANKING.

349. A **Bank** is an incorporated institution for the issue, custody, or loaning of money, for discounting commercial paper, and for facilitating the transmission of money from one place to another by means of checks, drafts, or bills of exchange.

REM.—Many banks perform only a part of the functions enumerated above.

350. **Bank Discount** is a deduction made from the sum due upon a note at maturity for cashing or buying such note *before maturity*. This deduction is the simple interest of the sum due at maturity, computed for the number of days between the date of discount and the date of maturity.

REM. 1.—Bank discount is computed on the face of a note which does not draw interest, and upon the face plus its accrued interest at maturity of a note which draws interest; since, in the former case, the face, and in the latter, the face plus its accrued interest, will be the sums respectively due at maturity. In allowing interest upon interest-bearing notes, compound subtraction is often employed by banks of discount in obtaining the time; but the discount itself is computed for exact days.

REM. 2.—The difference between *true discount* and *bank discount* is in the sum taken as the principal. In true discount the present worth is taken as the principal; and in bank discount, the future worth.

351. The **Term of Discount** is the time from the date of discount of a note to the date of its maturity.

REM.—Ordinarily, in computing the interval between two dates, the first day of the interval is omitted (Note, **219**); but in the banks of Philadelphia, Baltimore, and some other cities, it is customary to include both the day of discount and the day of maturity, which is equivalent to charging discount for one day more than the actual time.

352. The **Proceeds** of a note is the sum received for it at the date of discount, and is equal to the *sum due* at maturity minus the *discount.*

353. Computations in Bank Discount may be made in accordance with the principles of percentage, the sum due at maturity being regarded as the base; the bank discount and proceeds as percentages. Hence,

General Direction: *If the bank discount or proceeds are required, multiply its two given factors; if given, divide it by its given factor and the quotient will be its remaining factor.*

Rem. 1.—The two factors of a given or required discount are the *sum due at maturity* and the *rate of bank discount for the given time;* and of a given or required proceeds are the *sum due at maturity* and the *rate of proceeds for the given time.*

Rem. 2.—To find the rate of bank discount for the given time, mentally divide the rate per annum into 360, and the result into the term of discount in days *regarded as hundredths* (Note, **327**).

Rem. 3.—Since the proceeds of a note equals the sum due at maturity minus the discount for the given time, the rate of proceeds must equal the rate of the sum due at maturity (100%) minus the rate of bank discount for the given time.

354. To find the bank discount or proceeds of a note.

ILLUSTRATIVE EXAMPLE.

A note of $1200, dated June 13, 1883, and payable 4 months after date, was discounted at bank, July 2, 1883. What were the date of maturity, the term of discount, the bank discount, and the proceeds of the note, the rate of bank discount being 6%?

Solution.

June 13 + (4 mo. 3 da.) = Oct. 16, date of maturity (Rem. 4, **345**).
July 2 to Oct. 16 = 106 days, term of discount (**348**).
Interest of $1200 for 106 da. at 6% = $21.20, bank discount (**319**).
$1200 — $21.20 = $1178.80, proceeds (**352**).

Rule.—I. *Compute the interest of the sum due at maturity for the term of discount and at the given rate per annum; the result will be the bank discount.*

II. *Subtract the bank discount from the sum due at maturity to find the proceeds.*

EXAMPLES FOR PRACTICE.

Find the date of maturity, term of discount, bank discount, and proceeds.

	Face of note.	*Date of note.*	*Time.*	*Date of disc't.*	*Rate of disc't.*
1.	$1500	Jan. 5, 1885	60 days	Feb. 1, 1885	6%
2.	$950	Sept. 30, 1885	90 "	Nov. 8, 1885	7%
3.	$325	Mar. 9, 1886	30 "	Mar. 12, 1886	6%
4.	$150	Nov. 8, 1885	3 months	Jan. 1, 1886	8%
5.	$500	Feb. 5, 1884	60 days	Feb. 28, 1884	6%
6.	$1200	Oct. 31, 1883	4 months	Jan. 8, 1884	5%
7.	$275	Apr. 3, 1885	2 months	Apr. 15, 1885	6%
8.	$2800	Aug. 15, 1883	90 days	Sept. 6, 1883	7%
9.	$5200	May 6, 1886	6 months	Sept. 18, 1886	6%

To enable the learner to discriminate between the various forms of notes in general use, we append a specimen of each.

10. DEMAND NOTE.

218\frac{25}{100}$. BALTIMORE, MD., *Jan. 30, 1886.*

On demand, for value received, I promise to pay SAMUEL WOLVERTON, *or bearer, Two Hundred and Eighteen $\frac{25}{100}$ Dollars with interest.*

R. C. CLAY.

REM.—The above note is payable whenever presented (on demand); is negotiable (payable to bearer); and bears interest from the date of the note at the legal rate of Maryland. If "or bearer" had been omitted, it would have been non-negotiable, since it would then have been payable to Samuel Wolverton only.

What would have been the sum due upon the above note, if it had been presented for payment Feb. 19, 1886 ?

11. TIME NOTE (*without interest*).

$3000. DAYTON, OHIO, *June 3, 1885.*

Three months after date, for value received, I promise to pay GEO. C. ROUND, *or order, Three Thousand Dollars.*

JOSEPH S. SINCLAIR.

REM.—The above note is payable 3 mo. 3 da. from June 3, or on Sept. 6, 1885. Since it is payable to the "order of" Geo. C. Round, it is negotiable with Geo. C. Round's indorsement. No interest is mentioned in the note, hence it cannot draw interest until maturity, after which, if it is not then paid, it will draw interest at the legal rate in Ohio.

Find the term of discount, the discount, and the proceeds, if the preceding note had been discounted at bank July 15, 1885, at 8% ?

12. TIME NOTE (*with interest*).

790\frac{30}{100}$. NEW YORK, *May 3, 1884.*

Ninety days after date, I promise to pay JOHN ARMSTRONG, *or bearer, Seven Hundred and Ninety* $\frac{30}{100}$ *Dollars, with interest at 6%. Value received.* W. A. MARBURY.

REM.—The above note is payable 93 days after May 3, or on Aug. 4, 1884. Since it is payable to John Armstrong, *or bearer*, it is negotiable without John Armstrong's indorsement. It draws interest from May 3, until paid.

What would have been the discount and the proceeds, if the above note had been discounted June 24, 1884, at 8% ?

13. JOINT NOTE.

$538. CHICAGO, *Sept. 9, 1886.*

Four months after date, we promise to pay to the order of SAMUEL GUNN & CO., *Five Hundred and Thirty-eight Dollars, for value received.*

JAMES E. JONES.
ROGER A. PALMER.

REM.—The above is called a *joint* note because it is signed by more than one person, who are jointly responsible for its payment; that is, each maker is only *individually* responsible for the payment of his proportionate part (one-half) of the face of the note.

What are the date of maturity, the term of discount, the discount, and the proceeds of the above note, if discounted at bank Nov. 13, 1886, at 6% ?

14. JOINT AND SEVERAL NOTE.

489\frac{42}{100}$. BOSTON, *Dec. 8, 1885.*

One month after date, we jointly and severally promise to pay to the order of HENRY BLACK, *Four Hundred and Eighty-nine* $\frac{42}{100}$ *Dollars. Value received.*

J. E. PRYOR.
W. O. HOLMES.

REM.—The effect of a *joint note* is to hold its makers only jointly responsible for its payment; but that of a *joint and several note* is to hold either maker *individually* responsible for the payment of the full amount of the note if the other defaults.

What are the date of maturity, the term of discount, the discount, and the proceeds of the preceding note, if discounted at bank Dec. 12, 1885, at 8%?

15. NOTE PAYABLE AT A BANK.

$500. PHILADELPHIA, *May 2, 1886.*

Ninety days after date, I promise to pay to J. M. GAULDEN, *or order, Five Hundred Dollars, without defalcation, at the First National Bank. Value received.* GEO. W. KELLER.

REM.—In Pennsylvania the phrase "without defalcation" is required in a note to make it negotiable.

What were the discount and the proceeds of the above note, if discounted at the bank June 18, 1886, at 8%?

16. NOTE PAYABLE TO ONE'S OWN ORDER.

$600. NEW ORLEANS, *Oct. 15, 1885.*

Sixty days after date, I promise to pay to the order of myself, Six Hundred Dollars. Value received. W. E. GARIG.

REM.—Business men frequently draw notes payable *to their own order*, so that with their indorsement in blank (Rem. 4, **342**), they may be transferable without the further indorsement of the holder.

What are the proceeds of the above note if discounted at bank Nov. 2, 1885, at 9%?

17. A note of $1800, dated June 16, 1885, bearing 6% interest, and payable 4 months after date, was discounted at bank June 29, 1885, at 6%. What were the bank discount and the proceeds?

18. What is the difference between the bank discount and true discount of a note of $2500, dated Sept. 1, 1885, and payable 3 months after date with interest at 6%, if discounted on the day of issue at 8%?

19. Henry Gould & Co. offered at bank the following, which were passed and discounted Sept. 6, 1886, at 6%:

Note dated July 30, 1886, at 4 months, for $850.25.
" " Aug. 15, 1886, " 60 days, " 325.50.
" " Aug. 31, 1886, " 3 months, " 575.80.

What was the discount on each note, and the amount of proceeds passed to their credit?

20. At 5%, what are the proceeds of a 90-day note of $580.75, dated May 23, 1886, and discounted June 1, 1886?

354

21. J. K. Hopper's draft of $1500 drawn on J. E. Morse, favor of Wm. J. Robb, dated Oct. 31, 1884, payable 90 days after date, and accepted Nov. 3, 1884, was discounted for E. C. Williamson, Dec. 7, 1884. If the rate of discount was 5½%, what were the date of maturity, term of discount, discount, and proceeds?

22. A draft dated July 10, 1884, drawn by E. H. Norman, Milwaukee, on W. H. Devon, New York, payable sixty days after sight, and accepted July 16, 1884, was discounted Aug. 3, 1884. If the face of the draft was $1123.40, and the rate of discount 5%, what were the date of maturity, term of discount, discount, and proceeds?

23. A bank discounted, July 2, 1884, for Wm. H. Patrick, a note of A. M. Sellers for $1250, dated June 9, 1884, and payable 4 months after date. Rate of discount 6%. Find the date of maturity, unexpired time, discount, and proceeds.

24. A merchant discounted Aug. 19, 1885, at 7%, a note of G. W. H. Carr, endorsed by Wm. A. Heitmueller, for $439.50, dated July 7, 1885, and payable 90 days after date. Find the date of maturity, unexpired time, discount, and proceeds.

25. Discounted Nov. 5, 1885, for J. H. Elliott, the following notes:

No. 1. Dated Oct. 28, 1885, payable 4 months after date, $1000.
" 2. " " 29, " " 4 " " " 700.
" 3. " " 30, " " 4 " " " 560.
" 4. " " 31, " " 4 " " " 63.20.

Find the date of maturity, term of discount, discount, and proceeds of each, if the rate of discount was 4½%.

355. To find the face of a note.

ILLUSTRATIVE EXAMPLE.

For what sum must I draw a 60-day note that, when discounted at bank at 8% per annum, the proceeds will be $887.40?

Explanation.—If a note of $1 will yield $.986 proceeds in 63 da., to yield $887.40 proceeds it will require a note whose face is as many times $1, as the given proceeds ($887.40) are times the proceeds of $1 ($.986), that is 900 times $1, or $900.

Solution.

60 + 3 = 63 da., term of discount.
.63 ÷ 45 = .014, discount of $1 (Note, **327**).
$1 — $.014 = $.986, proceeds of $1.
$887.40 ÷ $.986 = 900 times ($1 or $900) face.

Approximate Solution.—Find the interest of $887.40 for 63 da. at 8%, obtaining $12.424 —; then the interest on this result ($12.424 —) for the same time and at the same rate, obtaining $.173+. The sum of the given proceeds ($887.40) + the interest on the given proceeds ($12.424) + interest on this interest ($.173) = face of note ($899.997, practically $900).

Rem.—To find the exact answer, by the above method, continue to compute interest upon interest until a result less than 1 cent is obtained.

Rule.—*Divide the given proceeds by the proceeds of $1 at the given rate and for the given time with grace.*

EXAMPLES FOR PRACTICE.

Find the face of note.

	Proceeds.	*Term of disc't.*	*Rate of disc't.*
1.	$673.88	54 days	6%.
2.	$717.80	108 "	10%.
3.	$364.56	90 "	8%.
4.	$394.40	72 "	7%.
5.	$118.20	60 "	9%.
6.	$237.20	84 "	5%.

Find the face of note.

	Proceeds.	*Term of disc't.*	*Rate of disc't.*
7.	$1390.20	36 days	7%.
8.	$534.06	66 "	6%.
9.	$1164	135 "	4%.
10.	$392	4 months	6%.
11.	$715.40	3 "	4%.
12.	$433.40	3 "	4½%.

13. I owed a firm $750, and, in payment thereof, drew a 90-day note, the proceeds of which, when discounted at 6%, equaled the debt. What was the face of the note?

14. The holder of a note dated Oct. 15, 1883, and payable 4 months after date, had it discounted at bank Dec. 17, 1883, at 8% per annum. What was the face of the note, if the bank discount was $19.60?

15. What must be the face of a note payable 4 months after date that, when discounted at 6%, the proceeds will exactly pay a debt of $1000?

16. For what sum must a 30-day note be drawn that, when discounted at 8%, the maker may realize $500 as proceeds?

17. I owed $6000, and, to pay it, I had a note of $3500, maturing in 18 days, discounted by a bank at 8%, and drew a second note, payable in 2 months, for such a sum that, when discounted at the same rate, the proceeds of both notes enabled me to pay the debt. What was the face of the second note?

356. To find the rate of discount is an application of **325**; and to find the term of discount is an application of **326.** No special illustrative examples are therefore deemed necessary.

1. I drew a 90-day note of $900 and had it discounted at bank, receiving $881.40 as proceeds. What was the rate % of discount?

Bank discount ÷ bank discount at 1% per annum = rate of discount.

2. A note of $700, dated May 12, 1883, and payable 90 days after date, was discounted at bank June 26, 1883. What was the rate % of discount per annum, if the proceeds were $694.40?

3. A note of $650, nominally due Oct. 6, 1883, was discounted at 7½% per annum, from which the holder derived $641.47 proceeds. At what date was the note discounted?

Total discount ÷ discount for one day = term of discount in days.

4. I received a note of $840, dated May 3, 1883, and payable 60 days after date. I subsequently had it discounted at 8% per annum and received $831.04 as proceeds. At what date was it discounted?

5. If, in each of the following columns, the terms marked √ are given, how may the term marked √ be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Face of non-interest-bearing note....	√	√	?	√	√	?	..	..	√	√	√	√	?
Rate of bank discount...........	..	..	..	√	√	√	√	√	?	√	√	?	√
Term of discount..............	..	..	..	√	√	√	√	√	√	?	?	√	√
Bank discount................	√	?	√	?	..	..	?	√	..	..	√	√	√
Proceeds of note...............	?	√	√	..	?	√	√	?	√	√	..	..	..

357. The following will illustrate the method of finding interest on daily balances, as practiced by banks; rate 4%.

Bank Account Current.			*Daily Balances.*		*Products.*	
1883.	*Dr.*	*Cr.*	*Dr.*	*Cr.*	*Dr.*	*Cr.*
Jan. 1.		$800		800 × 8 =		6400
" 9.		200		1000 × 9 =		9000
" 18.	$300			700 × 17 =		11900
Feb. 4.		500		1200 × 15 =		18000
" 19.	100			1100 × 16 =		17600
Mar. 7.	400			700 × 8 =		5600
" 15.		300		1000 × 6 =		6000
" 21.	200			800 × 11 =		8800
Interest		9.25				90) 83300
Balance	809.25					$9.25
	$1809.25	$1809.25				Interest on daily balances.

357

Explanation.—On Jan. 1, $800 were credited and remained unchanged until Jan. 9, or 8 days. $800 credited for 8 days=8 times $800, or $6400 credited for 1 day. On Jan. 9, $200 additional were credited, increasing the credit balance from Jan. 9 to Jan. 18 (9 da.) to $1000 ; and $1000 credited for 9 days = 9 times $1000, or $9000 credited for 1 day. On Jan. 18, $300 were withdrawn, leaving a credit balance of $700 which continued unchanged until Feb. 4, or 17 days ; and $700 credited for 17 days = 17 times $700, or $11900 credited for 1 day. Continue in this manner until the last item has been considered and credited for the number of days between its date (Mar. 21) and the close of the quarter (Apr. 1) ; when it will be found that the daily balances are equivalent to a total credit of $83300 for 1 day. At 4% per annum, the interest is 1% for every 90 days. Hence, divide $833.00 by 90, obtaining $9.25 as credit interest, which, added to the credit side of the account, produces $1809.25. Subtract the sum of the debit items ($1000) from the sum of the credit items ($1809.25) to find the balance due the depositor ($809.25).

Balance the following bank accounts current;

(1.)

Bank Account Current.		
1883.	*Dr.*	*Cr.*
Apr. 1		$500
" 15	$100	
" 28	200	
May 4		300
" 20	150	
June 2		400
" 8	200	
" 24		250
Int. @ 5%		***
Bal. July 1	*****	
	*****	*****

(2.)

Bank Account Current.		
1884.	*Dr.*	*Cr.*
Jan. 1		$1500
" 6		300
" 18	$600	
" 31		200
Feb. 8	100	
" 15		430
" 22	275	
Mar. 19	130	
" 30		475
Int. @ 4%		****
Bal. Apr. 1	*****	
	*****	*****

(3.)

Bank Account Current.		
1883.	*Dr.*	*Cr.*
July 1		$7000
" 20	$1500	
" 31		2000
Aug. 8	350	
" 12	840	
" 19		400
" 27	500	
Sept. 5		300
" 12		290
" 25	400	
Int. @ 6%		****
Bal. Oct. 1	*****	
	*****	*****

SAVINGS-BANKS.

358. A **Savings-Bank** is an institution in which the savings or earnings of individuals are deposited for safe-keeping.

REM.—A savings-bank supplies each of its depositors with a book, in which are entered the sums deposited and the sums withdrawn; and at certain regular periods interest at a fixed rate is computed on all sums that have been on deposit for a certain time. This interest, if not withdrawn, is at once placed to the credit of the depositor and considered an additional deposit, drawing interest during the next period. Savings-banks therefore allow compound interest on deposits.

359. **An Interest Term** is the period of time at the close of which interest is declared on deposits, and compounded if not withdrawn.

REM. 1.—The interest term is not uniform. With some savings-banks it is 1 year; with others it is 6 months, commencing Jan. 1 and July 1; or 3 months, commencing Jan. 1, Apr. 1, July 1, and Oct. 1.

REM. 2.—If there have been no withdrawals within an interest term, savings-banks usually allow interest on money deposited at or before the commencement of each month or each quarter of that term from the commencement of such month or quarter to the close of the interest term.

REM. 3.—If, however, there have been withdrawals, they are deducted from the last deposits, and interest is computed upon the smallest balance remaining to the depositor's credit within each month or each quarter of the interest term.

360. To find the balance of deposit when there are no withdrawals.

ILLUSTRATIVE EXAMPLES.

1. On Jan. 1, 1883, a depositor's balance in a savings-bank, whose interest term is 6 months, was $175. He subsequently made the following deposits: Feb. 12, 1883, $60; Mar. 13, 1883, $25; May 9, 1883, $30; June 24, 1883, $75; July 5, 1883, $90; Sept. 28, 1883, $125. What was the balance to his credit on Jan. 1, 1884, interest at 4% per annum being allowed on deposits from the first day of the calendar month on or before which they are made?

Solution.

Bal. Jan. 1. 1883............	$175 × 6 =	$1050	on deposit for 1 mo. of int. term.
Deposit Feb. 12..............	60 × 4 =	240	" " " " "
" Mar. 13..............	25 × 3 =	75	" " " " "
" May 9................	30 × 1 =	30	" " " " "
" June 24..............	75 × 0 =	0	" " " " "
Total deposit first term......	$365	=3) $1395	" " " " "
Interest deposit July 1.......	4.65		$4.65, interest accrued first term.
Bal. due depositor July 1....	$369.65 × 6 =	$2214	on deposit for 1 mo. of 2d term.
Deposit, July 5..............	90 × 5 =	450	" " " " "
" Sept. 28............	125 × 3 =	375	" " " " "
Total deposit................	$584.65	3) $3039	
Int. deposit, Dec. 31.........	10.18		$10.18, interest accrued second term.
Bal. due Jan. 1, 1884........	$594.78		

Explanation.—At the close of the first interest term (July 1, 1883), $175 had been on deposit 6 *full* calendar months, which is equivalent to 6 times $175, or $1050, on deposit for 1 month. The next deposit ($60) having been made Feb. 12, is not entitled to draw interest until the first day of the following month; and hence, being on deposit 4 full calendar months, is equivalent to 4 times $60, or $240, on deposit for 1 month. Proceed in this manner with the remaining deposits of the first interest term (ending July 1), finding that the total deposits for that term are equivalent to a single deposit of $1395 for 1 month, the interest of which at 4% per annum is ⅓ of 1% of $1395, or $4.65. Add this interest to the total deposit ($365), obtaining $369.65 as the balance to the depositor's credit on July 1. At the close of the second interest term (Jan. 1, 1884,) this balance ($369.65) had been on deposit 6 full calendar months, which is equivalent to 6 times $369 (fractions of a dollar being discarded in computing interest), or $2214, on deposit for 1 month. Continuing as in the first interest term, it will be found that the balance due the depositor on Jan. 1, 1884, was $594.78.

2. In the preceding example, what will be the balance to the depositor's credit on Jan. 1, 1884, if interest at 4% per annum had been allowed on the smallest balance within each quarter of the interest term?

Solution.

Balance Jan. 1, 1883.....	$175	× 2 =	$350	on deposit for 1 quarter of first term.
Deposit Feb. 12.........	60	× 1 =	85	" " " " " "
" Mar. 13.........	25			
" May 9..........	30	× 0 =	0	" " " " " "
" June 24.........	75			
Total deposit, first term .	$365	=	$435	" " " " " "
Interest deposit, July 1 ..	4.85			$435 × .01 = $4.85, accrued interest.
Bal. due depositor July 1,	369.85	× 2 =	$738	on deposit for 1 quarter of 2d term.
Deposit July 5..........	90	× 1 =	215	" " " " " "
" Sept. 28.........	125			
Total deposit, 2d term...	$584.85	=	$953	" " " " " "
Int. deposit, Jan. 1......	9.53			$953 × .01 = $9.53, accrued interest.
Bal. due Jan. 1, 1884....	$593.88			

Explanation.—At the close of the first interest term (July 1, 1883), $175 had been on deposit 2 quarters, which is equivalent to 2 times $175, or $350 on deposit for 1 quarter. During the first quarter of the interest term, two deposits had been made, that of Feb. 12 ($60) and of Mar. 13 ($25), which being made after the commencement of the first quarter (Jan. 1, 1883) do not draw interest until at the commencement of the next quarter (Apr. 1, 1883). $60 + $25, or $85, therefore draw interest for 1 quarter of the first interest term. The next two deposits, that of May 9 ($30) and of June 24 ($75) being made after the commencement of the *last quarter* of the interest term, do not draw interest during that term. $435 on

360

deposit for 1 full quarter at 4% per annum = 1% of $435, or $4.35 interest, which, added to the sum of the deposits in that term ($365), produces $369.35 as the balance due the depositor July 1, 1883 ; etc.

EXAMPLES FOR PRACTICE.

1. On Jan. 1, 1883, a depositor's balance in a savings-bank whose interest term is 6 months, was $94. He afterwards made the following additional deposits: Jan. 27, $12; Feb. 4, $8; Feb. 19, $15; Apr. 8, $25; May 16, $20; June 4, $30. What was the balance to his credit July 1, 1883, interest at 4% being computed from the first day of each calendar month?

2. A made the following deposits in a savings-bank whose interest term is 3 months: June 9, 1883, $100; July 12, 1883, $30; Sept. 4, 1883, $40; Nov. 15, 1883, $60; Jan. 8, 1884, $50; Mar. 9, 1884, $70. What was the balance due July 1, 1884, interest at 4% being computed from the first day of each quarter?

3. What was the balance due upon the following savings-bank account on Dec. 31, 1883, the interest term being 6 months, and computing interest at 4% from the first day of each calendar month? Balance Jan. 1, 1883, $150. Deposits: Jan. 28, 1883, $30; Mar. 5, 1883, $25; Apr. 16, 1883, $75; May 7, 1883, $45; July 5, 1883, $60; Sept. 8, 1883, $50; Oct. 1, 1883, $20; Dec. 15, 1883, $45.

4. On Jan. 1, 1883, a depositor's balance in a savings-bank, whose interest term is 6 months, was $175. He subsequently made the following deposits: Feb. 12, 1883, $60; Mar. 13, 1883, $25; May 9, 1883, $30; June 24, 1883, $75; July 5, 1883, $90; Sept. 28, 1883, $125. What was the balance to his credit on Jan. 1, 1884, the rate of interest being 4%, computed from the first day of each calendar month?

361. To find the balance of deposit when there are withdrawals.

ILLUSTRATIVE EXAMPLES.

1. A depositor's balance, Jan. 1, 1884, was $320, and his subsequent deposits were: Jan. 19, $50; Mar. 15, $40; Apr. 9, $75; Apr. 29, $25; May 6, $30; June 1, $60. His withdrawals were: Jan. 31, $75; Mar. 18, $20; Apr. 15, $10; May 28, $160. What was the balance due July 1, 1884, the interest term being 6 months, computing interest at 6% from the first day of each quarter?

Solution.

Date.	*Deposits.*	*Withdrawals.*	*Balances.*	*Smallest Balance in each quarter.*	*Interest on smallest Balance.*
Balance Jan. 1.	$320		***$320***		
Jan. 19,	$50		$370		
" 31,		$75	$295	$295	$295 × 1½% = $4.42
Mar. 15,	$40		$335		
" 18,		$20	$315		
Balance Apr. 1.			***$315***		
Apr. 9,	$75		$390		
" 15,		$10	$380		
" 29,	$25		$405		
May 6,	$30		$435		
" 28,		$160	$275	$275	$275 × 1½% = $4.12
June 1,	$60		$335		

Last balance, $335 + interest first quarter, $4.42 + interest second quarter, $4.12 = $343.54, balance due depositor July 1, 1884.

Explanation.—To ascertain the smallest balance in each quarter, arrange the deposits and withdrawals in separate columns according to their respective dates, drawing a double line at the close of each quarter to avoid mixing items belonging to different quarters. Then place in the column of balances the amount due the depositor after each deposit or withdrawal, repeating *under* the double line the last balance of deposit *over* such line, so that it, as the first balance of the next quarter, may be compared with the other balances of that quarter. The smallest balance in each quarter can then be readily noticed. Compute one quarter's interest upon the smallest balance in each quarter, and add the interest thus obtained to the last balance in the balance column, obtaining $343.54 as the balance due the depositor.

2. In the preceding example, what was the balance due July 1, 1884, if the interest term was 6 months, and interest at 6% per annum was allowed from the first day of each calendar month?

Explanation.—To ascertain the smallest balance in each month, arrange the deposits and withdrawals in separate columns according to their respective dates, and place in the third column the amount due the depositor after each deposit and withdrawal, as shown in the solution following. At the beginning of each month, repeat the last balance of the preceding month so that it, as the first balance of the new month, may be compared with the remaining balances of the same month. Since there is no deposit or withdrawal in February, the last balance in January ($295) will not only be the first but also the least balance in February; and it will also be the first balance in March. Thus arranged, the smallest balance in each month can then be readily noticed, and should be written in a fourth column. Adding the column of smallest balances, we find they are equal to $1810 on deposit for 1 month, and one month's interest of $1810, at 6% per annum, is ½% of $1810, or $9.05. Add the accrued interest $9.05 to the last balance in the balance column ($335), obtaining $344.05 as the balance due the depositor July 1, 1884.

Solution.

Date.	*Deposits.*	*Withdrawals.*	*Balances.*	*Smallest Balance in each month.*	*Interest on Smallest Balance.*
Bal. Jan. 1.....	$320		$320	$295	×½% = $1.475, Jan. int.
" 19.....	$50		370		
" 31.....		$75	295		
Bal. Feb. 1.....			295	$295	×½% = $1.475, Feb. int.
Mar. 1.....			295	$295	×½% = $1.475, Mar. int.
" 15.....	$40		335		
" 18.....		$20	315		
Bal. Apr. 1.....			315	$315	×½% = $1.575, Apr. int.
" 9.....	$75		390		
" 15.....		$10	380		
" 29.....	$25		405		
Bal. May 1.....			405	$275	×½% = $1.375, May int.
" 6.....	$30		435		
" 28. ...		$160	275		
Bal. June 1.....			275	$335	×½% = $1.675, June int.
" 1.....	$60		335		
					$9.05, total int.
				Or, $1810	×½% = $9.05, total int.

Last balance ($335) + 1 month's interest on the sum of the monthly balances ($9.05) = $344.05, balance due depositor July 1, 1884.

EXAMPLES FOR PRACTICE.

(In the following examples, unless otherwise stated, the interest term is understood to be 6 months, commencing Jan. 1 and July 1.)

1. Balance the following savings-bank account to July 1, 1883, interest at 4% being allowed on monthly balances. Balance due Jan. 1,

1883, $90. Deposits: Jan. 30, $40; Feb. 18, $50; Apr. 8, $60; May 1, $20; June 4, $40. Withdrawals: Jan. 15, $25; Feb. 8, $15; Mar. 13, $20; May 8, $12.

2. What was due on the following savings-bank account, Jan. 1, 1884, interest at 5% per annum being allowed on quarterly balances? Balance due July 1, 1883, $150. Deposits: Aug. 15, $80; Aug. 31, $20; Sept. 15, $50; Oct. 9, $35; Nov. 1, $50. Withdrawals: July 8, $30; Aug. 4, $20; Sept. 3, $25; Sept. 15, $40; Oct. 8, $10.

3. A person deposited in a saving-bank, Jan. 1, 1883, $125; Jan. 20, $30; Feb. 5, $5; Mar. 19, $75; Apr. 12, $40; May 30, $50; June 18, $20. He withdrew Mar. 1, $25; Apr. 1, $20; June 5, $40. What was the balance due the depositor July 1, 1883, interest at 6% per annum being allowed on monthly balances?

4. A depositor's balance Jan. 1, 1884, was $320, and his subsequent deposits were: Jan. 19, $50; Mar. 15, $40; Apr. 9, $75; Apr. 29, $25; May 6, $30; June 1, $60. His withdrawals were: Jan. 31, $75; Mar. 18, $20; Apr. 15, $10; May 28, $160. What was the balance due July 1, 1884, computing interest at 6% from the first day of each calendar month?

PARTIAL PAYMENTS.

362. A **Partial Payment** is a payment of only a part of a debt and its accrued interest.

REM.—A partial payment of a note is usually acknowledged by indorsing on its back the date and amount of the payment.

363. When partial payments have been made on an interest-bearing note, the balance unpaid may be found by the following method, adopted by the Supreme Court of the United States, and hence called the United States rule.

364. United States rule for partial payments.

ILLUSTRATIVE EXAMPLE.

On a note of $3000, dated March 12, 1881, and drawing simple interest at 6% per annum, are indorsed the following payments: August 18, 1881, $325. April 9, 1882, $75. July 3, 1882, $400. Dec. 4, 1882, $150. How much was due February 25, 1883?

Solution.

Face of note or original principal	$3000
Interest from Mar. 12 to Aug. 18 (5 mo. 6 da.)	78
Amount due at the time of the first payment (Aug. 18)	$3078
First payment to be deducted	325
Debt unpaid, forming a new principal from Aug. 18	$2753
Interest of new principal from Aug. 18 to Apr. 9 (7 mo. 21 da.)	105.99
Since the 2d payment is less than this interest, compute the	
Interest of same principal from Apr. 9 to July 3 (2 mo. 24 da.)	38.54
Amount due at the time of the third payment (July 3)	$2897.53
Second and third payments to be deducted ($75 + $400 =)	475
Debt unpaid, forming a new principal from July 3	$2422.53
Interest of above principal from July 3 to Dec. 4 (5 mo. 1 da.)	60.97
Amount due at the time of the fourth payment (Dec. 4)	$2483.50
Fourth payment to be deducted	150
Debt unpaid, forming a new principal from Dec. 4	$2333.50
Interest of above principal from Dec. 4 to Feb. 25 (2 mo. 21 da.)	31.50
Amount due at the date of settlement (Feb. 25, 1883)	$2365.00

REM.—It is usually apparent whether a payment is less than the accrued interest, as with the second payment of the solution ; in which case, interest could have been more conveniently computed in *one operation* from the date of the preceding payment (Aug. 18) to the date of the third payment (July 3), or for 10 mo. 15 da., obtaining $144.53+, or the same interest as in the solution was obtained in *two* operations ($105.99 + $38.54 = $144.53).

Rule.—I. *Find the interest of the principal from the date of the note to the date of the first partial payment. If the first payment equals or exceeds the interest thus found, subtract the first payment from the sum of the principal and its accrued interest, and consider the remainder as a new principal.*

II. *If the first payment is less than the interest thus found, find the interest of the same principal to the date of the next or of such subsequent partial payment at which the sum of the payments will equal or exceed the interest due at such date. Subtract the sum of the payments to that date from the sum of the principal and its accrued interest to that date, and consider the remainder as a new principal.*

384

III. *Similarly find the interest of the new principal to the date of the next partial payment. If the next partial payment is greater than the interest thus found, proceed as in* I; *but if less, proceed as in* II. *So continue to the date of settlement.*

NOTE.—It will be observed in the foregoing rule (1) that a partial payment in excess of the accrued interest will have the effect of reducing the principal, since, after discharging such accrued interest, there will remain a surplus to be so applied; (2) that a payment less than the accrued interest cannot have the effect of reducing the principal, since such payment is not sufficient to discharge the accrued interest, *which must first be paid*; (3) that no new principal should exceed a preceding principal, since that would, in effect, be compounding interest on such excess, which it is the purpose of the U. S. Rule to prevent.

EXAMPLES FOR PRACTICE.

(1.)

$750. WASHINGTON, D. C., *July 15, 1882.*

Three years after date, for value received, I promise to pay M. F. SINCLAIR, *or order, Seven Hundred and Fifty Dollars, with interest at 6% per annum.* J. F. O'BANNON.

What was the balance due July 18, 1885, if the following partial payments had been previously made: Oct. 18, 1883, $175; Jan. 12, 1885, $200?

(2.)

$1200. NEW ORLEANS, *Aug. 8, 1880.*

Three years after date I promise to pay T. C. WILL, *or order, Twelve Hundred Dollars, with interest at 8%. Value received.*

E. BENSEL.

The above note contained indorsements as follows: Sept. 13, 1881, $75; Oct. 12, 1882, $80; Jan. 1, 1883, $25. What was the balance due Sept. 12, 1883?

(3.)

$890. MOBILE, *Mar. 18, 1884.*

Two years after date I promise to pay to W. A. FRANK & CO., *or order, Eight Hundred and Ninety Dollars, value received, with interest at 6%.* R. G. WALKER.

The following payments were indorsed on the above note:

May 13, 1884, $35.	Feb. 3, 1885, $75.
Aug. 5, 1884, $10.	Dec. 19, 1885, $150.

What was due Mar. 21, 1886?

(4.)

$3500. BOSTON, *May 15, 1881.*

Three years after date, for value received, I promise to pay R. L. THOMAS, *or order, Three Thousand Five Hundred Dollars, with interest at 6%.* F. S. HOLMES.

The following indorsements were made on the above note ·

Sept. 12, 1881, $350. Jan. 18, 1883, $400.
May 9, 1882, $75. Aug. 5, 1883, $90.
Oct. 8, 1882, $250. Dec. 12, 1883, $350.

What was the amount due July 3, 1884 ?

5. On a mortgage for $5200, dated June 16, 1880, there were paid Aug. 12, 1880, $300 ; June 15, 1881, $125 ; Aug. 19, 1882, $1200 ; May 4, 1883, $50 ; and Dec. 24, 1884, $2000. What was the balance due Feb. 12, 1885, at 6% ?

365. Mercantile Rule for partial payments.

ILLUSTRATIVE EXAMPLES.

1. By the merchants' rule, find the balance due July 18, 1886, on a note of $500, at 6%, dated Sept. 15, 1885, and upon which were the following indorsements : Oct. 12, 1885, $75 ; Dec. 18, 1885, $128 ; Mar. 9, 1886, $250.

Solution.

Face of note, dated Sept. 15, 1885		$500.00
Interest of $500 from Sept. 15 to July 18 (306 da.) at 6%		25.50
Sum of principal and accrued interest at date of settlement		$525.50
First payment made Oct. 12, 1885	$75.	
Interest of $75, from Oct. 12 to July 18 (279 da.)	3.49	
Second payment, made Dec. 18, 1885	128.	
Interest of $128 from Dec. 18 to July 18 (212 da.)	4.52	
Third payment, made Mar. 9, 1886	250.	
Interest of $250 from Mar. 9 to July 18 (131 da.)	5.46	
Sum of payments and accrued interest at settlement		466.47
Balance due at date of settlement		$59.03

Explanation.—By mercantile usage, if the time of settlement is one year or less from the date of a note, or other interest-bearing debt, the balance due is found by increasing the debt by its accrued interest to the date of settlement, and diminishing the result by the sum of the partial payments and the interest accrued upon each payment from the time it was made to the date of settlement.

2. By the merchants' rule, find the balance due Sept. 8, 1884, on a note of $1600, at 6%, dated Nov. 12, 1881, and upon which the following payments were indorsed: Jan. 4, 1882, $125; July 13, 1882, $150; Dec. 20, 1882, $300; Jan. 4, 1884, $400.

Solution

Face of note, dated Nov. 12, 1881		$1600.
Interest of $1600 from Nov. 12, '81, to Nov. 12, '82, at 6%		96.
Sum of principal and interest at first yearly settlement		$1696.
First payment, made Jan. 4, 1882	$125.	
Interest of $125 from Jan. 4 to Nov. 12, 1882 (312 da.)	6.50	
Second payment, made July 13, 1882	150.	
Interest of $150 from July 13 to Nov. 12, 1882 (122 da.)	3.05	
Sum of payments and interest at first yearly settlement		$284.55
Balance due, and new principal formed at first yearly settlement		$1411.45
Interest of $1411.45 from Nov. 12, '82, to Nov. 12, '83		84.69
Sum of new principal and interest at 2d yearly settlement		$1496.14
Third payment, made Dec. 20, 1882	$300.	
Interest of $300 from Dec. 20, '82, to Nov. 12, '83, (327 da.)	16.35	
Sum of payment and its interest at 2d yearly settlement		316.35
Balance due and new principal formed at 2d yearly settlement		$1179.79
Interest of $1179.79 from Nov. 12, '83, to final settlement, Sept. 8, '84 (301 da.)		59.19
Sum of new principal and interest at date of final settlement		$1238.98
Fourth payment made Jan. 4, 1884	$400.	
Interest of $400 from Jan. 4 to Sept. 8 (248 da.)	16.53	
Sum of payment and its interest at final settlement		416.53
Balance due at date of final settlement		$822.45

Explanation.—On notes running longer than one year, merchants usually charge interest upon the principal from the time it commenced to draw interest to the end of the first interest year, and allow interest on partial payments from the time they were made to the end of the same interest year; then, striking a balance, charge interest upon the unpaid balance to the end of the second interest year, and from the amount deduct the payments made within the second interest year and the interest upon each payment from the time it was made; and so continue *until the note* is fully paid. If the date of the final settlement is less than one year

365

from the date of the next preceding settlement, the balance due is found as explained in Ill. Ex. 1, and shown in the final settlement of Ill. Ex. 2.

EXAMPLES FOR PRACTICE.

1. On a note of $800, at 6%, and dated Mar. 4, 1883, are the following indorsements: Apr. 12, 1883, $75; July 9, 1883, $150; Sept. 5, 1883, $90; Dec. 8, 1883, $200. What was due Jan 1, 1884?

2. On a note of $1500, at 5%, dated July 5, 1882, were indorsed the following payments: Sept. 19, 1882, $300; Nov. 5, 1882, $200; Jan. 18, 1883, $175; Mar. 3, 1883, $350. What was due June 15, 1883?

3. What was the balance due Aug. 3, 1884, on a note of $500, at 7%, dated Dec. 15, 1883, and upon which the following payments had been made: Jan. 30, 1884, $150; Mar. 31,1884, $80; May 2, 1884, $160; July 4, 1884, $75?

4. On Sept. 9, 1881, I gave my note of $2000, at 6%, upon which I subsequently made the following payments: Nov. 20, 1881, $325; Jan. 13, 1882, $75; May 25, 1882, $472; and July 21, 1882, $200. What did I owe upon the note Aug. 17, 1882?

5. What was the balance due Mar. 15, 1885, upon a note of $1725.40, at 5%, dated Apr. 19, 1884, and upon which the following payments were indorsed: June 7, 1884, $150; July 1, 1884, $136; Aug. 12, 1884, $75; Sept. 23, 1884, $125; Nov. 18, 1884, $85; Dec. 20, 1884, $225; Mar. 5, 1885, $178?

6. On a note of $1800, at 6%, dated Nov. 13, 1878, were indorsed the following payments: Jan. 16, 1879, $100; Mar. 12, 1879, $50; July 3, 1879, $250; Dec. 8, 1879, $400; Aug. 15, 1880, $300. What was the balance due Jan. 1, 1881?

7. What was the balance due Sept. 8, 1883, upon a note of $5600, at 6%, dated Oct. 3, 1882, if it contained the following indorsements: Dec. 15, 1882, $500; Feb. 28, 1883, $800; June 13, 1883, $450; Aug. 7, 1883, $1200?

8. A note of $2500, at 6%, dated Mar. 4, 1881, bore the following indorsements: Feb. 8, 1882, $800; July 9, 1883, $500. What was the balance due May 9, 1884?

9. What was the balance due June 13, 1885, upon a note of $1325, at 5%, dated May 6, 1882, and containing the following indorsements: Aug. 5, 1882, $90; Oct. 8, 1883, $325; Jan. 1, 1884, $150; July 6, 1884, $215; Oct. 8, 1884, $45; and May 13, 1885, $286?

385

10. A note of $3500, at 5%, dated Feb. 5, 1879, bore the following indorsements: July 15, 1879, $300; Oct. 7, 1879, $125; Jan. 8, 1880, $50; Aug. 3, 1880, $130; Dec. 19, 1880, $175; May 3, 1881, $200; Aug. 16, 1881, $400; Jan. 1, 1882, $150. What was the balance due Apr. 12, 1882?

368. By the **New Hampshire Rule**, when notes or other obligations specify that the interest is payable annually (**330**), partial payments are applied, first, to discharging the simple interest which has accrued upon the annual interest; second, to discharging the annual interest; and third, to reducing the principal.

369. By the **Vermont Rule**, whether the time of final settlement is more or less than one year from the time the note begins to draw interest, only one balance is struck, and that at the time of final settlement as in Ill. Ex. 1 of the mercantile rule, by increasing the principal by its accrued interest to the date of final settlement, and diminishing the result by the sum of the partial payments and the interest accrued on each payment from the time it is made to the date of final settlement.

Rem.—If the interest is contracted to be paid annually, the Vermont method of striking balances is the same as the New Hampshire (**368**).

370. By the **Connecticut Rule**, if the time between the date of the note and the first payment, or between any considered payment and the one next preceding it, is *one year or more*, and the considered payment equals or exceeds the interest of the principal to the date of such payment, the balance is struck at the time of making the payment, as by the U. S. rule; but if the time is *less than one year*, and the considered payment equals or exceeds the interest of the principal to the date of such payment, the balance due is found by adding one year's interest to the principal, and deducting from this amount the sum of the partial payments made within that interest year and their accrued interest as in Ill. Ex. 2 of the mercantile rule. If any payment is less than the interest of the principal to the date of such payment, no balance is struck for that payment, since to do so would be compounding interest on the balance of interest due.

To the Teacher.—If considered necessary, suitable examples under the other partial payment rules can be selected for solution by the New Hampshire, Vermont, or Connecticut rules.

STOCKS AND BONDS.

371. A Corporation consists of several persons who are authorized by law to transact business as a single individual.

REM.—The special act which legalizes the existence of a corporation and specifies its powers and obligations, is called its *Charter*.

372. The Capital Stock of a corporation is so much of its capital as is represented by its shares.

REM.—A *Share* is one of the equal parts into which the capital stock is divided. It usually represents $100 of the capital stock, though shares sometimes represent as low as $1 and as high as $1000 of the capital stock. Shares of $50 and $25 are respectively called *half-stock* and *quarter-stock*. The owner of one or more shares of a corporation is called a stockholder of that corporation.

373. The **Par Value** of a share is the value specified upon its "face," and represents the capital stock for which it was originally issued.

374. The **Market Value** of a share is the sum for which it can be sold.

REM.—When the market value of a share is *greater* than its par value, it is said to be *above par* or *at a premium;* and when its market value is *less* than its par value, it is said to be *below par* or *at a discount*.

375. The **Gross Earnings** of a corporation embrace the *total receipts* from its business before any expense has been deducted.

376. The **Net Earnings** of a corporation are its profits; that is, what is left of its gross earnings after all expenses have been paid.

377. A **Dividend** is so much of the net earnings of a corporation as is divided among its stockholders.

378. An **Assessment** is a sum levied upon the stockholders of a corporation to make up its losses or deficiencies.

REM.—Dividends and assessments are declared at a certain % of the par value of the stock or at so much per share.

379. The **Preferred Stock** of a corporation is so much of its capital stock as has advantages over the remainder, or common stock, when dividends are declared.

Rem.—To prevent the management of an embarrassed corporation from passing out of the hands of its officers into those of a *receiver* appointed by a court, some of its stockholders or friends may extend pecuniary aid, for which new stock is issued, called preferred stock, because it is entitled to receive a dividend of a specified per cent. out of the net earnings of the corporation before any dividend can be declared upon the remainder of the capital stock.

380. **Watered Stock** is stock which has been inflated by distributing among the stockholders extra shares for which no additional payment is required.

Rem.—When the charter of a corporation forbids it to declare a dividend exceeding a certain rate % of the par value of its capital stock, or if its stockholders wish to keep the public in ignorance of its prosperity, its capital stock can be watered by issuing additional shares, so that a low rate of dividend upon the watered stock will produce as much income as a high rate of dividend upon the original capital stock.

381. A **Bond** is an obligation to pay a certain sum of money at a specified time, with interest at a fixed rate % payable at regular intervals.

Rem. 1.—Bonds are issued for money borrowed by the General Government, States, counties, cities, and business corporations.

Rem. 2.—A *coupon bond* is one with certificates attached, representing the successive installments of interest. Each coupon is detached and presented for payment when the interest for the period mentioned therein becomes due.

Rem. 3.—A *registered bond* is one which is payable to the owner, whose name is registered in the books of the corporation or government which issued it. Registered bonds can be transferred only by a properly acknowledged assignment.

Rem. 4.—*Government bonds* usually derive their name from the rate of interest which they bear and the date at which they are due; as 4's of 1907; 4½'s of '91.

382. **Quotations** usually denote the price at which $100 par value of stocks or bonds can be bought or sold.

Rem. 1.—Mining securities are usually quoted at so much per share, without regard to their par value. Thus, mining stock, the par value of which is $5 per share, may be quoted at 3, which denotes that its market value is $3 per share. In some cities, *all* stocks whose par value is less than $100 are thus quoted.

Rem. 2.—*Bonds* of all denominations are quoted at the market value of each $100 of their face value. This is also the prevailing practice with stocks, except mining stocks. Quotations thus expressed are theoretically *rates per cent.* Half-stock worth $57 per share would thus be quoted at 114, the price of 2 shares or of $100 par.

383. Computations in Stocks and Bonds may be made in accordance with the principles of percentage, the par value being the base; and all other terms, as dividend, assessment, premium, discount, brokerage, income, and market value, are percentages. Hence,

General Direction: ***If any of the above percentages is required, multiply its two given factors; if given, divide it by its given factor, and the quotient will be its remaining factor.***

Rem. 1.—The two factors of any of the preceding percentages are the par value of the stocks or bonds and the rate per cent. of that percentage.

Rem. 2.—It should be remembered that the quotation is theoretically the rate % of market value (Rem. 2, **382**). Hence, since securities are bought and sold at their market value, the quotation regarded as hundredths must be the rate per cent. of prime cost if securities are bought, or of gross proceeds if securities are sold.

Rem. 3.—If securities are bought through a broker, the rate % of gross cost equals the rate % of prime cost (quotation) plus the rate % of brokerage; and if sold through a broker, the rate % of net proceeds equals the rate % of gross proceeds (quotation) minus the rate % of brokerage.

Rem. 4.—In stock computations, it is usually more convenient to operate upon shares (one-hundredth of the par value), and to regard all rates per cent. as *dollars per share.* Thus, a premium of 15% = $15 per share; a quotation of 115 = $115 per share; a brokerage of ⅛% = $⅛ per share; etc.

EXAMPLES FOR PRACTICE.

1. What is the dividend on $7500 par value of stock, if the rate of dividend is 4%?

Par value ($7500) × rate of dividend (.04)=dividend ($300).
Number of shares (75) × dividend per share ($4) = dividend ($300).

What is the

2. Dividend on $3200 par of stock, if the rate of dividend is 3½%?
3. Dividend on 87 shares of stock, if the rate of dividend is 12½%?
4. Assessment on $10500 par of stock; rate of assessment 6¼%?
5. Assessment on 123 shares of stock; rate of assessment 7⅛%?
6. Premium on $1800 par of bonds, if the rate of premium is 18%?
7. Discount on 73 shares of stock, if the rate of discount is 8⅝%?
8. What is the market value of $12500 par of stocks, if the rate of premium is 9⅛%?

Par value ($12500) × rate of market value (1.09⅛)=market value ($13640.63—).
No. of shares (125) × market value per share ($109.125)=market value ($13640.63—).

What is the market value

9. Of $18600 par of bonds, if the rate of premium is $3\frac{5}{16}\%$?
10. Of 30 shares of stock, if the rate of premium is $8\frac{5}{8}\%$?
11. Of $2800 par of stock, if the rate of discount is $4\frac{1}{4}\%$?
12. Of 159 shares of stock, if the rate of discount is $3\frac{1}{16}\%$?
13. Of $1700 par of bonds, if the quotation is $91\frac{3}{8}$?
14. Of $2900 par of stocks, if the quotation is $100\frac{7}{8}$?
15. What is the cost of $7600 par of stocks bought at 25% premium. including $\frac{1}{8}\%$ brokerage?

Par value ($7600) × rate of cost ($125\frac{1}{8}\%$)=cost ($9509.50).
No. of shares (76) × cost of 1 share ($125.125)=cost of all the shares ($9509.50).

Including $\frac{1}{8}\%$ brokerage, what is the

16. Cost of $8500 par of stock bought at $15\frac{3}{4}\%$ premium?
17. Cost of 23 shares of stock bought at $8\frac{1}{8}\%$ discount?
18. Cost of $3500 par of stock bought at $105\frac{3}{8}$?
19. Net proceeds of 78 shares sold at $75\frac{1}{4}$?
20. Net proceeds of $5700 par of bonds sold at $18\frac{5}{8}\%$ premium?
21. Net proceeds of 85 shares of stock sold at $26\frac{7}{8}\%$ discount?
22. If the par value of stocks is $6100, and the dividend $427, what is the rate % of dividend?

Dividend ($427)÷ par value ($6100)= rate of dividend (7%).
Dividend ($427)÷ No. shares (61)= rate per share ($7=7%).

If the par value of stock is

23. $6800, the assessment $204, what is the % of assessment?
24. $5000, the dividend $318.75, what is the % of dividend?
25. $8300, the premium $1182.75, what is the % of premium?
26. $5400, the discount $681.75, what is the % of discount?
27. $3600, and the market value $4243.50, what is the quotation?
28. $1400, and the market value $1515.50, what is the rate % of premium?
29. $8000, and the market value $5370, what is the % of discount?
30. If the cost of 80 shares of stock, including $\frac{1}{4}\%$ brokerage, is $8760, at what quotation were they bought?

Cost of all the shares ($8760)÷ No. of shares (80)= cost of 1 share ($$109\frac{1}{2}$):
Gross cost per share ($$109\frac{1}{2}$)—brokerage per share ($$\frac{1}{4}$)=prime cost per share ($$109\frac{1}{4}$).

If, allowing $\frac{1}{8}\%$ brokerage,

31. The cost of 52 shares of stock is $5369, at what quotation were they bought?

383

32. The cost of 47 shares is $5804.50, at what % premium were they bought?

33. The cost of 175 shares is $14743.75, at what % discount were they bought?

34. The proceeds of 92 shares are $7693.50, at what quotation were they sold?

35. The proceeds of 180 shares are $21465, at what % premium were they sold?

36. The proceeds of 58 shares are $5031.50, at what % discount were they sold?

37. What is the par value of stock bought for $7712, at 20½% premium?

Market value ($7712) ÷ rate of market value ($1.20½) = par value ($6400).
Market val. ($7712) ÷ market val. per sh. ($120.50) = no. sh. (64 = $6400).

What is the par value of stock if the

38. Dividend is $170.50, and the rate of dividend 5½%?
39. Assessment is $1031.25, and the rate of assessment 8¼%?
40. Premium is $3293.75, and the rate of premium 19⅜%?
41. Discount is $1550.25, and the rate of discount 13¼%?
42. Market value is $7193.25, and the rate of premium 4¼%?
43. Market value is $7182, and the rate of discount 14½%?
44. Market value is $4812.50, and the quotation 68¾?
45. Cost is $1770.75, brokerage ¼%, and quotation 98⅛?

Cost ($1770.75) ÷ rate of cost (98⅛ + ¼% = 98⅜%) = par value ($1800).
Cost of all the sh. ($1770.75) ÷ cost per sh. ($98.375) = no. sh. (18).

46. Cost is $6631.25, brokerage ⅛%, and quotation 132½?
47. Cost is $13530, brokerage ⅛%, and premium 12⅝%?
48. Cost is $12487.50, brokerage ¼%, and discount 17%?
49. Net proceeds are $10436.25, brokerage ⅛%, and quotation 75¾?
50. Net proceeds are $4051.75, brokerage ¼%, and premium 6⅞%?
51. Net proceeds are $27320, brokerage ⅛%, and discount 14½%?

52. A gentleman sold 135 shares of railroad stock at 18¾% premium, paying ⅛% brokerage. What were the net proceeds of the sale?

53. A man bought 150 shares of steamboat stock at 2⅜% discount, and sold the same at 1⅝% premium. What was his net gain, allowing ⅛% brokerage each way?

54. A speculator sold through a broker 74 shares of canal stock, and received $6336.25 as the net proceeds of the sale. At what quotation was the stock sold, allowing ¼% for brokerage ?

55. I bought stock at 4⅝% discount and sold the same at 2⅛% premium, making a net profit of $975. The brokerage each way was ⅛%. How many shares were involved in the transaction ?

56. A speculator sold 40 shares of stock at 15⅝% discount, paying ¼% brokerage. What were the net proceeds of the sale ?

57. A retired merchant bought 124 shares of stock for $14337.50, including ¼% brokerage. At what % premium were they bought ?

58. A broker, on his own account, sold stock at 6⅜% above par, and obtained $7908.75. How many shares did he sell ?

59. A man bought canal stock at 98¾ and sold the same at 101⅝, receiving $6090 as the net proceeds of the sale. What was his net gain, allowing ⅛% brokerage each way ?

60. A broker, on his own account, sold 160 shares of B. & O. R. R. stock for $31800, realizing a profit of $340. At what quotation did he buy the stock ?

61. A man bought 200 shares of manufacturing stock at 135⅜, sold 75 shares of the same at 136⅞, and the remainder at 138¼. What was his net gain, allowing ⅛% brokerage each way ?

62. I bought stock at 28⅜% discount, received a dividend thereon of 6%, and then sold out at 31⅛% discount, netting a gain of $384 after paying my broker ⅛% for buying and the same for selling. What was the par value of the stock ?

63. A man bought 80 shares of N. C. R. R. stock for $5270, including brokerage at ¼%. At what % discount was the stock bought ?

64. A capitalist drew a semi-annual dividend of 5¼% on his banking stock, and received $420. He afterwards sold 30 shares of that stock at 125⅜, and the remainder at 127⅞, paying a broker ⅛% for selling. What were the net proceeds of the sale ?

384. The par value is always understood as the *normal* base in stock computations *when no other base is mentioned*; but in many problems, the investment, an *abnormal* base, is taken as the standard, in which case such abnormal base is always mentioned.

Rem. 1.—In the expression "8% stocks," "8% bonds," the base of 8% is understood to be the par value, since no other base is mentioned; but in the expression "stocks paying 8% on the investment," the base of 8% is the investment *mentioned.*

384

Rem. 2.—When the rate % on par value and rate % on investment are both given in the same problem, the rate on par value should be considered as dollars of income per share (Rem. 4, **383**), and the rate on investment should be considered as the rate of this income.

EXAMPLES FOR PRACTICE.

1. At what price must 6% stock be purchased to realize an income of 8% on the investment?

Income per share ($6) ÷ rate of income on investment (8%) = investment per share ($75).

At what price must

2. 9% stock be bought to realize 12% on the investment?

3. 7% bonds be bought to secure 10% on the investment?

4. 6% stock be purchased to secure 4% on the investment?

5. At what quotation must 5% stocks be bought to realize an income of 4% on the investment, brokerage for buying ¼%?

Income per share ($5) ÷ rate of income on investment (4%) = investment per share ($125).

Investment per share ($125) — brokerage per share ($¼) = quotation ($124¾).

If in each case the brokerage is ⅛%, at what quotation must

6. 7% stock be bought to realize 8% on the investment?

7. 3% stock be bought to realize 5% on the investment?

8. 8% stock be purchased to secure 10% on the investment?

9. What % on investment is realized by buying 9% stock at 112½?

Income per share ($9) ÷ investment per share ($112.50) = rate of income on investment (8%).

What % on investment will be realized

10. By purchasing 4% stock at 80?

11. By purchasing 6% stock at 75?

12. By purchasing 8% stock at 120?

13. By purchasing 5% stock at 112⅜, brokerage ⅛% additional?

Income per share ($5) ÷ investment per share ($112⅜ + $⅛) = rate of income on investment (4⁴⁄₉%).

14. By purchasing 7½% stock at 59⅜, brokerage ⅛% additional?

15. By purchasing 6% stock at 112, brokerage ⅛% additional?

16. By purchasing 5% stock at 139⅞, brokerage ⅛% additional?

17. What % stock can be bought at 80 to yield 5% on investment?

Investment per share ($80) × rate on investment (5%) = income per share ($4 = 4% on par, Rem. 4, **383**).

What per cent. stock can be bought

18. At 87½, and yield 8% income on the investment?
19. At 125, and yield 4% income on the investment?
20. At 60, and yield 5% income on the investment?
21. At 112¼, brokerage ¼% additional, to yield 8% on investment?

Investment per share ($112¼ + $¼) × rate on investment (8%) = income per share ($9 = 9%).

22. At 119⅞, brokerage ⅛% additional, to yield 5% on investment?
23. At 139¾, brokerage ¼% additional, to yield 8¾ % on investment?

24. Canal stock paying 7% annual dividends was purchased at a price that yielded 8% on the investment. What was the cost of 80 shares bought at that rate, brokerage ⅛%. At what quotation was the stock bought?

25. What per cent. stock can be bought at 79⅞, brokerage ⅛% additional, to yield 5% on the investment?

26. If I buy 5% bonds at 80 and 7% bonds at 115, which investment will yield the greater per cent. of income on investment?

27. An investor purchased railroad bonds at 40% below par and thus realized an income of 10% on his investment. He purchased canal bonds bearing the same rate of interest at 20% below par, and received on the canal bonds an annual income of $2400. What did he pay for the canal bonds?

28. I bought 90 shares of stock at 125 and realized 6% per annum on my investment. What % dividend was annually declared thereon?

29. An investor bought sufficient 6% railroad bonds at 40% below par to secure a semi-annual income of $240. What was the amount of his investment, including ¼% brokerage?

30. A speculator bought 150 shares of 4% stocks at a price that paid 5% on his investment, and sold the same to a broker at 10% profit. How much did the broker pay for the stock?

31. If a quarterly dividend of 2% be received on stock bought at 75, what is the % of annual income on the investment?

32. A gentleman paid $8960 for 112 shares of stock, and realized 5% on his investment when the annual dividends thereon were paid. What was the rate % of dividend received?

33. I bought U. S. 4% bonds at 119⅞, brokerage ⅛% additional, and derive from the purchase a quarterly income of $300. How much did I invest?

34. At what quotation must U. S. 4½% bonds be bought to enable the buyer to realize 3% on his investment, allowing ⅛% for brokerage ?

35. I paid $10600 for U. S. 4½% bonds and realized $4\frac{13}{53}$% per annum on my investment. What was the par value of the bonds ?

36. How much must 50 shares of 9% stocks decline in market value to secure 10% on the investment, if they are at present paying 8% ?

37. Bank stock bought at 110 yielded the purchaser semi-annual dividends of 5%. What per cent. of income, per annum, did he realize on his investment ?

38. A merchant retired from business with a capital of $20916.25, and with it purchased $8000 par of 5% bonds at 106¼, $5000 par of 6% bonds at 103⅝, and invested the remainder in manufacturing stock at 96⅛, paying 8% dividends. What was his annual income, the brokerage for buying each class of securities being ⅛% ?

39. On 94 shares of stock, I receive two semi-annual dividends, one of 5%, and the other of 4%, by which I realize an annual income of 10% on my investment. What did I pay for the stock ?

40. A gentleman bought 82 shares of stock at 106⅜, and sold the same at a net profit of $266.50. At what quotation were they sold, brokerage each way ⅛% ?

41. A capitalist drew the quarterly interest on his U. S. 4% bonds, amounting to $540, and afterwards sold the same at 124⅝. What were the net proceeds of the sale, brokerage ⅛% ?

42. A speculator purchased manufacturing stock at 105¾, and sold the same at 108⅝, netting a profit of $236.25. The brokerage each way was ⅛%. How many shares, par $500, were involved in the transaction ?

First find how many $100 shares, and divide by 5.

43. A lady invested $20948.75 as follows: $6160 in Maryland State 6% bonds at 96⅛; $8225 in manufacturing stock at 87⅜, paying 8% annual dividends; and the remainder in steamboat stock at 73⅝, paying 10% annual dividends. The brokerage was ⅛% for purchasing each kind of securities. What was her total annual income ?

44. I sold $8600 par of 5% stock at 108⅛, and with the proceeds purchased 4% stock at 53⅞. What was my annual gain, ⅛% being charged for brokerage each way ?

45. If stock actually worth $80 per share be watered by issuing a stock dividend of 10%, what is the actual value of the watered stock ?

46. If money is worth 3% per annum, what should be the quotation of stock which declares dividends of 5% regularly every year?

47. A steamboat company divided $14000 of its net earnings among its stockholders. What was the rate % of dividend, the capital stock being $350000? How much did a stockholder receive who held a certificate for 50 shares (par $25)?

48. A broker bought 140 shares of canal stock (par $50) at 98⅜, and sold the same at 102⅛. What was his gain?

Change the shares of half-stock to equivalent shares of whole stock, and then proceed as usual.

49. The par value of a certain stock is $25 per share, and yields 8% dividends. How much greater % of income would be realized on the same if purchased at $20 per share than if bought at par?

50. A gentleman paid 86⅝ for canal stock paying 8% annual dividends, and received each year $480. What did the stock cost, including brokerage at ⅛%?

51. I invested $7002.25 in 5% bonds bought at 5⅝% discount, brokerage ¼%. What is my annual income therefrom?

52. I realized 5% per annum on an investment in stock bought at 119⅞, brokerage ⅛%. What semi-annual rate of dividend was regularly declared thereon?

53. A gentleman bought U. S. 4's of 1901 at 118⅝; U. S. 4½'s of '91 at 108¼; and U. S. currency 6's at 127. If bought at par, the sums invested in each kind of bonds would have been equal. His total income was $3480 per annum. What did he pay for the bonds, brokerage ⅛%?

54. What is the value of a ground rent of $160 per annum, capitalized at 5%?

REM.—To capitalize a ground rent means to find a sum of money which, if placed at interest at the current rate % per annum, will produce the same income as the ground rent.

55. What is the value of a ground rent of $120 per annum, if capitalized at 6%?

56. Do I make a profit or suffer a loss by buying a $90 ground rent at a capitalization of 5% and afterwards selling it at a capitalization of 4½%, and how much?

57. I bought a ground rent of $80 per annum for $2000. At what rate % was the ground rent capitalized?

58. An investor purchased a ground rent for $6000. He afterwards sold it at a capitalization of 4%, and realized a gain of $1500. What annual income did the ground rent produce?

59. At what price must 5% bonds, maturing in 1887, be purchased in 1884, to enable the investor to realize 8% on his investment?

Solution.

1887−1884=3 years, time the bonds have to run before maturity.
$100+($5 × 3)=$115, principal and interest of a $100-bond at maturity.
$1+($.08 × 3)=$1.24, value in 1887 of $1 invested in 1884 at 8% per annum.
$115÷$1.24=$92\frac{23}{31}$, investment in 1884, which at 8% per annum will amount to $115 in 1887.

NOTE.—In the above operation no allowance has been made for interest on the annual payments of interest. Various labor-saving tables, anticipating compound interest, etc., are in use by investors in obtaining results in bond investments.

60. At what price must 4% bonds, maturing in 8 years from date of investment, be purchased to yield 6% on the investment?

61. What must be paid for 6% bonds maturing in 15 years from date of purchase, so that 7% may be realized on the investment?

62. At what quotation must 5% bonds, maturing in 12 years, be purchased so that 4% per annum may be realized on the investment, allowing ⅛% for brokerage?

63. If I purchase 5% bonds, maturing in 8 years from date of investment, at 85⅜, brokerage ⅛%, what rate % on my investment will I realize?

Solution.

$100+($5 × 8)=$140, principal and interest of a $100-bond at maturity.
$140−($85⅜+$⅛)=$54.50, total income in 8 years from an investment of $85.50.
($85.50 × .01) × 8=$6.84, interest on an investment of $85½, at 1% per annum, for 8 years.
$54.50÷$6.84=$7\frac{331}{342}$%, required rate % per annum on the investment.

NOTE.—In the above operation, as in that of Ex. 59, no allowance is made for interest on the annual payments of interest.

64. What rate % per annum do I realize on my investment if I buy 6% bonds, maturing in 20 years, at 112?

65. I can purchase 4% bonds, having 20 years to run, at 120, and 6% bonds, having 3 years to run, at 105. Which is the better investment, and how much greater rate % per annum will I realize?

66. What rate % per annum is realized on an investment in 5% bonds maturing in 12 years, purchased at 93⅛, brokerage ⅛%?

67. If, in each of the following columns, the terms marked √ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Par value............	√	√	√	√	√	?	?	?	?	?	√	√	..	..
Rate of premium......	√	..	..	√	..	√	..	..	√	..	?	..	?	..
" " discount......	..	√	..	..	√	..	√	..	..	√	..	?	..	?
" " quotation.....	..	..	√	..	..	..	..	√	..	..	..	..	√	√
Premium............	?	..	..	..	..	√	..	..	..	..	√	..	..	..
Discount............	..	?	..	..	..	..	√	..	..	..	..	√	..	..
Market value.........	..	..	?	?	?	..	..	√	√	√	..	..	.	..

	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
Par value.................	..	..	..	..	..	..	?	..	..	√	..	?	..
Rate of brokerage...........	..	√	..	√	..	√	..	..	√	√	..	√	√
" " quotation..........	..	..	√	√	√	√	..	..	√	√	..	√	√
" " income on par......	√	√	√	√	?	?	√	..	..	..	..	..	√
" " income on investm't.	√	√	?	?	√	√	..	√	..	..	?	..	..
Income..................	..	..	..	..	..	..	√	√	..	..	√	..	√
Investment per share.......	?	?	..	..	..	..	..	..	?	..	..	..	..
Total investment..........	..	..	..	..	..	..	..	?	..	?	√	√	?

EXCHANGE.

385. Exchange is the system by which debts to persons in distant places are paid by means of written orders, called *Drafts* or *Bills of Exchange.*

REM. 1.—A in Baltimore may have a bill against B in Cincinnati of $5000 for canned fruits, and C in Baltimore may owe D in Cincinnati $5000 for bacon. In this case, A in Baltimore can draw on B in Cincinnati for $5000, and, either himself or through the medium of a banker or broker of Baltimore, sell the draft to C, who transmits it to D. D of Cincinnati presents it to B of Cincinnati for payment; and both debts are thus paid without incurring the risk or inconvenience of transmitting money to and fro. This is called *exchange*, because the debt of B to A in a *distant* city is exchanged for the more convenient debt to D in his *own* city.

REM. 2.—Exchange is usually conducted through bankers or brokers, who buy commercial bills on distant cities and mail them for collection to their correspondents in those cities. Drafts or bills of exchange are then drawn on their correspondents for the whole or any required part of the sum thus placed to their credit, and sold to persons who have debts to pay in those cities.

386. A **Draft** or **Bill of Exchange** is an order written by one person (called the *drawer*) directing a second person (called the *drawee*) to pay a specified sum of money to a third person (called the *payee*), or to the payee's order.

REM. 1.—A *Foreign Bill of Exchange* is one in which the drawer and drawee reside in *different* countries; and a *Domestic* or *Inland Bill of Exchange* is one in which the drawer and drawee reside in the *same* country.

REM. 2.—Domestic or inland bills of exchange are usually called *drafts;* and are distinguishable as *sight drafts* when they are payable upon presentation, and as *time drafts* when they are payable at a certain time after the date of the draft, or after the date of its presentation for acceptance.

387. An **Acceptance** is an agreement by the drawee to pay the sum specified in the bill of exchange; and is denoted by writing the word "accepted" across the face of the instrument, with his signature; and if payable a given number of days *after sight,* writing the date of his acceptance.

388. Bills of exchange are negotiable or non-negotiable upon the same conditions, and are subject to the same indorsements, and to the same protest if not paid at maturity, as promissory notes (see **341** to **343,** inclusive).

REM.—The date of maturity of bills of exchange is ascertained in the same manner as that of notes (**345**); and, in the United States, excepting sight drafts, are allowed 3 days of grace, unless the phrase "without grace" is mentioned. In a few States and Territories, even sight drafts are allowed days of grace.

389. The **Face** or **Par** of a bill of exchange is the sum expressed therein in words (or both words and figures). It is the *original obligation* exclusive of interest, premium, discount, etc.

REM. 1.—When exchange on a given place sells for *more* than its face or par value, exchange on that place is said to be *above par* or *at a premium;* and if it sells for *less* than its face, it is said to be *below par* or *at a discount.*

REM. 2.—If there is an equality of debts between two cities, say Mobile and Philadelphia, the course of exchange on either city at the other will be at par; but if Philadelphia owes Mobile more than Mobile owes Philadelphia, exchange on Mobile will be above par at Philadelphia, since there will be in Philadelphia more buyers of Mobile exchange than there are sellers; and in Mobile, exchange on Philadelphia will be below par, since there will be in Mobile more sellers of Philadelphia exchange than there are buyers. The variation of exchange, whether at a premium or discount, between any two cities, will not exceed the cost of safely transmitting the money from the debtor city to the creditor city.

DOMESTIC EXCHANGE.

390. **Domestic or Inland Exchange** is exchange between two cities of the same country.

SIGHT DRAFT.

Drovers and Mechanics' National Bank.

$700. BALTIMORE, *Feb. 25, 1886.*

............*At Sight*..................................

Pay to the order of............*F. A. Sadler,*..............

............*Seven Hundred*......................*Dollars.*

To FIRST NATIONAL BANK, J. D. WHEELER,

No. 4935. *Philadelphia.* *Cashier.*

Explanation.—F. A. Sadler, a merchant of Baltimore, owes $700 to S. A. Morris of Philadelphia, and in payment of the debt F. A. Sadler procures from the Drovers and Mechanics' National Bank a draft of $700 on the First National Bank of Philadelphia, payable to the order of himself. He then indorses on the back of the draft "Pay to the order of S. A. Morris" with his (F. A. Sadler's) signature, and sends it in a letter to S. A. Morris. If the letter is lost or stolen, the draft will be valueless to any other party than S. A. Morris (see restrictive indorsement, Rem. 4, **342**). When S. A. Morris receives the draft, he can indorse it in blank and present it to the First National Bank of Philadelphia for payment, and in this manner receive the $700 due him by F. A. Sadler.

TIME DRAFT.

Banking House of Alex. Brown & Sons.

$900. BALTIMORE, *Feb. 27, 1886.*

............*At thirty days sight*..........................

Pay to the order of........ *W. W. Thornton & Co.*..........

.............*Nine Hundred*....................*Dollars.*

Value received, charge to acct.

To BROWN BROS. & CO. A. BROWN & SONS.

No. 7839. *Chicago, Ill.*

Explanation.—The above time draft was purchased by W. W. Thornton &

390

Co., who indorsed upon its back "Pay to the order of W. T. Blair & Co.," with signature, and forwarded it to the indorsee. When W. T. Blair & Co. receive the draft, they present it for acceptance to Brown Bros. & Co. The draft then becomes legally due 30 + 3, or 33 days after date of acceptance.

391. In domestic exchange the face of the draft is regarded as the base; and the premium, discount, interest if a time draft, and market value (= prime cost if a draft is purchased or gross proceeds if one is sold), are regarded as percentages. Hence,

General Direction: *If any of the preceding percentages is required, multiply its two given or obtainable factors; if given, divide it by its given or obtainable factor and the quotient will be its other factor.*

Rem. 1.—The two factors of any of the preceding percentages are the face of the draft and the rate % of that particular percentage.

Rem. 2.—The rate % of market value (if a sight draft) is the rate of the face (100%) + the rate % of premium, or − the rate % of discount.

Rem. 3.—The rate % of market value (if a time draft) is the rate % of market value of a sight draft (Rem. 2) − the rate % of interest for the given time increased by 3 days for grace (Note, **327**).

Rem. 4.—Since drafts are bought and sold at their market value, the rate % of market value will also be the rate % of prime cost if drafts are bought; or the rate % of gross proceeds if they are sold.

Rem. 5.—Brokerage, if included, is usually estimated upon the *market value* of a draft, that is upon its prime cost if purchased, or upon its gross proceeds if sold. Brokerage is added to the market value to find the gross cost of a purchase, or subtracted from the market value to find the net proceeds of a sale.

Rem. 6.—Interest on the face of a time draft should be computed for the given time + 3 days for grace, at the legal rate of the State in which the draft is purchased if no special rate is designated, and deducted from the face of the draft. Hence, the cost of a time draft is less than the cost of a sight draft (by the interest on its face), if both are purchased at the same rate of exchange.

EXAMPLES FOR PRACTICE.

1. What is the cost of a sight draft of $960 at $\frac{5}{8}\%$ premium?

Face of draft ($960) × rate of premium ($\frac{5}{8}\%$) = premium ($6).
Face of draft ($960) + premium ($6) = cost of draft ($966).

What is the cost of a sight draft

2. Of $7200 at $\frac{1}{2}\%$ premium?
3. Of $8500 at $\frac{3}{4}\%$ discount?
4. Of $3200 at $\frac{5}{8}\%$ discount?
5. Of $1500 at $\frac{3}{8}\%$ premium?

6. What is the cost of a 60-day draft of $1600 at ⅜% discount, interest 6%?

Face of draft ($1600) × rate of discount (⅜%) = discount ($6).
Interest of $1600 for 63 days at 6% = $16.80, discount for interest.
Face of draft ($1600) — total discount ($6 + $16.80) = cost ($1577.20).

7. What is the cost of a 30-day draft of $800 at ¾% premium, interest 6%?

Face of draft ($800) × rate of premium (¾%) = premium ($6).
Interest of $800 for 33 days at 6% = $4.40, discount for interest.
Face of draft ($800) + net premium ($6 — $4.40) = cost ($801.60).

What is the cost of

8. A 60-day draft of $3500 at ¾% premium, interest 6%?
9. A 30-day draft of $1800 at ½% premium, interest 8%?
10. An 18-day draft of $2500 at ¾% discount, interest 10%?
11. A 15-day draft of $750 at ⅝% premium, interest 5%?
12. A 30-day draft of $5000 at ⅞% discount, interest 6%?

13. What is the face of a sight draft which can be bought for $1492.50, if the course of exchange is ½% discount?

Cost of draft ($1492.50) ÷ rate % of cost (100% — ½%) = face ($1500).

What is the face of a sight draft which can be bought

14. For $829.15, if exchange is at ⅝% premium?
15. For $350.68, if exchange is at ⅜% discount?
16. For $7518.75, if exchange is at ¼% premium?
17. For $134.98, if exchange is at ¾% discount?
18. For $1210.50, if exchange is at ⅞% premium?

19. What is the face of a 30-day draft which can be bought for $4416.16, at ⅞% discount, interest 6%?

.33 ÷ 60 = .0055, rate of interest for 33 da. (Note, **327**).
Rate of face (100%) — rate of exchange disct. (⅞%) — rate of disct. for interest (.0055) = rate of cost (.98575).
Cost ($4416.16) ÷ rate of cost (.98575) = face of draft ($4480).

20. At ⅞% premium, what is the face of a 69-day draft which can be bought for $165.27, interest 10%?

.72 ÷ 36 = .02, rate of interest for 72 da. (Note, **327**).
Rate of face (100%) + rate of exchange premium (.00⅞) — rate of discount for interest (.02) = rate of cost (98⅞%).
Cost ($165.27) ÷ rate of cost (.98875) = face of draft ($168).

391

What is the face of a time draft which can be bought

21. For $120.84, if payable 30 days after sight; premium 1¼%, interest 6%?

22. For $1579.20, if payable 45 days after sight; discount ½%, interest 6%?

23. For $711.90, if payable 77 days after sight; premium ⅞%, interest 9%?

24. For $1179.90, if payable 60 days after sight; discount ⅝%, interest 6%?

25. A merchant of Baltimore bought a quantity of dressed beef from a firm in Chicago, and remitted in payment a sight draft for $1836. What did the merchant pay for the draft at ⅝% discount?

26. A firm in Nashville bought a 60-day draft on Boston of $3600 at ⅞% discount. What was the cost, allowing interest at 6%?

27. A resident of Philadelphia found that to remit a 60-day draft in payment of a debt in Milwaukee, at ⅞% premium and 6% interest, would cost $1197.90. What sum did he owe in Milwaukee?

28. A commission merchant of Charleston sold for a firm in Boston a consignment of English crockery. The sales amounted to $12240, and his commission 3%. With the proceeds, the commission merchant bought a 30-day draft on Boston at ⅝% discount and 6% interest. What was the face of the draft?

29. I bought a 45-day draft at ¾% premium, and paid $2395.60. What was the face of the draft, interest 6%?

30. A cotton broker of New York bought a 30-day draft on Savannah of $5600 at ½% discount, paying $\frac{1}{16}$% additional for brokerage, rate of interest 5%. How much did he pay for the draft?

31. A commission merchant sold a consignment of 15380 bushels of wheat at $1.40 per bushel, charging 2% for commission, and $94 for other expenses. If exchange is at ⅛% discount and interest 6% per annum, how large a draft, payable 20 days after sight, can he buy with the net proceeds, allowing 4 days for the acceptance of the draft?

32. A commission merchant of San Francisco bought for a firm in Philadelphia a consignment of tea costing $15000. The rate of commission for buying was 2%. The firm in Philadelphia remitted a 15-day draft for the amount due, at ⅞% premium. What did the firm pay for the draft, rate of interest 6%?

33. If, in each of the following columns, the terms marked √ are given, how may the term marked ? be found; and why?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Face of draft........	√	√	√	√	√	√	√	√	..	..	?	?	?	?
Rate of premium.....	√	..	√	..	√	√	..	..	..	..	√	..	√	..
" " discount......	..	√	..	√	..	..	√	√	..	..	..	√	..	√
" " interest.......	..	..	√	√	√	√	√	√	..	..	..	..	..	..
" " brokerage....	..	..	..	..	√	√	√	√	√	√	..	..	√	√
Days to maturity.....	..	..	√	√	√	√	√	√	..	..	..	..	..	..
Market value.........	?	?	?	?	..	..	..	..	?	?	√	√	..	..
Gross cost (purchase)..	..	..	..	..	?	..	?	..	√	..	..	..	√	..
Net proceeds (sale)....	..	..	..	..	..	?	..	?	..	√	..	..	..	√

FOREIGN EXCHANGE.

392. Foreign Exchange is that which is carried on between *different* countries.

Rem.—Foreign bills of exchange are usually drawn at sight (sometimes called "short exchange"), or at 60 days after sight (sometimes called "long exchange"). Three days of grace are usually allowed on foreign bills of exchange. The price of long exchange is less than that of short exchange by the interest on the face of the draft for the difference in time between the two dates of maturity.

393. The **Par of Exchange** is the established value of the monetary unit of one country, expressed in the currency of another.

Rem. 1.—The *intrinsic par* of exchange is the real or intrinsic value of coins estimated by the weight and purity of the metals of which they are composed. Thus, the intrinsic par of the sovereign or £ of Great Britain compared with the monetary unit of the United States (the gold dollar) is $4.8665; the sovereign containing 113 grains of pure gold, and the dollar 23.22 grains, and 113+23.22 = 4.8664[illegible], or practically 4.8665. The intrinsic value of the standard coins of foreign countries is estimated by the Director of the U. S. Mint, and on the 1st of January of each year is proclaimed by the Secretary of the Treasury.

Rem. 2.—The *commercial par of exchange* is the market value of the coin or currency of one country estimated by the requirements of trade between it and another country.

394. A **Set of Exchange** is a bill of exchange usually drawn in sets of three, of the same tenor and date, and so worded that when one of the set is paid, the others become void.

Rem.—The object of a set of exchange is to provide against loss in transmitting from one country to another. The bills are sent either by different routes, or by the same route at different dates. Sometimes only the first and second of the set are sent, and the third is preserved.

SET OF EXCHANGE.

1

Exchange for £500. NEW YORK, *March 10, 1886.*

Sixty days after sight of this First of Exchange..........

(Second and Third of the same tenor and date unpaid),

Pay to the order of Messrs. J. E. MORSE & Co.............

Five Hundred Pounds Sterling..........................

Value received, and charge the same to account of

To J. S. MORGAN & Co.,
London.

No. 149. DREXEL, MORGAN & Co.

2

Exchange for £500. NEW YORK, *March 10, 1886.*

Sixty days after sight of this Second of Exchange........

(First and Third of the same tenor and date unpaid),

Pay to the order of Messrs. J. E. MORSE & Co.............

Five Hundred Pounds Sterling..........................

Value received, and charge the same to account of

To J. S. MORGAN & Co.,
London.

No. 149. DREXEL, MORGAN & Co.

3

Exchange for £500. NEW YORK, *March 10, 1886.*

Sixty days after sight of this Third of Exchange.........

(First and Second of the same tenor and date unpaid),

Pay to the order of Messrs. J. E. MORSE & Co.............

Five Hundred Pounds Sterling..........................

Value received, and charge the same to account of

To J. S. MORGAN & Co.
London.

No. 149. DREXEL, MORGAN & Co.

395. A **Letter of Credit** is a circular letter issued by a banking house to persons who wish to travel abroad, and addressed to bankers generally, and to its own correspondents particularly, in the different countries which the traveler is about to visit, requesting them to supply the traveler with money in sums to suit his convenience until the total amount specified in the letter has been obtained.

Rem.—The difference between a *bill of exchange* and a *letter of credit* is, that the former is paid at one designated place, at one fixed time, and in one stated amount, and the latter is payable at several places, at different times, and in variable amounts.

396. The **Rate of Foreign Exchange** is the market value of the monetary unit of one country estimated in the currency of another.

Rem. 1.—When the cost of exchange on a country is greater than the intrinsic value of the coin represented by its face, to a point beyond the cost of safely transmitting such coin to that country, gold is *exported* at a profit; and when less than its intrinsic value, beyond the cost of shipment and insurance, gold is imported at a profit. Hence, the commercial par of exchange cannot be greater or less than the intrinsic par to a point beyond the express charges and insurance of shipping coin or bullion from one country to the other.

Rem. 2.—American exchange on Great Britain is quoted by giving the exchange value of £1 in dollars; on France, Belgium, and Switzerland by giving the exchange value of $1 in francs; on Holland by giving the exchange value of 1 guilder in cents; on Germany by giving the exchange value of *four* reichsmarks in cents.

397. To find the cost of a foreign bill of exchange.

ILLUSTRATIVE EXAMPLES.

1. How much must be paid for a bill of exchange on Liverpool of £320 17s. 6d., purchased at 4.86?

Solution.—£320 17s. 6d. = £320.875. If £1 of exchange is worth $4.86, £320.875 of exchange must be worth 320.875 times $4.86, or $1559.45+.

2. What is the cost of a bill of exchange on Paris of 1766.30 francs, bought at 5.19½?

Solution.—1766.30÷5.195 = 340. If 5.19½ francs of exchange are worth $1, 1766.30 francs of exchange are worth as many times $1 as 1776.30 fr. are times 5.19½ fr., that is, 340 times $1, or $340.

3. What is the cost of a bill of exchange on Frankfort of 800 reichsmarks, purchased at 94⅞?

Solution.—($.94875 × 800) ÷ 4 = $189.75. If 4 marks of exchange are worth $.94⅞, 1 mark is worth ¼ of .94⅞ ; and 800 marks, 800 times ¼ of $.94⅞, or $189.75.

Rule.—*Divide the face of the bill by the quotation if both express similar monetary units; or multiply the quotation by the face of the bill if they express different monetary units. The result in either case will denote the market value in U. S. money.*

Note 1.—In German exchange, divide the result of the rule by 4.

Note 2.—Bills of exchange are sometimes bought and sold through brokers; in which case brokerage is computed on the market value (=prime cost if a purchase, or gross proceeds if a sale), and added to the market value to find the total cost to the purchaser, or deducted from the market value to find the net proceeds due the seller.

Examples for Practice.

Find the cost.

	Face of draft.	*Quotation.*
1.	£340	4.85¼
2.	£800	4.86⅛
3.	£350 2s. 6d.	4.85⅝
4.	£730 12s. 6d.	4.85⅞
5.	3175.30 francs	5.18⅞
6.	418.75 "	5.19⅛
7.	5200.25 "	5.19⅝

Find the cost.

	Face of draft.	*Quotation.*
8.	750 guilders	40⅛
9.	3820 "	39⅞
10.	1830 "	40¼
11.	5700 "	40⅜
12.	1500 reichsmarks	94⅝
13.	1800 "	95¼
14.	3200.36 "	94⅞

15. A merchant of New York bought of Glenn & Co., Manchester, Eng., 840 yards cassimere at 16s. 8d. per yard; 500 yards tweed suitings at 12s. 6d. per yard; and paid by bill of exchange on Liverpool at 4.84¾. What was the cost of the bill of exchange?

16. What is the cost of a bill of exchange on Geneva of 3475.75 francs, exchange being 5.19⅛?

17. An exchange banker sold a draft on Amsterdam of 2732 guilders at 40⅜. What did he receive for the draft?

18. What is the cost of a draft on Berlin at 60 days' sight of 9325 reichsmarks, exchange 94⅞?

19. I purchased, through a broker, a bill of exchange on London of £750 5s. at 4.85⅜. What was the total cost, brokerage ⅛%?

20. I sold, through a broker, a bill of exchange on Bremen of 815 reichsmarks at 95⅜. What were the net proceeds due me, brok. ⅛%?

398. **To find the face of a foreign bill of exchange.**

ILLUSTRATIVE EXAMPLES.

1. I paid $285 for a bill of exchange on Bremen bought at 95. What was the face of the bill?

Solution.—Since the quotation denotes the market price of 4 marks, as many times 4 marks can be bought for $285 as $.95 is contained times in $285, that is, 300 times 4 marks or 1200 marks.

2. I sold, through a broker, a bill of exchange on Havre and realized $3196 as net proceeds. What was the face of the bill, exchange being 5.19¼, and brokerage ⅛%?

Solution.—$3196+(100%−⅛%)=$3200, gross proceeds or market value. 5.19¼ fr. × 3200=16616 francs, face of bill.

Rule.—*Divide the market value by the quotation, if both express similar monetary units; or multiply the quotation by the market value, if they express different monetary units.*

NOTE 1.—If brokerage is included, divide the gross cost in purchasing a draft by 100% + rate of brokerage; or the net proceeds in selling a draft by 100% − rate of brokerage. The quotient in either case will be the market value of the draft.

NOTE 2.—**397** is applied when the remitter owes money which is payable where the payee resides, in which case the face of the bill should equal the debt, and all expenses of transmission should be defrayed by the remitter. **398** is applied when the remitter owes money which is payable where the remitter lives, in which case the total cost of the bill should equal the sum owed, and all expenses of transmission are deducted therefrom to find the face of the bill.

EXAMPLES FOR PRACTICE.

Find the face of a bill of exchange.

	Market value.	*Course of exchange.*		*Market value.*	*Course of exchange.*
1.	$6126.75	$4.86¼ to £1.	5.	$1489.50	41⅜¢ to 1 guilder.
2.	$2730	$4.87⅛ " "	6.	$181.44	40⅛¢ " "
3.	$800	5.20⅛ fr. to $1.	7.	$304.80	95¼¢ to 4 marks.
4.	$324	5.19¼ " " "	8.	$852.75	94¾¢ " "

What is the face of a bill of exchange which can be bought

9. For $5807.25, if quoted at 4.85 to the £, brokerage ⅛%?
10. For $1834.80, if quoted at 5.19⅜ fr. to the dollar, brok. ¼%?
11. For $7584.32, if quoted at 40¾¢ to the guilder, brokerage $\frac{1}{16}$%?
12. For $8500, if quoted at 95⅛¢ to 4 marks, brokerage ⅛%?

398

13. I bought, through a broker, a bill of exchange on Sheffield, and paid $3889.70. What was the face of the bill, the course of exchange being 4.85, and the brokerage ¼% ?

14. I sold, through a broker, a bill of exchange on Bristol, Eng., at 4.85, and received $3580.02¾ as net proceeds. What was the face of the bill, brokerage ¼% ?

15. The cost of a bill of exchange on Bordeaux, bought at 5.20, was $941.175, including ⅛% brokerage. What was the face of the bill ?

16. A merchant sold 362 Mexican dollars at 88½ cents each, and invested the proceeds in a bill of exchange on Amsterdam at 40. What was the face of the bill ?

17. What is the face of a bill of exchange on Hamburg which can be bought for $756.945, when the course of exchange is 94½, and the rate of brokerage for buying ⅛% ?

18. If Illinois Central R. R. stock is quoted in New York at 142, what should be the equivalent London quotation, the course of exchange being 4.85½ ?

Explanation.—American securities are quoted in London at an assumed valuation of $5 to the £. Hence, *multiply the American quotation by 5, and divide the product by the rate of exchange.*

Solution.

142% of $5 = $7.10.

$7.10 ÷ $4.855 = 146.24 + %.

19. What is the equivalent London quotation of Lake Shore R. R. stock, quoted in New York at 109, the course of exchange being 4.84¾ ?

20. St. Paul and Omaha is quoted in New York at 102. What is the equivalent London quotation, the course of exchange being 4.85 ?

21. Missouri Pacific is quoted in New York at 87. What is the equivalent London quotation, course of exchange 4.86 ?

22. What is the equivalent London quotation of New York Central R. R. stock, which is quoted in New York at 89½, the course of exchange being $4.86½ ?

23. Baltimore and Ohio R. R. stock is quoted in London at 200. What is the equivalent New York quotation, course of exchange 4.85 ?

Solution.—(200% of $4.85) ÷ $5 = 194%, equivalent New York quotation.

24. The course of exchange being 4.85½, what is the equivalent New York quotation of Illinois Central R. R. stock, quoted in London at 130 ?

25. What is the equivalent New York quotation of stock, quoted in London at 118, when the course of exchange is 4.86 ?

26. New York Central Railroad stock is quoted in London at 106. What is the equivalent American quotation, if the course of sterling exchange is 4.85¼ ?

27. What is the equivalent New York quotation of U. S. 4% bonds, quoted in London at 118, sterling exchange being 4.84¾ ?

28. If, in each of the following columns, the terms marked √ are given, how may the term marked ? be found; and why ?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Face of draft............	√	√	√	√	√	?	?	?	?	√	√	√	√	√
Quotation of English exch.	√	..	..	√	..	√	..	..	√	?	..	..	?	..
" " French "	..	√	..	..	√	..	√	..	..	..	?	..	..	?
" " German "	..	..	√	..	..	..	..	√	..	..	..	?	..	..
Brokerage..............	..	..	..	√	√	..	..	√	√	..	..	..	√	√
Market value............	?	?	?	..	..	√	√	..	..	√	√	√	..	..
Gross cost (purchase).....	..	..	..	?	..	..	..	√	..	..	..	..	√	..
Net proceeds (sale).......	..	..	..	..	?	..	..	..	√	..	..	..	..	√

CUSTOM-HOUSE BUSINESS.

399. **Duties** or **Customs** are taxes levied by the general government on goods imported from foreign countries, to produce a revenue for the support of the government, to protect home industries, or for both purposes.

400. **Custom-Houses** are branches of the Treasury Department established by Congress at the principal ports of the United States for the collection of duties, and the entry and clearance of vessels.

REM.—The ports at which custom-houses are established are called "ports of entry;" in addition to which there may be other designated ports in the same collection district called "ports of delivery." Duties are levied and paid, and vessels entered and cleared, at the port of entry; but the cargoes of vessels properly entered may afterward be discharged at one or more of the ports of delivery.

401. An **Invoice** or **Shipper's Manifest** is an itemized list of merchandise shipped or sold, containing the name of the purchaser or consignee, and the seller or consignor, the place and date of purchase, the quantity, quality, price, incidental charges, etc.

REM.—Invoices are required to be made out in the weights, measures, and currency of the country from which the importation is made; the value in U. S.

money of the foreign currency of the principal nations of the world being annually proclaimed for this purpose by the Secretary of the Treasury (**183**). If the value of a foreign currency has not been thus proclaimed, invoices made out in such currency must be accompanied by a consular certificate denoting its value in U. S. money.

402. A **Tariff** is a list or schedule of goods, and the rates of duty imposed by law on the same.

403. Duties are classified as *Ad Valorem* and *Specific*.

404. An **Ad Valorem Duty** is a duty levied upon imported goods at a certain per cent. of their market value at the time and place of purchase or of shipment; as, china ware, plain, 55%, ornamented 60%, silks 50%, jewelry 25%, etc.

Rem. 1.—By market value of goods is meant their invoice value after all discounts have been deducted and before any extra charges (such as commission, cartage, boxing, freight, etc.) have been added. Prior to the Tariff Act of 1883 these charges were included in estimating the dutiable value of goods.

Rem. 2.—Duties are not computed on fractions of a dollar. If the cents are less than 50 they are rejected, if 50 or more the dollars are increased by 1.

405. A **Specific Duty** is a duty levied upon imported goods according to their weight or measure, and without reference to their value; as, camphor, 5¢ per pound; linseed oil, 25¢ per gallon; hemp carpeting, 6¢ per square yard; bituminous coal, 75¢ per ton, etc.

Rem. 1.—In computing specific duties the ton is estimated at 2240 lb., the hundredweight at 112 lb., and the quarter at 28 lb.

Rem. 2.—Some goods are subject to both a specific and an ad valorem duty; as, Axminster carpets, 45¢ per square yard and 30% ad valorem; cigars, $2.50 per pound and 25% ad valorem, etc.

406. **Allowances** are deductions made in estimating specific duties, and are distinguished as *Tare*, *Breakage*, and *Leakage*.

Rem. 1.—**Tare** is a deduction from the gross weight of goods, allowed for the *weight* of the cask, box, bag, etc., in which they are contained.

Rem. 2.—**Breakage** is a deduction allowed for the *waste* of liquids imported in bottles.

Rem. 3.—**Leakage** is a deduction allowed for the waste of liquids imported in casks or barrels.

407. Ad valorem duties are computed according to the principles of percentage, the market value at the place of purchase or export (Rem. 1, **404**) being regarded as the base, and the ad valorem duty as the percentage.

ILLUSTRATIVE EXAMPLE.

What is the duty upon 800 yards of Brussels carpeting 27 inches wide, and invoiced at 11.25 francs per yard, the rate of duty being 30 cents per square yard and 30% ad valorem?

Solution.

$.193 × (11.25 × 800) = $1737.
$1737 × .30 = $521, ad valorem duty

27 in. = $\frac{27}{36}$ = $\frac{3}{4}$ yd., width.
$\frac{3}{4}$ × 800 = 600 sq. yd.
$.30 × 600 = $180, specific duty.

$521 + $180 = $701, combined duty.

Explanation.—Multiply 11.25 fr. by 800 to find the cost at the place of purchase (9000 fr.); this result by the custom-house value of 1 franc as shown in **183** ($.193) to find the dutiable value in U. S. money ($1737), and this result by the rate of ad valorem duty (30%) to find the ad valorem duty ($521).

Multiply the width of the carpet ($\frac{3}{4}$ yd.) by its length (800 yd.) to find the number of square yards (600), and this result by the specific duty per sq. yd. (30¢) to find the specific duty ($180). Add the ad valorem duty ($521) to the specific duty ($180) to find the total duty ($701).

Rule.—AD VALOREM DUTY.—*Reduce the net invoice price to U. S. money* (**183**); *deduct all allowances* (**406**); *and multiply the remainder by the rate of ad valorem duty.*

SPECIFIC DUTY.—*Deduct the total allowance from the total quantity specified in the invoice; reduce the remainder to the denomination on which the rate of specific duty is given, and multiply the result by the rate of specific duty.*

NOTE.—By net invoice price is meant the net cost or market value of the importation at the place of purchase (Rem. 1, **404**).

EXAMPLES FOR PRACTICE.

What is the ad valorem duty upon an importation invoiced

1. At £340 12s. 6d., allowing 10% for breakage, rate of duty 30%?
2. At 15378.40 marks, allowing 5% for leakage, rate of duty 15%?
3. At 9506.35 guilders, allowing 2% for tare, rate of duty 50%?
4. At £835 15s., rate of duty 10%?
5. At 8195 lira, allowing 8% for breakage, rate of duty 25%?
6. At 927.75 francs, if the rate of duty is 75%?
7. At 3500 yen, if the rate of duty is 12%?
8. At 475 pesos (Cuba), if the rate of duty is 40%?

407

What is the specific duty upon an importation of

9. 7835 sq. yd. of dress goods, rate of duty 25¢ per sq. yd.?

10. 300 tons of medicinal roots, rate of duty 1¢ per pound?

11. 1200 tons guano, rate of duty 75¢ per ton?

12. 30 doz. bottles wine, breakage 10%, duty $2.50 per doz.?

13. 650 gallons brandy, leakage 2%, rate of duty $1.50 per gal.?

14. 160 gross bottles, breakage 5%, rate of duty 10¢ per doz.?

15. A merchant in Paris consigned to a firm in Boston 3500 yards of 20-in. bunting, the market price of which in Paris, at the time of shipment, was 1.15 francs per yard. What was the duty at 10 cents per square yard and 35% ad valorem?

16. At 50% ad valorem, what is the duty upon an importation of 200 dozen kid gloves, invoiced at 73 francs per dozen?

17. What is the duty upon a consignment of watches, invoiced at £3560 10s., at 20% ad valorem?

18. If subject to a specific duty of $2 per gallon, what must be the selling price per gallon of an importation of 500 gallons French cognac to enable the importer to realize 15% profit, if the net invoice price is 18 francs per gallon, and the charges other than duty aggregate 95 francs in France, and $50 in the United States, the course of exchange on Paris being 5.20 francs to the dollar?

19. At 30% ad valorem, what is the duty upon an oil painting the net invoice of which is 3200 guilders?

20. At 1¼ cents per pound, what is the duty upon 3 T. 2 cwt. 1 qr. of cast-iron stove plates?

21. At 65 cents per cubic foot, what is the duty upon a block of marble 8 ft. × 2 ft. × 18 in.?

22. What is the duty upon an importation of 1500 yards flannel weighing 350 pounds net, and valued at 60 cent per yard, the rate of duty thereon being 24 cents per pound and 35% ad valorem?

23. A merchant of Baltimore imported from Sheffield a lot of pocket knives invoiced as follows: 50 doz. at 12s. 6d. per doz.; 25 doz. at 17s. 6d. per doz.; 15 doz. at £1 5s. 6d. per doz.; 20 doz. at £1 12s. 6d. per doz.; 8 doz. at £2 5s. per doz.; 15 doz. at £2 10s. per doz.; and 9 dozen at £3 12s. per doz. The charges in Sheffield amounted to £3 10s. What was the duty upon the importation at 50% ad valorem? What was the cost in Baltimore of remitting a bill of exchange in payment, the course of English exchange being 4.86½, brokerage ⅛% additional?

TAXES.

408. A **Tax** is a sum of money levied on the person, property, or income of an individual for the support of the government or for other public purposes.

409. A **Property Tax** is a sum levied on real estate or personal property.

REM.—*Real estate* means lands, buildings, or other *fixed* property; *personal property* means horses, cattle, furniture, merchandise, money, notes, or other *movable* property.

410. A **Capitation** or **Poll Tax** is a sum levied, irrespective of property, on the person of every adult male citizen not specially exempt by law.

411. An **Assessor** is a public officer whose duty it is to discover persons and property subject to taxation within his assessment district, and to estimate the taxable value of each piece of property.

412. A **Tax Collector** is a public officer whose duty it is to receive or collect the taxes of his collection district.

413. A **License Fee** is a sum exacted by a city, state, or general government for the privilege of conducting a specified business.

414. Property taxes may be computed according to the principles of percentage, the assessed value of the property being regarded as the base, and the property tax as the percentage. Hence,

General Direction: *If the property tax is required multiply its two given factors; if the property tax is given, divide it by its given factor and the quotient will be its remaining factor.*

REM.—The two factors of a given or required property tax are its assessed value and the rate % of taxation.

415. To find the property tax.

ILLUSTRATIVE EXAMPLE.

The rate of taxation in a certain city is $1.15 on the $100. What tax should be paid by a person whose property is worth $18654, and is assessed at ⅔ of its value?

Solution.—⅔ of $18654 = $12436, assessed value. $1.15 on $100 = $.0115 on $1. Hence, $.0115 × 12436 = $143.01 +, required tax.

Rule.—*Multiply the assessed value by the rate of taxation on $1.*

EXAMPLES FOR PRACTICE.

What is the tax, if the assessed value of the property is

1. $3500, and the rate of taxation 1% ?
2. $7200, and the rate of taxation ⅜% ?
3. $9800, and the rate of taxation 12½ mills on the dollar ?
4. $25300, and the rate of taxation 9 mills on the dollar ?
5. $6300, and the rate of taxation 83 cents on $100 ?
6. $10500, and the rate of taxation $1.25 on $100 ?
7. $8758, and the rate of taxation $1.18 on $100 ?
8. $2940, and the rate of taxation 95 cents on $100 ?
9. I have property worth $19600, which was assessed at ¾ of its value. What was my entire tax at 7½ mills on the dollar, including 3 polls at 75 cents each ?
10. My real estate was assessed at $7500, and my personal property $12500. What amount of tax did I pay thereon at $1.18 per $100, having been allowed a discount of 3% for prompt payment ?
11. What is my entire tax for the present year upon real estate worth $12500, and personal property worth $5000, the real estate having been assessed at ⅗ of its value, the rate of taxation being $1.40 on the $100 ?
12. The assessed value of the real estate of a town is $784300, and of the personal property $35800. If the rate of taxation is 35 cents on the $100, what will be the total revenue of the town, including $730 from other sources ?

416. **To find the rate of taxation.**

ILLUSTRATIVE EXAMPLE.

The assessed value of real estate in a town is $986000, and of personal property $450000. The sum required by taxation is $12145.63 net, after allowing 2% for collection. What should be the rate of taxation on the entire property, if the number of polls be 375, and the tax for each poll 50 cents ?

416

Solution.

\$986000 + \$450000 = \$1436000, property subject to taxation.
\$12145.63 ÷ (100% − 2%) = \$12393.50, total tax to be levied.
\$12393.50 − (\$.50 × 375) = \$12206, total tax on property.
\$12206 ÷ \$1436000 = .0085, rate of taxation on \$1.
.0085 of \$1 = 8½ mills on \$1, or 85 cents on \$100.

Rule.—*Divide the property tax by the assessed valuation. The quotient carried to hundredths will be the rate per cent. of taxation; carried to thousandths will be the rate in mills on \$1; and carried to ten-thousandths will be the rate in cents on \$100.*

NOTE 1.—If a poll tax is included, deduct it from the total tax, and then apply the rule.

NOTE 2.—The collector's commission is computed upon the gross amount of tax collected. Hence, in allowing for commission, regard the net or available tax as the percentage; the gross tax as one of its factors (the base); and the rate % of net tax (= 100% − rate % of collector's commission) as its other factor.

EXAMPLES FOR PRACTICE.

Find the rate of taxation in mills on \$1 if the assessed valuation is

1. \$514000, and the gross tax \$6233, including 130 polls at 50¢.
2. \$15000, and the gross tax \$137.25, including 3 polls at 75¢.
3. \$15387200, and the total property tax \$169259.20.

Find the rate of taxation in cents on \$100 if the assessed valuation is

4. \$34000, and the total property tax is \$469.20.
5. \$1275000, and the total property tax is \$12495.
6. \$78000, the net tax \$837.33, and the collector's commission 5%.
7. \$187500, the net tax \$2793, and the collector's commission 2%.
8. The assessed valuation of the property of a village is \$832000. The estimate of corporate expenses includes \$1500 for schools, \$1600 for streets, \$1200 for salaries and commissions, and \$692 for sundry other expenses. What will be the rate of taxation expressed as mills on the dollar? What taxes will be required of A, whose real estate is assessed at \$8000, and personal property at \$450?
9. The cost of certain public improvements in a town is estimated at \$4578.40; and to raise this sum, \$1475000 worth of property is subject to taxation. The preceding estimate does not include the

collector's commission of 3%. How much on $100 will each property holder be required to pay? What will be the tax of B, whose property is valued at $8300?

10. In a county containing property assessed at $15480000 there were 6750 polls. The county expenses for a certain year included $12000 for schools, $7300 for interest on the county debt, $9000 for roads and bridges, $5000 for salaries and commissions, and $20015 for sundry other purposes. If the county poll tax was 50 cents, and the estimated revenue from licenses for that year $8500, what tax rate on the $100 must be declared to defray these expenses and provide a sinking fund of $5000? What was the county tax of C, who owned property assessed at $15300, and who was liable for 3 polls?

INSURANCE.

417. **Insurance** is a contract by which one party, for a stipulated sum, agrees to indemnify another for a specified loss or damage, if sustained within a given time.

REM.—Insurance is of two kinds: *property insurance* and *personal insurance.*

418. **Property Insurance** is that which indemnifies the owner of real estate, merchandise, or other property for its loss or damage by fire, water, wind, or other specified casualty.

REM.—Property insurance is distinguished as *Fire Insurance* (indemnity for loss or damage by fire); *Marine Insurance* (indemnity for the loss or damage of vessels or their cargoes by storms or other dangers of navigation); *Stock Insurance* (indemnity for the loss of horses, cattle, or other live stock); *Transit Insurance* (indemnity for the loss of, or damage to, merchandise during its transportation from one place to another); and *Accident Insurance* (indemnity for the breakage of plate glass, etc.).

419. **Personal Insurance** is the insurance of persons.

REM.—Personal insurance is distinguished as *Life Insurance*, which secures the payment of a specified sum to a designated party at the death of the insured; *Health Insurance*, which secures the payment of a weekly sum during sickness; and *Accident Insurance*, which secures the payment of a stipulated sum in case of accident.

420. The **Insurer** or **Underwriter** is the party who assumes the risk or agrees to indemnify for loss. The **Insured** is the party protected from loss.

Rem. 1.—Insurance is usually effected through a company or corporation called an *Insurance Company*. Most insurance companies make a specialty of *one* class of risks, but many combine two or more classes.

Rem. 2.—Insurance companies are distinguished as *Stock Insurance Companies*, of which the capital is owned by stockholders, who alone share the profits or losses; *Mutual Insurance Companies*, which have no stockholders, and in which the policy-holders (parties insured) share the profits or losses; and *Mixed Insurance Companies*, which are combinations of stock and mutual insurance, that is, in which the profits, beyond a specified dividend to the stockholders, are divided among the policy-holders.

421. A **Policy** is the written agreement or contract between the insurance company and the party insured.

Rem.—A policy contains a description of the person or property therein insured, the conditions upon which the insurance was effected, the amount of indemnity to be paid, etc., etc.

422. The **Premium** is the sum paid by the insured to the insurer for the risks which the latter assumes.

Rem. 1.—The premium is a certain rate per cent. of the amount of insurance, as ½%, ¾%, etc. Such rates are usually expressed by giving the cost of $100 of insurance; as 50 cents per $100, 75 cents per $100, etc.

Rem. 2.—Rates of premium vary with the nature of the risk assumed, and the time for which the policy is issued. The more hazardous the risk, or the longer its continuance, the higher will be the rate of premium demanded.

423. The **Term of Insurance** is the time for which the risk is assumed, or the property insured.

Rem.—The term of fire insurance is usually from 1 to 5 years, though some companies issue policies for longer periods. The term of marine insurance may be for the entire voyage, or only from one designated port to another.

FIRE AND MARINE INSURANCE.

424. **Fire Insurance** is a contract by which one party, for a stipulated payment, agrees to indemnify another for any loss or damage to a described piece of property, if caused by fire within a given time.

Rem. 1.—Fire insurance companies are also liable for loss or damage caused by water, when applied for the purpose of extinguishing a fire; and for the loss of buildings which have been torn down or otherwise damaged or destroyed to prevent a fire from spreading.

Rem. 2.—Fire insurance companies do not usually insure property for its *full* value; that the party insured may be interested in its protection. They also reserve the privilege of rebuilding or repairing damaged property, if it be to their interest.

REM. 3.—In an ordinary fire policy, if the loss is only *partial*, the insurance company promises to pay the full value of the property destroyed, or the full depreciation of the property damaged, if it does not exceed the sum covered by insurance; but if insured in two or more companies, each company will pay its *pro rata* share of the loss.

425. **Marine Insurance** is a contract wherein the insurer, for a stipulated payment, agrees to indemnify the insured for any loss or damage to a certain vessel, cargo, or freight, by storms or other perils or risks of navigation to which it may be exposed during a particular voyage, or within a specified time.

REM. 1.—*Transit Insurance* (Rem., **418**) is a form of marine insurance, whether the transportation be effected by rail or water route, or both; and so is *Inland Insurance*, or insurance on steamboats or other vessels navigating rivers, lakes, or other inland waters.

REM. 2.—Marine policies contain an "average clause," under the terms of which the indemnity to be paid for a *partial loss* of property *partially insured* will be such a part of the loss, as the sum covered by insurance is of the full value of the property. Thus, if a vessel valued at $60000 is insured for one-half its value ($30000), and is *totally* destroyed, the owners will be entitled to the full amount covered by insurance ($30000); but if it is damaged to the extent of $8000, the owners will be entitled to only one-half of the partial loss, or $4000.

426. **Salvage** is a compensation allowed parties for their voluntary assistance in saving a ship, cargo, or other property from perils of fire or water.

427. **Short Rates** are certain rates of premium which are charged by companies when the period of insurance is less than one year.

REM.—It often happens that goods are insured for short periods of time, especially consignments. The rates for such periods are taken from the *Short Rate Tables*, which are higher, proportionately, than the rates for one year.

428. **To Cancel a Policy** is to annul the contract between the insurance company and the party insured.

REM. 1.—When a policy is canceled before the expiration of the time for which it was issued, a *Return Premium* is paid by the insurance company to the party insured. When canceled at the instance of the insurance company, the return premium will be such a part of the entire premium, as the unexpired term is of the whole term; but when canceled at the request of the policy-holder, the return premium will be what is left of the total premium after deducting the "short rate premium" for the time the policy had been in force.

Rem. 2.—After a partial loss has been paid, the return premium will be such a part of the entire premium as the remainder of the policy (after deducting the partial loss paid) is of the entire or original policy.

429. Computations in fire and marine insurance are made in accordance with the principles of percentage; the sum insured being regarded as the base, and the premium as the percentage. Hence,

General Direction: *If the premium is required, multiply its two given factors; if given, divide it by its given factor and the quotient will be its other factor.*

Rem.—The factors of a premium are the sum insured and the rate of premium (usually called rate of insurance).

EXAMPLES FOR PRACTICE.

1. A merchant insured his warehouse worth $10800 at $\frac{2}{3}$ of its value. What was paid for premium if the rate of insurance was $\frac{5}{8}$% ?

Sum insured ($\frac{2}{3}$ of $10800) × rate of premum ($\frac{5}{8}$%)= premium (?).

2. The stock of a manufacturing company was insured for $75000 at the following rates: $\frac{1}{3}$ of the stock at $\frac{5}{8}$%, $\frac{2}{3}$ of the remainder at $\frac{3}{4}$%, and the remainder at $\frac{7}{8}$%. What was the total premium paid ?

3. A stock of goods valued at $45000 was insured in the Eagle Fire Insurance Company for $10000, in the Manhattan F. I. Co. for $15000, and in the Globe F. I. Co. for $9000. If the stock was damaged by fire to the extent of $18000, what proportion of the loss should be paid by each company (Rem. 3, **424**) ?

4. A store and contents were insured as follows: $10000 in the Home Insurance Co., $6000 in the Essex Insurance Co., and $8000 in the Howe Insurance Co. A partial loss of $6000 having been caused by fire, how much should each company contribute ?

5. A stock of goods was insured for one year for $12000 at $\frac{5}{8}$%. Six months thereafter the policy was canceled at the request of the insured. What was the return premium, the short rate for six months being $\frac{3}{8}$% (Rem. 1, **428**) ? What would have been the return premium if the policy had been canceled at the request of the underwriters ?

6. A fire insurance company took a risk of $75000 at $\frac{7}{8}$% premium, and reinsured $\frac{1}{3}$ of the risk in another company at $\frac{5}{8}$%, and $\frac{1}{2}$ of it in another at $\frac{3}{4}$%. What did the company gain by reinsurance ?

7. A ship worth $75000 was insured for $20000 in one insurance company at $\frac{3}{4}$% premium, and for $35000 in another at $\frac{7}{8}$% premium.

She was afterwards damaged by a storm to the extent of $3000. What insurance did the owners of the ship receive from each company?

8. A vessel worth $30000 was insured for $5000 in each of four companies. What insurance would its owners receive from each company if the vessel sustain a partial loss of $8000? If the vessel be totally destroyed?

9. I paid $175 premium on a schooner worth $25000, which was insured at ⅘ of its value. What was the rate % of insurance? What was the rate of insurance in cents on the $100?

Premium ($175) ÷ sum insured (⅘ of $25000) = rate of premium (?).

10. An insurance company received $120 premium on property insured for $16000. If a building be insured for $40000 in another company at the same rate, what will be the premium on the latter?

11. A marine insurance company received $484.50 for insuring a vessel worth $80000 at ⅗ of its value. What was the rate % of insurance if $4.50 was charged for the policy and survey?

12. The stock of a wholesale notion house is insured in several companies for $12000, and is damaged by water, while extinguishing a fire, to the extent of $960. What % of its risk should be paid by each company? What amount should be paid by a company which had a risk of $5000 upon the stock?

13. An insurance company accepted a risk of $32000 at 1⅛% premium; and insured ⅓ of the risk in another company at ⅞% premium, and ¼ of it in another at ¾%. What rate % of premium did the company net on the remainder of the risk?

14. A merchant owned a steamboat worth $40000, and had ⅝ of its value insured in one company at ¾% premium, and ⅜ of its value in a second company at ⅞%. What was the average rate % of premium paid on the whole?

15. A fire insurance company charged $129.60 for insuring a house for $13500. What was the rate of insurance per $100?

16. If the premium paid for insuring a house was $43.20, and the rate of insurance 80 cents per $100, for what sum was it insured?

17. For what sum is a house insured if the premium paid is $75, and the rate of insurance ⅝%?

Premium ($75) ÷ rate of premium (⅝%) = sum insured (?).

18. The London Insurance Company charged $150 for insuring property at 1¼% premium, and the Manhattan Ins. Co. charged ⅞%

premium for property of the same valuation, though less hazardous. What premium was paid to the Manhattan Company?

19. I paid $19.20 for having my house insured, the rate of insurance being 60 cents per $100. For what sum was my house insured?

20. A man owns ⅜ of a steamboat, and insures ¾ of his interest at 1¼%, paying $112.50 premium. What was the value of his interest in the boat? If, before the policy lapses, the boat should be damaged by fire to the extent of $4000, what sum will he receive from the insurance company?

21. The invoice price of a shipment of goods is $7930. The shipper wishes to have them insured at such a sum as will cover both the invoice price and the premium in case of loss. What should be the sum insured, if the rate of insurance is ⅞%?

Value ($7930) ÷ 100% − rate % of insurance (99⅛%) = sum to insure which will include both value and premium (?).

This kind of insurance is not applicable to real estate or other risks in which insurance for a greater sum than the value of the property will offer an incentive to negligence in its protection. It is allowed in transit and some other kinds of marine insurance.

22. The value of a schooner's cargo is $8372.50, and the owner wishes to insure it for a sum which will cover the value of the cargo and the cost of insurance. If the rate of insurance is 1⅛%, and the cost of the policy $1.75 additional, what should be the sum insured?

23. The value of a certain consignment is $711.72. For what sum must the shipper have it insured to obtain both the value of the consignment and the premium thereon if it should be totally lost, the rate of insurance being $1.15 per $100?

24. A ship is insured for enough more than its value to include the cost of insurance at 1¼%, and $6 for the policy and survey. If the ship is worth $51350, what should be the sum insured?

LIFE INSURANCE.

430. **Life Insurance** is insurance upon the *life* of persons.

Rem. 1.—Life insurance policies are distinguished as *Life Policies* (insurance payable at the death of the person insured); *Term Policies* (insurance payable at the death of the party insured if it occur within a specified number of years); *Joint Life Policies* (insurance payable to the survivor at the death of the party with whom he had been jointly insured); Endowment Policies (insurance payable to the person insured if he should survive a specified number of years, or to a designated party if the person insured should die before the expiration of such period); *Reserve Endowment Policies* (insurance payable at the death of the person insured, but which will entitle him to surrender the policy at any time he may desire, and receive therefor an endowment varying with the number of years such policy had been in continuance); and *Tontine Policies* (insurance payable at the death of the party insured, and which will further entitle him, if he survives to a certain specified time, to a *pro rata* distribution of the tontine premiums paid during that period after deducting therefrom the tontine death claims and other expenses of that period).

Rem. 2.—Life insurance may be effected by the payment of a single premium (single payment policies); of five, ten, fifteen, or twenty consecutive annual premiums (five-payment policies, ten-payment policies, etc.); or of one premium annually during the life of the person insured (annual payment policies). Annual premiums are frequently subdivided into semi-annual and quarterly payments.

431. The **Expectation of Life** is the average number of years which persons of a given age have been found to live.

Rem.—The expectation of life is based upon a carefully prepared table of mortality, extending through a series of years, and showing the *average* number of deaths in every thousand persons of the same age. While the duration of life of any single individual cannot be determined with accuracy, it has been found that for every person of a given age who lives longer than the average of persons of that age, some other person of the same age will die correspondingly earlier. The following is the

AMERICAN EXPERIENCE TABLE OF MORTALITY.

Expectation of Life, in Years.

Age.	*Expectation of Life.*	*Age.*	*Expectation of Life.*	*Age.*	*Expectation of Life.*	*Age.*	*Expectation of Life.*	*Age.*	*Expectation of Life.*	*Age.*	*Expectation of Life.*
10		28	36.73	37	30.35	46	23.80	55	17.40	64	11.68
20	42.20	29	36.03	38	29.62	47	23.08	56	16.72	65	11.10
21	41.53	30	35.33	39	28.90	48	22.36	57	16.05	66	10.54
22	40.85	31	34.62	40	28.18	49	21.63	58	15.39	67	10.00
23	40.17	32	33.92	41	27.45	50	20.91	59	14.74	68	9.48
24	39.49	33	33.21	42	26.72	51	20.20	60	14.09	69	8.98
25	38.81	34	32.50	43	25.99	52	19.49	61	13.47	70	8.48
26	38.11	35	31.78	44	25.27	53	18.79	62	12.86	71	8.00
27	37.43	36	31.07	45	24.54	54	18.09	63	12.26	72	7.54

431

432. Rates of Life Insurance are expressed at a given sum on each $1000 of the amount insured. The following is a

TABLE OF INSURANCE RATES.

Showing premiums for an insurance of $1000 upon the mutual plan.

	Life Policies. Insurance payable at death only.					***Endowment Policies.*** Premiums payable annually for 10 years.			
	Premiums payable annually for				*Single Payment.*	*Insurance payable as stated, or at death if it occur earlier.*			
Age.	*Life.*	*20 years.*	*15 years.*	*10 years.*		*In 20 years.*	*In 15 years.*	*In 10 years.*	*Age.*
25	$19.89	$27.39	$32.34	$42.56	$326.58	$75.39	$87.92	$103.91	**25**
26	20.40	27.93	32.97	43.37	332.58	75.54	88.05	104.03	**26**
27	20.93	28.50	33.62	44.22	338.83	75.71	88.19	104.16	**27**
28	21.48	29.09	34.31	45.10	345.31	75.89	88.34	104.29	**28**
29	22.07	29.71	35.02	46.02	352.05	76.08	88.50	104.43	**29**
30	22.70	30.36	35.76	46.97	359.05	76.29	88.67	104.58	**30**
31	23.35	31.03	36.54	47.98	366.33	76.52	88.86	104.75	**31**
32	24.05	31.74	37.35	49.02	373.89	76.76	89.06	104.92	**32**
33	24.78	32.48	38.20	50.10	381.73	77.02	89.27	105.11	**33**
34	25.56	33.26	39.09	51.22	389.88	77.30	89.50	105.31	**34**
35	26.38	34.08	40.01	52.40	398.14	77.62	89.75	105.53	**35**
36	27.25	34.93	40.98	53.63	407.11	77.95	90.02	105.75	**36**
37	28.17	35.83	42.00	54.91	416.21	78.33	90.31	106.00	**37**
38	29.15	36.78	43.06	56.24	425.64	78.73	90.63	106.28	**38**
39	30.19	37.78	44.17	57.63	435.42	79.18	90.98	106.58	**39**
40	31.30	38.83	45.33	59.09	445.55	79.68	91.37	106.90	**40**
41	32.47	39.93	46.56	60.60	456.04	80.23	91.79	107.26	**41**
42	33.72	41.10	47.84	62.19	466.89	80.83	92.26	107.65	**42**
43	35.05	42.34	49.19	63.84	478.11	81.49	92.78	108.08	**43**
44	36.46	43.64	50.61	65.57	489.71	82.22	93.34	108.55	**44**
45	37.97	45.03	52.11	67.37	501.69	83.03	93.97	109.07	**45**
46	39.58	46.50	53.68	69.26	514.04	83.93	94.67	109.65	**46**
47	41.30	48.07	55.35	71.25	526.78	84.92	95.46	110.30	**47**
48	43.13	49.73	57.10	73.32	539.88	86.02	96.31	111.01	**48**
49	45.09	51.50	58.95	75.49	553.33	87.23	97.27	111.81	**49**
50	47.18	53.38	60.91	77.77	567.13	88.54	98.33	112.68	**50**

NOTE.—To find the semi-annual rate per $1000, add 4% interest to any one of

432

the preceding annual rates, and divide the result by 2. To find the quarterly rate, add 6% interest and divide the result by 4.

433. EXAMPLES FOR PRACTICE.

1. A man, aged 38 years, takes out an endowment policy of $8000, payable to himself in 15 years if living, or to his heirs at his death if he die before the expiration of 15 years. What annual premium will he be required to pay for ten years?

Solution.—In the Table of Rates it is found that an endowment policy of $1000, payable in 15 years, will cost a person aged 38 years ten annual premium payments of $90.63 each. Hence, a similar endowment policy of $8000 must cost 8 times $90.63, or $725.04 per annum for ten years.

2. A person aged 27 years took out an ordinary life policy of $15000. How much premium had he paid at the age of 50 years?

3. I am 32 years of age and wish to secure a 20-year endowment policy of $5000. What premium will I be required to pay annually for 10 years?

4. What semi-annual payment for life will secure to a person aged 36 years a policy of $2000?

5. What quarterly payment for 10 years will secure to a person aged 30 years an endowment policy of $4000, payable in 20 years?

6. Mr. A, aged 32, obtained a life policy of $3000, requiring 10 annual premium payments. What annual premium did he pay?

7. What single payment will secure to a person aged 45 years a life policy of $6000?

8. What quarterly payment for 10 years will secure to a person aged 40 years a life policy of $5000?

9. What sum of money should be laid aside monthly by a person, aged 29, to pay the annual premium of an ordinary life policy of $5000?

10. A man, aged 48 years, obtained a 20-year endowment policy of $7000, but he died after paying the 8th annual premium. What sum did his heirs realize above the cost of insurance?

Solution.—By reference to the Table of Rates, it is found that, to a man aged 48 years, the annual premium per $1000 on a 20-year endowment policy is $86.02. Hence, a similar policy of $7000 will cost 7 times $86.02, or $602.14 per annum, and 8 annual premiums on such policy will amount to 8 times $602.14, or $4816.72. Deduct the total premium paid ($4816.72) from the amount of the policy ($7000), obtaining $2183.28 as the sum realized above the cost of insurance.

11. A man, aged 35 years, took out a 10-year endowment policy of $5000, and died after paying the 5th annual premium. What sum did his heirs realize above the cost of insurance?

12. A gentleman, aged 43 years, died after making 7 semi-annual payments of premium on an ordinary life policy of $8000. How much greater was the sum received by his heirs than that paid as premiums?

RATIO.

434. Ratio is the relative value of one quantity as compared with another quantity of the same kind.

Rem. 1.—The ratio of two quantities is obtained by dividing the quantity compared by the quantity taken as the standard of comparison.

Rem. 2.—The two quantities compared, taken separately, are called *terms* of the ratio; and taken together, are called a couplet.

Rem. 3.—The left-hand term of the couplet, or quantity compared, is called its *antecedent;* and the right-hand term, or quantity taken as the standard of comparison, its *consequent.* When the terms of comparison are so arranged, the couplet denotes a *direct ratio.*

Rem. 4.—When the terms of comparison are arranged in reverse order, the standard of comparison being placed as the antecedent, and the quantity compared as the consequent, the couplet is said to express an *inverse ratio.*

Rem. 5.—The *symbol of ratio* is the sign of the operation (division) with the horizontal line omitted (:). It is placed between the antecedent and consequent, means *divided by,* and is read *is to.*

Rem. 6.—Since a ratio is the result of division, and division may be expressed in the form of a fraction, the ratio of two quantities may be similarly expressed by writing the antecedent for the numerator and consequent for the denominator. Thus, the ratio of $8 to $13 may be written $8 : 13, or $\frac{8}{13}$.

435. A **Simple Ratio** is a ratio of two quantities; as $6 : $2.

Rem. 1.—The learner should discriminate between a ratio and its mere expression. Thus, $6 : $2 is simply the expression of the ratio, the ratio itself being the resulting quotient (3).

Rem. 2.—The terms of a ratio are usually *concrete.* If concrete, they should be reduced to the same denomination, if necessary. The ratio is *abstract.*

436. A **Compound Ratio** is the product of two or more simple ratios; as $\begin{array}{l}\$8 : \$4 \\ \$3 : \$2\end{array}$. It is read, abstractly, 8 times 3 is to 4 times 2.

437. Since the antecedent is a dividend, the consequent a divisor, and the resulting quotient the ratio, the principles of division (47) are applicable to ratios. Hence,

434—437

General Principle.—*Any change in the antecedent by multiplication or division produces a like change in the ratio; any change in the consequent by multiplication or division produces an opposite change in the ratio; and any similar change, by multiplication or division, of both antecedent and consequent produces no change in the ratio.*

Summary. (1) *Quantity compared ÷ standard of comparison = direct ratio.* (2) *Standard of comparison ÷ quantity compared = inverse ratio.* (3) *Antecedent ÷ consequent = ratio.*

EXAMPLES FOR PRACTICE.

Find the direct ratio.		**Find the inverse ratio.**	
Quantity compared.	*Standard.*	*Quantity compared.*	*Standard.*
1. 84 feet.	12 feet.	6. 111 gallons . . .	37 gallons.
2. 1980 yards . . .	792 yards.	7. 3⅓ tons	7⅞ tons.
3. 6 feet	18 inches.	8. $3.45	$2.76.
4. 2 bu. 3 pk. . .	3 pk. 5 qt.	9. 2 miles	440 yards.
5. ⅝ gallon.	¾ gallon.	10. £8 6s. 3d.	£2 7s. 6d.

11. The antecedent is 5⅓ years and the consequent 2½ years. What is the ratio?

12. What is the ratio of 16.80 miles : 3.15 miles?

13. What is the ratio of $445.05 to $19.35?

14. What is the ratio of 25 days to 3⅓ days?

15. What is the inverse ratio of $23.4375 to $3.125?

16. What is the difference between the inverse ratio of 7.92 bushels : 47.52 bushels, and the direct ratio of 3¾ feet : 30 feet?

PROPORTION.

438. **A Proportion** is an expression of equality between two ratios.

REM. 1.—The *symbol* of proportion is the double colon (::); and it is placed between two ratios to show that they are equal. The sign of equality is often used as a substitute for the symbol of proportion.

REM. 2.—*Simple Proportion* is the expression of equality between two simple ratios. Thus, 8 lb. : 24 lb. :: 5 da. : 15 da. is a simple proportion, consisting of the two simple and equal ratios (8 lb. : 24 lb.) and (5 da. : 15 da.) united by the symbol (::). It may be read, 8 lb. are to 24 lb. as 5 da. are to 15 da.; or, the ratio of 8 lb. to 24 lb. equals the ratio of 5 da. to 15 da. This proportion is sometimes written $\frac{8}{24} = \frac{5}{15}$.

438

Rem. 3.—*Compound Proportion* is the expression of equality between two ratios, one or both of which are compound. Thus $\left\{\begin{matrix}12\text{ da.} : 8\text{ da.}\\ 5\text{ hr.} : 6\text{ hr.}\end{matrix}\right\} :: \$150 : \$120$ is a compound proportion. It is usually read, 12 times 5 is to 8 times 6 as $150 is to $120.

439. The **Terms** of a proportion are the four simple or compound quantities used in the comparison. The terms of a proportion are sometimes called *proportionals*.

Rem. 1.—The *antecedents* of a proportion are its first and third terms, or the antecedents of the ratios of which it is composed; and the *consequents* of a proportion are its second and fourth terms, or the consequents of its two ratios.

Rem. 2.—The *extremes* of a proportion are its first and fourth terms, that is, its two outside terms; and the *means* of a proportion are its second and third terms, that is, its two inside terms.

Rem. 3.—In the proportion 8 lb. : 3 lb. :: 64¢ : 24¢, 8 lb. and 64¢ are the antecedents, 3 lb. and 24¢ the consequents; 8 lb. and 24¢ are the extremes, 3 lb. and 64¢ the means; 8 lb. : 3 lb. is the first couplet, 64¢ : 24¢ the second couplet.

440. Three quantities are proportional when the ratio of the first to the second is equal to the ratio of the second to the third.

Thus, 8 lb., 4 lb., and 2 lb. are proportional, since $8 \div 4 = 4 \div 2$. The second quantity (4 lb.) is called a *mean proportional* of the other two. The mean proportional of two given quantities is a quantity which multiplied into itself will equal the product of the two given quantities.

441. Principle.—*The product of the extremes of a proportion is equal to the product of its means.* Hence,

1. *If the product of the extremes of a proportion be divided by one of its means, the quotient will be the other mean.*
2. *If the product of the means of a proportion be divided by one of its extremes, the quotient will be the other extreme.*

442. To find an omitted term of a proportion.

ILLUSTRATIVE EXAMPLE.

Find the omitted term in the proportion, 9 yd. : 12 yd. :: $6 : ($?).

Explanation.—Applying **441**, take the product of the two given means ($12 \times 6 = 72$) as the product of the two extremes. Hence, divide the known product of both extremes (72) by the given *extreme* (9), and the quotient must equal the other *extreme* (8).

Solution.

$12 \times 6 = 72.$

$72 \div 9 = 8,$ Hence,

9 yd. : 12 yd. :: $6 : $8.

EXAMPLES FOR PRACTICE.

Find the omitted term of the following proportions :

1. 4 oz. : 9 oz. :: ? : 25¢.
2. $3 : ? :: 9 yd. : 12 yd.
3. ? : 21 mi. :: $4 : $12.
4. 10 gal. : 9 gal. :: $30 : ?.
5. $\frac{3}{4}$ bu. : $\frac{7}{8}$ bu. :: ? : $\$\frac{5}{8}$.
6. 9.4 bbl. : 10.25 bbl. :: $4.70 : ?.
7. £4 : £36 :: ? : 18 doz.
8. ? : $65 :: 9 men : 45 men.
9. $\left\{\begin{array}{l}\text{30 men : 5 men}\\ \text{4 da. : 8 da.}\end{array}\right\} :: \$18 : ?.$
10. $\left\{\begin{array}{l}\text{75 rd. : 25 rd.}\\ \text{54 bu. : 9 bu.}\end{array}\right\} :: ? : \text{11 men.}$
11. $\left\{\begin{array}{l}\text{3 mi. : ?}\\ \text{28 hr. : 6 hr.}\end{array}\right\} :: \$160 : \$80.$
12. $\left\{\begin{array}{l}\text{9 da. : 15 da.}\\ \text{? : 3 hr.}\end{array}\right\} :: \$28.80 : \$18.$

SINGLE RULE OF THREE.

443. The **Single Rule of Three** is a process of obtaining a result which depends upon the comparison of one pair of quantities of the same kind.

REM.—The *Single Rule of Three* is applicable to all problems containing *three* given quantities, two of which are of the same kind and form a certain ratio, and it is required to find a fourth quantity which will form an equal ratio with the remaining quantity. For this reason it is also called Simple Proportion (Rem. 2, **438**).

ILLUSTRATIVE EXAMPLES.

1. If 50 bu. wheat cost $48, what will 73 bu. of the same wheat cost ?

Solution.

$$\begin{array}{r|l} & \$48 \\ 50 & 73 \end{array}$$

$$(48 \times 73) \div 50 = \$70.08.$$

Explanation.—Since a *cost* is required, place the given cost ($48) at the right of a perpendicular line. There are now only two terms (50 bu. and 73 bu.) remaining to be arranged.

Since the cost of 73 bu. is required, and this required cost is *greater* than the given cost, *place the greater* of the two remaining terms (73 bu.) *at the right* of the line, and the less (50 bu.) on the opposite side.

Multiply the two right-hand terms ($48 × 73 = $3504), and divide the product by the left-hand term (50), obtaining $70.08 as the required cost.

REM.—As a proportion, the required cost is to the given cost as the quantity of which the cost is required is to the quantity of which the cost is given; that is, $? : $48 : : 73 bu. : 50 bu. Since two means and one extreme are given, apply 2, **441**, to find the other extreme, obtaining $70.08.

2. If 300 acres of a field will produce 16500 bushels of corn, how many bushels of corn will 93 acres of the same field produce?

Explanation.—Since bushels are required, place the given bushels (16500) at the right of the line.

Since bushels from 93 acres are required, and the required bushels must be *less* than the given bushels, *place the less* of the two remaining terms (93) on the right of the line, and the other term (300) on the opposite side.

Solution.

$$\begin{array}{r|l} & 165 \\ 1 & \cancel{16500}\text{ bu.} \\ \cancel{3} & \cancel{93} \\ \cancel{300} & 31 \end{array}$$

$$165 \times 31 = 5115 \text{ bu.}$$

Cancel and proceed with the uncanceled terms as in Ill. Ex. 1, obtaining 5115 as the required number of bushels. If 1 is obtained as the result of cancellation, omit it from the right-hand terms, and retain it in the left-hand.

3. If 24 men can do a given piece of work in 9 days, in how many days can 18 men do it?

Explanation.—Since days are required, place the given days (9) at the right of the line.

Since days for 18 men to do the work are required, and the required days are *greater* than the given days (for when *fewer* men are employed on a piece of work, the *greater* must be the time required to complete it), *place the greater* of the two remaining terms (24) on the right of the line, and the other term (18) on the left.

Solution.

$$\begin{array}{r|l} 1 & \cancel{9}\text{ da.} \\ \cancel{2} & \cancel{24} \\ \cancel{18} & 12 \end{array}$$

Ans. 12 da.

Canceling out all the terms, it is found that 12 is the only uncanceled factor. Therefore the required answer is 12 times 1 da. or 12 da.

Rule.—I. *At the right of a perpendicular line, place that one of the three given terms which is of the same kind as the required answer.*

II. *If, from the nature of the question, it is seen that the required answer will be greater than the term already written, place the greater of the two remaining terms underneath the term already written.*

III. *If, however, it is seen that the required answer will be less than the term already written, place the less of the two remaining terms underneath the term already written.*

IV. *After placing one of the two remaining terms at the right of the line, place the other term on the left.*

V. *Multiply the two right-hand terms, and divide the product by the left-hand term, first canceling factors which are common to the left-hand term and either of the right-hand terms.*

EXAMPLES FOR PRACTICE.

1. What will be the cost of 13 pounds of sugar if 8 pounds cost 72 cents ?

2. If 12 pounds of beef cost $1.80, how much will 9 pounds cost ?

3. A farmer raised 1750 bushels of wheat on 70 acres of land. At the same rate how many bushels could he have raised on 87 acres ?

4. If 50 bushels of wheat cost $75, what will 80 bushels cost at the same rate ?

5. If 25 pounds of coffee are worth 5 pounds of tea, how many pounds of tea are worth 35 pounds of coffee ? How many pounds of coffee are worth 3 pounds of tea ?

6. If I am charged $12.25 for 35 yards of goods, how much should I be charged for 47 yards of the same goods ?

7. How many oranges will cost as much as 180 lemons, if 20 lemons cost as much as 15 oranges ?

8. If a garrison consume 102 barrels of flour in 17 days, how many barrels of flour will it consume in one year ?

9. What will 30 pounds of butter cost, if 18 lb. 3 oz. cost $5.82 ?

10. If 82 sheep cost $287, how many sheep at the same price will cost $52.50 ?

11. If $320 will accrue $9.60 interest, how much interest will $240 accrue in the same time and at the same rate % ?

Sometimes, as in the preceding example, all of the three given terms express the same kind of units; but in such cases one of the three given terms will be found to agree more specifically with the required answer. Thus, *dollars of interest* are required, and the given dollars of interest ($9.60) will be the upper right-hand term. The two remaining terms specifically agree as *dollars of principal.*

12. In weighing out 176 pounds of sugar, a retail grocer lost $2\frac{1}{5}$ pounds by wastage. At that rate, how much would he have lost in weighing out 260 pounds ?

13. If $\frac{4}{5}$ of a yard of goods cost $\frac{3}{8}$ of a dollar, what part of a dollar will $\frac{4}{7}$ of a yard cost ?

14. What principal will accrue $1.59 interest, if $820 will accrue $5.30 interest in the same time and at the same rate % ?

15. A received a dividend of $420 on $8400 of stock. What would he have received if he had owned $9300 of the same stock ?

16. In the preparation of a certain compound, 12 gills of water

were mixed with 7 gills of other ingredients. How many gills of other ingredients should be mixed with 42 gills of water?

17. How long a shadow will be cast by a vertical staff 11 feet high, if at the same moment a staff 6 feet high cast a shadow 4 feet long?

18. A pole 18 feet high casts a shadow 6 feet long. How high is a tree which, at the same time, casts a shadow 13 feet long?

19. Two houses cast shadows respectively 15 feet and 18 feet in length. If the height of the higher house is 27 feet, what is the height of the lower?

20. How many men can do the same work in 20 days that 45 men can do in 16 days?

21. A contractor agreed to build a house in 45 days, and estimated that 6 men would be sufficient to do the work. Should he desire to complete the job in 30 days, how many men must he employ?

22. A invested $5400 in a partnership, and B $3600; and the contract between them specified that each partner's gain or loss should be proportional to his investment. If the total gain is $1500, what should be each partner's share?

23. A merchant sells $3600 worth of goods and gains $648. At the same % of profit, what amount of goods must he sell to gain $1620?

24. What should be the tax on a farm assessed at $32500, if a farm assessed at $7500 is taxed $18.21?

25. In what time can the whole of a piece of work be done, if ⅘ of it can be done in 12 days?

26. A merchant sold an invoice of goods at 15% profit and gained $450. What % would he have gained by selling the invoice at a profit of $750?

27. A is given a start of 12 rods in a foot-race, and can run at the rate of 30 rods per minute. In what time will he be overtaken by his opponent, who can run at the rate of 33 rods per minute?

28. If, in Ex. 27, A had a start of 20 seconds, in what time would his opponent have overtaken him?

29. A grocer's pound weight weighs only 15¼ ounces, and his other weights in the same proportion. Out of what amount does he defraud his customers by using these weights in selling groceries for which he receives $320?

30. A merchant has a yard-stick which measures only 34⅓ inches, and finds that by its use during a certain week he has defrauded his customers out of $23. What were his receipts during that week?

31. A borrowed $1200 from B, and kept it 1 year and 3 months. For what time should A lend $1500 to B to offset B's claim for interest ?

32. If 15 men can plow a field in 12 days, in how many days can 18 men plow it ?

33. An army of 12000 men has provisions for 14 days. How long will these provisions last if the army is reinforced by 9000 men ?

34. What part of a piece of work can be performed in 30 days, if with the same force of men $\frac{5}{11}$ of it can be performed in 25 days ?

35. If $\frac{3}{4}$ of the marked price of an article yield a profit of $\frac{1}{8}$ of its cost, what part of its cost would be gained by selling the article at $\frac{4}{5}$ of its marked price ?

36. If a man can make a journey in 18 days by traveling 12 hours each day, in what time can he make the same journey by traveling 10 hours each day ?

37. If 3289 stones, each $2\frac{1}{3}$ feet in length, are required to build a certain wall, how many stones of the same width and depth, but $2\frac{3}{4}$ feet long, will be required to build the same wall ?

38. A contractor could have executed a certain contract in 30 days by employing his full force of 24 men ; but he detached a portion of his men to perform another piece of work, and with the remainder executed the contract in 40 days. How many men did he detach ?

39. If A lend B $800 for 18 days, how long should B lend A $300 to offset his claim for interest ?

40. If the rate of freight is 18¢ per bushel for 1326 miles, what will be the proportionate rate for 884 miles ?

41. If a vessel is provisioned for a crew of 30 men and a voyage of 168 days, how long a voyage can she make with the same provisions if the crew be increased to 35 men ?

DOUBLE RULE OF THREE.

444. The **Double Rule of Three** is a process of obtaining a result which depends upon the comparison of two or more pairs of quantities of the same kind.

Rem.—If the terms of a problem in Double Rule of Three are arranged in the form of a proportion, the first couplet will contain as many simple ratios as there are pairs of given quantities of the same kind. For this reason it is also called *Compound Proportion* (Rem. 3, **438**).

ILLUSTRATIVE EXAMPLE.

If 4 men can earn $84 in 7 days by working 10 hours a day, how many dollars can 6 men earn in 10 days by working 9 hours a day?

Solution.

$$\begin{array}{r|l} & \$\cancel{84}\ \overset{3}{\cancel{12}} \\ 1\ \cancel{4} & 6 \\ 1\ \cancel{7} & \cancel{10}\ 1 \\ 1\ \cancel{10} & 9 \end{array}$$

$$\$3 \times 6 \times 9 = \$162$$

Explanation.—Since dollars are required, place the given dollars ($84) at the right of the perpendicular line.

Since dollars are required that 6 men can earn, and 6 men can earn more dollars than 4 men, place the greater of these two similar terms (6 men) on the right of the line, and the other (4 men) on the left.

Since dollars are required that 6 men can earn in 10 days, and 6 men in 10 days can earn more dollars than in 7 days, place the greater of these two similar terms (10 days) on the right of the line, and the other (7 days) on the left.

Since dollars are required that 6 men can earn in 10 days by working 9 hours, and 6 men in 10 days by working 9 hours will earn less dollars than by working 10 hours, place the less of these two similar terms (9 hours) on the right of the line and the other (10 hours) on the left.

Cancel all factors common to terms which are on opposite sides of the line, and divide the product of the uncanceled numbers on the right ($3 × 6 × 9) by the product of the uncanceled numbers on the left (1), obtaining $162 as the required answer.

Rule.—I. *At the right of a perpendicular line, place that given term which is of the same kind as the required answer.*

II. *With the remaining terms form pairs of the same kind, and place the terms of each pair on the right and left of the line by II, III, and IV of the Single Rule of Three, as if the required answer depended on that pair alone.*

III. *Cancel all factors which are common to terms on opposite sides of the line, and divide the product of the uncanceled numbers on the right of the line by the product of the uncanceled numbers on the left.*

NOTE.—In deciding upon the position of the terms of each pair, uniformly compare the term in the question with the similar term in the statement or supposition.

EXAMPLES FOR PRACTICE.

1. If $400 will accrue $5.60 interest in 84 days at 6% per annum, how much interest will $320 accrue in 96 days at 9% per annum?

2. A printer cut 30 pages of a certain book from a sheet of paper 18 in. wide and 25 in. long. How many pages of a book of the same size would a sheet yield which is 20 in. wide and 24 in. long?

444

3. A lumber dealer purchased two piles of wood. One pile was 24 feet long, 20 feet high, 16 feet wide, and contained 60 cords. The other pile was 32 feet long, 12 feet wide, and 18 feet high. Find, by the Rule of Three, the number of cords in the latter pile.

4. An excavator contracted to dig two cellars at the same price per cubic yard, and obtained $120 for one which was 40 feet long, 30 feet wide, and 9 feet deep. The other cellar was 36 feet long, 27 feet wide, and 12 feet deep. How much was received for excavating the latter?

5. If 300 men can perform a certain work in 45 days by laboring 10 hours a day, how many hours a day will 600 men have to labor to perform the same work in 25 days.

6. If 20 men can earn $1260 in 15 days by working 12 hours a day, how many dollars can 17 men earn in 23 days by working 10 hours a day?

7. How many half-barrels of flour will supply a gang of 9 laborers for 20 weeks, if 3 barrels will supply a gang of 5 laborers for 15 weeks and 3 days?

8. A livery-stable keeper's expenses for feeding 27 horses for 17 weeks were $91.80. What would be his expenses for feeding 23 horses for 13 weeks?

9. What should be the cost of pasturing 73 head of cattle for 7 weeks, if the cost of pasturing 95 head of cattle for 9 weeks is $513?

10. The freight on 12250 pounds of goods was $73.50 for 240 miles. A *pro rata* sum was charged on 28250 pounds of the same class of goods for 90 miles. How much was paid for freight for the shorter distance?

11. If $600 will accrue $9.60 interest at 6% per annum in 3 months and 6 days, at what rate per annum will $240 accrue $7.20 interest in 4 months?

12. If $840 will accrue $19.60 interest in 105 days at 8% per annum, in what time will $576 accrue $1.84 interest at 5%?

13. If from 650 reams of paper 4000 copies of a book containing 420 pages can be printed, how many reams will be required to print 7000 copies of a book containing 528 pages of the same size?

14. A contractor purchased a pile of stone 24 feet long, 12 feet high, and 9 feet wide for $120, and paid $300 for another pile of the same quality of stone which was 12 feet wide and 36 feet long. How high was the latter pile?

15. If 3 pipes of equal size will fill a cistern 5 feet long, 4 feet wide, and 9 feet deep in 3 hours and 20 minutes, in what time will 4 such pipes fill a cistern which is 3 feet long, 2½ feet wide, and 5 feet deep?

16. If 250 men can dig a ditch 48 rods long, 2½ feet wide, 3½ feet deep, and 5 degrees of hardness, in 8 days of 10 hours each, in how many days of 8 hours each can 40 men dig a ditch 341 yards long, 3½ feet wide, 4 feet deep, and 3 degrees of hardness?

17. If 7 men can plow a field 144 rods long and 105 rods wide in 9 days by using horses whose working gait is 5 miles per hour, in how many days can 5 men plow a field 216 rods long and 60 rods wide with horses whose gait is 4 miles per hour?

ALLIGATION.

445. Alligation is the process of averaging mixtures of different values.

REM. 1.—Alligation means to bind or link, because examples of this kind are usually solved by linking the quantities.

REM. 2.—The process of finding the average value of a mixture is distinguished as *Alligation Medial;* and of finding the proportional quantities to be used in a mixture of a given average price, as *Alligation Alternate.*

446. To find the average value of a mixture when the quantity and value of each ingredient are given.

ILLUSTRATIVE EXAMPLE.

A dealer mixed 50 lb. tea worth 30¢ a pound, 40 lb. worth 50¢ a pound, 80 lb. worth 75¢ a pound, and 30 lb. worth $1 a pound. What was the average value of the mixture?

Explanation.—Since 50 lb. at 30¢ = $15, 40 lb. at 50¢ = $20, 80 lb. at 75¢ = $60, and 30 lb. at $1 = 30, the entire mixture of 50 + 40 + 80 + 30, or 200 lb. tea must be worth $15 + $20 + $60 + $30, or $125. Hence, the average price of the mixture must be $125 ÷ 200, or 62½¢ per pound.

Solution.

50 lb. @ 30¢ = $15.
40 " " 50¢ = 20.
80 " " 75¢ = 60.
30 " " $1 = 30.
200 lb. @ ? = $125.
$125 ÷ 200 = 62½¢.

Rule.—*Divide the total value of the mixture by the sum of the ingredients.*

EXAMPLES FOR PRACTICE.

1. A grocer mixed 32 pounds Rio coffee worth 12 cents per pound, 24 pounds Rio worth 15 cents per pound, and 40 pounds Java coffee worth 25 cents per pound. What was the average price per pound?

2. Find the average price per pound of the following mixture: 40 pounds of tea at 60 cents per pound, 60 pounds at 80 cents per pound, and 60 pounds at $1.20 per pound?

3. A liquor dealer mixed 30 gallons of whiskey of 60% proof, 50 gallons of 70% proof, and 20 gallons of 90% proof. What was the % of proof of the mixture?

4. A goldsmith mixed 14 ounces of gold, 23 carats fine; 6 ounces, 16 carats fine; 6 ounces, 18 carats fine; and 6 ounces, 19 carats fine. How many carats fine was the mixture?

447. To find the quantity of each ingredient required in a mixture of a given average value.

ILLUSTRATIVE EXAMPLE.

A grocer wishes to mix four kinds of coffee worth respectively 17, 18, 24, and 25 cents per pound in such proportions that the mixture may be worth 20 cents per pound. How many pounds of each kind must the mixture contain?

Explanation.—Link each kind of coffee which is worth less than the given average price (20¢), with a kind which is worth more than the average price.

Comparing the linked prices 17¢ and 25¢ with the average price (20¢), it is seen that a pound of the kind worth 17¢, if sold at 20¢, will produce a gain of 3¢; hence, to produce a gain of 1¢, $\frac{1}{3}$ of a pound must be sold at 20¢; and a pound of the kind worth 25¢, if sold at 20¢, will produce a loss of 5¢; hence, to produce a loss of 1¢, $\frac{1}{5}$ of a pound must be sold.

Solution.

20	17	$\frac{1}{3}$		5		5 lb. @ 17¢
	18		$\frac{1}{2}$		4	4 " " 18"
	24		$\frac{1}{4}$		2	2 " " 24"
	25	$\frac{1}{5}$		3		3 " " 25"

Proof.

5 lb.	@ $.17	=	$.85
4 "	" .18	=	.72
2 "	" .24	=	.48
3 "	" .25	=	.75
14 lb.	" .20	=	$2.80

Similarly comparing the remaining linked prices (18¢ and 24¢) with the average price (20¢), it is seen that, at 20¢ per pound, it will require ½ of a pound of the kind worth 18¢ to produce a gain of 1¢, and ¼ of a pound of the kind worth 24¢ to produce a loss of 1¢. Place these fractions in a separate column.

To express these proportionals *integrally*, multiply the fractions of a pound in each separate column by the least common multiple of their denominators. Thus, ⅓ lb. and ⅕ lb., if multiplied by 15, will produce the equivalent integral proportionals 5 lb. and 3 lb.; for if ⅓ lb. of one kind and ⅕ lb. of the other will produce a counterbalancing gain and loss of 1¢, 15 times each of those quantities must also produce a counterbalancing gain and loss of 15¢. Proceed similarly with the fractions in the second column, obtaining the equivalent integral proportionals 4 lb. and 2 lb.

Hence, to obtain a mixture worth 20¢ per pound, the quantities may be mixed in the proportion of 5 lb. at 17¢, 4 lb. at 18¢, 2 lb. at 24¢, and 3 lb. at 25¢.

Principle.—*A gain in the price of one ingredient must be balanced by a corresponding loss in the price of some other ingredient, to produce a mixture of a given average price.*

Rule.—I. *Write the prices of the several ingredients in a column, and place the price of the mixture at their left.*

II. *Link a price which is less than the price of the mixture with a price which is greater, until all the prices are thus linked.*

III. *Compare the two prices of each linked pair with the price of the mixture, placing in a separate column for each linked pair and opposite the less price that part of the ingredient of less price which must be used to gain 1 unit of the price of the mixture, and opposite the greater price that part of the ingredient of greater price which must be used to lose 1 unit of the price of the mixture.*

IV. *If only one of the obtained proportionals of a column is fractional, multiply both proportionals of that column by the denominator of the fraction; and if both proportionals are fractional, multiply both by the least common multiple of their denominators.*

NOTE 1.—If the price of an ingredient is linked more than once, the *sum* of the results of all its comparisons will be the required quantity of that ingredient.

NOTE 2.—If one or more of the prices are fractional, multiply all the prices by the least common multiple of the denominators to obtain integral terms of comparison.

NOTE 3.—Since any quantity whose price is below the average may be linked with *any other* whose price is above the average, it is evident that a problem can have as may different but correct answers as there are different ways of thus linking them. Besides, both proportionals of any couplet may be multiplied or divided by *any* number, thus changing the results without affecting their correctness.

EXAMPLES FOR PRACTICE.

1. How many pounds of tea, worth respectively 60, 70, 85, and 90¢ per pound, must be taken to form a mixture worth 75¢ per pound ?

2. To form a mixture worth 8$\frac{3}{4}$ cents per pound, how many pounds of sugar worth 8, 9, and 10 cents per pound should be combined ?

3. A dealer combined a quantity of alcohol 80% proof from 4 different barrels, respectively 60%, 70%, 75%, and 90% proof. How many gallons were drawn from each barrel ?

4. How many barrels of flour at $6, 6\frac{1}{2}$, $7, and $8 per barrel, must be sold to realize an average price of 6\frac{3}{4}$ per barrel ?

5. In what proportion can vinegar worth 20, 25, and 40 cents per gallon be mixed to form a compound worth 30 cents per gallon ?

6. What proportion of water, whose respective temperatures are 50°, 70°, 80°, and 100° Fahrenheit, must be combined to obtain water of 95° Fahrenheit ?

7. A liquor merchant compounded a kind of whiskey worth $1.20 per gallon by mixing water with 3 kinds of whiskey worth respectively $1.10, $1.40, and $1.50 per gallon. What amount of each did he take ?

448. To find the quantity of each ingredient if the relative quantity of two or more of the ingredients is given.

ILLUSTRATIVE EXAMPLE.

A man bought Rio coffee at 14 cents per pound, Laguayra at 16 cents, Maracaibo at 18 cents, and Java at 24 cents. His purchase averaged 20 cents per pound, and the Rio was to the Laguayra as 2 to 5. What number of pounds of each did he purchase ?

Explanation.—Since the price of only one ingredient (Java, 24¢) is greater than the price of the mixture (20¢), link all the remaining prices which are less with the price of Java.

Solution.

									Ans.
20	14	$\frac{1}{6}$			2				2 lb. Rio.
	16		$\frac{1}{4}$			1 × 5			5 " Laguayra.
	18			$\frac{1}{2}$			2		2 " Maracaibo.
	24	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	3	1 × 5	1		9 " Java.

448.

Applying **447**, it is seen that, integrally, the Rio and Java coffees were as 2 lb. to 3 lb., the Laguayra and Java as 1 lb. to 1 lb., and the Maracaibo and Java as 2 lb. to 1 lb. The proportions thus obtained are 2 *lb. Rio*, 1 *lb. Laguayra*, 2 lb. Maracaibo, and 3+1+1, or 5 lb. Java, but the proportion of Rio to Laguayra, instead of being as 2 to 1, as found, was really as 2 to 5. Hence, multiply the quantity of Laguayra as found (1 lb) by 5 to obtain the true proportion. But if one term of a couplet (1 lb. Laguayra) be increased fivefold, the other term of that couplet (1 lb. Java) must be similarly increased, to preserve the counterbalancing gain and loss of that couplet.

Principle.—*To multiply or divide* BOTH *proportionals of a comparison by the same number preserves their counterbalancing gain and loss, and produces any desired relation of those proportionals to other quantities.*

Rule.—*Find the proportional quantities by Rule,* **447**, *and multiply or divide the quantities thus found which are to be of a given relation by any number or numbers that will produce such relation.*

EXAMPLES FOR PRACTICE.

1. A grocer combined teas worth 50, 60, 75, and 80 cents a pound in such proportions as to produce a mixture worth 70 cents a pound. The quantity of tea worth 75 cents was to that worth 60 cents as 4 to 1. What proportion of each kind of tea did he take?

2. A dairyman averaged 32 cents per pound upon 5 kinds of butter which he sold at 20, 25, 30, 35, and 40 cents per pound. He sold $\frac{1}{2}$ as much butter at 30 cents as he sold at 35 cents. What proportional quantity of each kind did he sell?

3. A jeweler combined three kinds of silver respectively $\frac{2}{3}$ pure, $\frac{4}{5}$ pure, and $\frac{5}{6}$ pure, in such proportions that the mixture was $\frac{7}{9}$ pure. What proportion of each did he take, if the mixture contained 5 times as much silver $\frac{4}{5}$ pure as of silver $\frac{2}{3}$ pure?

4. A liquor dealer mixed a quantity of whiskey 65% proof by combining whiskeys which were respectively 50%, 55%, 60%, 70%, and 75% proof. What proportion of each did he take, if the 50% proof in the mixture was to the 75% proof as 3 to 2?

5. A farmer sold turkeys at $1.25 each, ducks at 75 cents, chickens at 50 cents, and geese at 90 cents. He sold 3 times as many ducks as turkeys, and realized an average price of 80 cents. How many of each did he sell?

448

449. To find the quantity of each ingredient of a mixture if the quantity of one or more ingredients is limited.

ILLUSTRATIVE EXAMPLE.

A dealer sold 10 barrels of flour at \$6.25 per barrel. What is the least number of barrels which he must sell at \$6.30, \$6.75, and \$7.00 per barrel to realize an average price of \$6.50 per barrel ?

$$\$6.50 \left\{ \begin{array}{r|c|c|c|c|l} \$6.25 & \frac{1}{25} & & 2 & & 2 \times 5 = 10. \\ 6.30 & & \frac{1}{20} & & 5 & 5 \\ 6.75 & & \frac{1}{25} & & 4 & 4 \\ 7.00 & \frac{1}{50} & & 1 & & 1 \times 5 = 5. \end{array} \right.$$

$10 \div 2 = 5$, multiplier of the couplet containing the limited proportional.

Explanation.—It is found by Rule, **447**, that the proportionals, if unlimited, would have been 2 bbl. at \$6.25, 5 bbl. at \$6.30, 4 bbl. at \$6.75, and 1 bbl. at \$7. But, instead of 2 bbl., there were 5 times 2, or 10 bbl., sold at \$6.25. Hence, multiply 2 and the other proportional of its couplet (1 bbl. at \$7) by 5, obtaining as the required proportionals 10 bbl. at \$6.25, 5 bbl. at \$6.30, 4 bbl. at \$6.75, and 5 bbl at \$7.

Rule.—*Find the proportional quantities by Rule,* **447.** *Divide the limited quantity by the obtained proportional of the limited quantity; and multiply the quotient by the proportionals of such ingredients as are linked to the limited quantity.*

EXAMPLES FOR PRACTICE.

1. A clothier bought 24 coats at \$12 each. How many must he buy at \$5 and \$7 each that the average price will be \$8 ?

2. A merchant sold 15 yards of muslin at 18 cents per yard. What is the least number of yards at 14, 10, and 9 cents per yard which he can sell to realize an average price of 12 cents per yard ?

3. A dealer so mixed 8 pounds of coffee at 18 cents per pound with other grades at 14, 17, 21, and 22 cents per pound, that the price of the mixture could be fixed at 19 cents per pound. What is the least number of pounds of each which he could take ?

4. A broker purchased 90 shares of railroad stock at 70. How many additional shares at 90 must he purchase that the average price per share may be 75 ?

5. A farmer bought 20 calves at \$15 each, and a sufficient number at \$8 and \$10 each to make the average price of the lot \$12 each. How many calves at \$8 and \$10 did he buy ?

450. To find the quantity of each ingredient of a mixture if the total quantity of the ingredients is limited.

ILLUSTRATIVE EXAMPLE.

A broker sold mining stock at 60, railroad stock at 65, steamboat stock at 80, and bank stock at 88. Altogether, he sold 243 shares, and received an average price of $75 per share. How many shares of each did he sell?

Explanation.—It is found by Rule, **447**, that the proportionals, if unlimited, would have been 1 share at 60, 13 shares at 65, 3 shares at 80, and 10 shares at 88, or 27 shares of all. But, instead of 27 shares of all, there were 9 times 27, or 243 shares sold; hence, multiply each of the obtained proportionals by 9 to find the required proportionals.

Solution.

75	60	$\frac{1}{15}$		1		1 × 9 =	9
	65		$\frac{1}{10}$		13	13 × 9 =	117
	80	$\frac{1}{5}$		3		3 × 9 =	27
	88		$\frac{1}{13}$		10	10 × 9 =	90
						27	243

243 ÷ 27 = 9, multiplier.

Rule.—*Find the proportional quantities by Rule,* **447.** *Divide the limited quantity of all the ingredients by the sum of the obtained proportionals, and multiply each proportional by the quotient.*

EXAMPLES FOR PRACTICE.

1. A miller mixed 240 bushels from four grades of wheat, costing $.90, $.95, $1.10, and $1.20 per bushel, so that the average cost was $1 per bushel. How many bushels of each grade did he take?

2. What quantity of water must be added to 16 quarts of acid 90% proof and 24 quarts 95% proof, that the mixture may be an acid 80% proof?

3. A produce merchant bought 130 barrels of 3 kinds of onions at an average price of $2 per barrel. If he had purchased each kind separately, the prices would have been respectively $1.90, $2.25, and $2.50 per barrel. How many barrels of each kind did he buy?

4. A liquor dealer diluted a quantity of rum for which he paid $2.50 per gallon with a quantity of water, making 20 gallons in all. How many gallons of each did the mixture contain, if it was sold at *$2 per gallon* without loss to the dealer?

BANKRUPTCY.

451. A **Bankrupt** or **Insolvent** is a person who fails in business or is unable to meet his pecuniary obligations.

REM.—Legal proceedings in bankruptcy are usually instituted either at the instance of the bankrupt himself, by his formal acknowledgment before a court of proper jurisdiction of his inability to pay his debts; or at the instance of one or more of his creditors, by their formal application to the same court to have him adjudged a bankrupt. Firms and corporations unable to meet their pecuniary obligations are also said to be bankrupt, and are subject to the same legal proceedings in bankruptcy as a single individual in a similar condition.

452. The **Assets** of a bankrupt are his entire property, including debts due to him by others.

REM.—Assets are distinguished as *Available* and *Nominal*, the former including items that can be converted into cash at their stated valuation, and the latter such as are not supposed to be immediately available except at a discount upon their stated value.

453. The **Liabilities** of a bankrupt are the pecuniary obligations due by him to others.

REM.—A *debtor* of a bankrupt is a person or firm who is indebted to the bankrupt; and a *creditor* is one to whom the bankrupt is indebted. Some creditors, such as employees, etc., are paid in full of all their claims, if the assets are sufficient for this purpose, and hence are called *preferred creditors*.

454. A **Schedule** or **Statement** is a list setting forth the assets and liabilities of the bankrupt, containing the name of each of his creditors and debtors, their places of business or residence, and the sum due to or from each.

455. An **Assignee** is a person appointed to take charge of the property of a bankrupt for the purpose of converting it into cash and, after deducting the expenses of the assignment, to pay such a proportion of the bankrupt's liabilities as the available assets will allow.

456. A **Dividend** is the proportion of available assets which is paid by the assignee to the creditors of a bankrupt.

457. A **Discharge** is a decree issued by a court of bankruptcy, on the compliance of a bankrupt with the requirements of the insolvent laws, relieving him from further obligations to his creditors.

REM.—Upon receiving a *discharge*, the bankrupt is at liberty to recommence business legally free from future responsibility for any debt enumerated in his schedule.

458. Computations in Bankruptcy may be made in accordance with the principles of percentage; the total liability being the base, and the dividend or available assets as the percentage.

General Direction: *If the dividend is required, multiply its two given factors; if given, divide it by its given factor and the quotient will be its other factor.*

REM. 1.—The two factors of a total dividend are the total liability and the rate per cent. of dividend; of a dividend to a particular creditor are the total claim of that creditor and the rate per cent. of dividend.

REM. 2.—The net assets when distributed among the creditors being called a dividend (**456**), the rate % of net assets is called the rate of dividend.

ILLUSTRATIVE EXAMPLE.

A merchant failed in business and rendered the following schedule: Assets—cash, $1346.10; real estate, $3578.64; personal accounts, $2125; merchandise on hand, $1517.36. Liabilities—promissory notes outstanding, $3021; due J. H. Baldwin & Co., $1673; D. S. Rogers, $1177; S. R. White, $6052. The expenses of the assignment were $221. How much should each creditor receive?

Solution.

Assets.		*Liabilities.*	
Cash	$1346.10	Notes outstanding . .	$3021.00
Real estate	3578.64	J. H. Baldwin & Co. .	1673.00
Personal accounts .	2125.00	D. S. Rogers	1177.00
Merchandise	1517.36	S. R. White	6052.00
Total assets . .	$8567.10		
Less expenses .	221.00		
Net assets . . .	$8346.10	Total liabilities .	$11923.00

$8346.10 ÷ $11923 = .70 or 70%, rate of dividend.

$1673 × .70 = $1171.10, dividend to Baldwin & Co.
1177 × .70 = 823.90, " " D. S. Rogers.
6052 × .70 = 4236.40, " " S. R. White.

Rule.—I. *From the total assets deduct all expenses, and divide the remainder by the total liabilities. The quotient carried to hundredths will be the rate per cent. of dividend.*

II. *Multiply the amount of each creditor's claim by the rate per cent. of dividend.*

NOTE.—The same result may be obtained by the "Rule of Three" by taking the net assets and liability due each creditor as right-hand terms, and the total liability as the left-hand term (443).

EXAMPLES FOR PRACTICE.

1. J. H. Bell & Co. failed with liabilities amounting to $150000 and assets amounting to $93147. How much should each creditor receive on the dollar, and what sum was allowed J. E. Arnold & Bro., whose claim was $17314, if the expenses of settlement were $3147?

2. A coal firm failed with liabilities amounting to $49000. The assets were, cash, $2160; real estate, $16500; notes on hand, $2340. The expenses of the assignment were 2% of the assets. How much should Wright & Co. receive, whose claim against the firm was $12617?

3. G. W. Smith's claim against a bankrupt firm was $2400, and J. M. Owens' $1680. After the expenses of the assignment were deducted from the assets, there remained $6120. If the total liabilities were $8160, how much did Smith and Owens receive?

4. A lumber firm failed with assets amounting to $23072, and liabilities amounting to $56000. The firm owed Joel Manning & Co. $7327.50; M. J. Whiting & Co., $3725; and Robert French & Sons, $2375. How much did each of these creditors receive, if the expenses of bankruptcy were $672?

5. The Empire Knitting Mills Company failed with liabilities amounting to $274320, and assets amounting to $125912.88. If the expenses for settling were $2468.88, how much should W. A. Miles, J. R. Thomas, and W. G. Lewis receive, whose claims were respectively $34620, $15234, and $8750?

6. A firm failed with liabilities amounting to $52250, and with assets, exclusive of real estate, amounting to $2042.50. The assignee obtained for the real estate $31350, and deducted for settling the bankruptcy $475. W. H. Devon's claim against the firm was $7284; W. A. Heitmueller's, $6382; W. H. Patrick's, $5794; S. F. Lackland's, $4767; and J. K. Hopper's, $3964. How much did each of these creditors receive?

7. A bankrupt firm owed J. P. Jordan & Co. $1874; W. H. Henson, $675; T. A. Smith & Son, $385; and C. E. Sinclair, $284. The assets of the firm were $48700, and their liabilities $59048.75. How much did each of the above creditors receive, if the expenses of settling the bankruptcy were 3% of the assets?

8. The real estate of a bankrupt firm was sold by their assignee for $12000, and their goods for $6122. There were collected on notes due the firm $2107, and on personal accounts $2673. The total liabilities of the firm are $27033, and the expenses of settling $734.94. How much on the dollar can be paid, and what should M. E. Whitney receive, whose claim is $6215?

GENERAL AVERAGE.

459. General Average is the method of apportioning among all the interests endangered, the loss sustained by a vessel, its freight, its cargo, or any part thereof, when, in case of a common peril, it has been voluntarily sacrificed or damaged to secure the safety of the rest.

REM.—The essentials of a loss, before it can be apportioned by a general average, are: 1. An *imminent* and *common* peril; 2. A *voluntary* and *indispensable* loss; 3. The *success* of the sacrifice in saving the remaining interests.

460. Particular Average is any loss or damage which is not voluntarily encountered, or which lacks some other essential of a general average loss as specified in Rem., **459**.

REM.—A loss or damage to a vessel or her cargo, *if involuntary*, is borne exclusively by the owner of the property so lost or damaged; if voluntary, and the peril is *not common*, the loss or damage is apportioned among the particular interests benefited.

461. Jettison is a term applied to that part of the cargo or equipment of a vessel which is thrown overboard for the safety of the remainder.

462. Marine Salvage is the remuneration allowed to persons who are instrumental in saving vessels or their cargoes from the perils of the sea.

463. An **Adjuster of Averages** is one who is employed to adjust and apportion the losses and expenses of a general average.

464. The losses and expenses apportioned by general average are:

1. Jettison; freight on the jettison; damage to cargo from water running down the open hatches during jettison; chafing, breaking, or other damage to the loosened cargo after jettison.

2. Sacrifice of a vessel's equipments, or any part thereof, such as masts, spars, sails, cables, etc. Only two-thirds of the cost of a vessel's repairs are apportioned by general average, the remaining third being an exclusive charge on the vessel, as the new equipments are supposed to be worth that much more than the old.

3. Expense incurred in floating a stranded vessel, for towing a disabled one, or salvage (**462**) paid another vessel for affording any other relief.

4. Port charges of a disabled vessel, incurred in entering or leaving a harbor of refuge, whether such disability be caused by accident or sacrifice, provided it cannot continue its voyage without imminent and common peril; expenses of unloading the cargo to make repairs, storing the cargo in a warehouse, and its subsequent reloading; and the expense of so much of the repairs as were rendered necessary by a *sacrifice* for the common safety.

5. Port charges of an undamaged vessel, if incurred to avert an imminent and common peril; as, for instance, to escape from an enemy's vessel, to avoid stranding during an adverse wind, etc.

6. Wages and provisions of the crew from the time the vessel changes its course to make a harbor of refuge until it can safely resume its voyage.

465. The **Contributory Interests** to a loss or expense apportioned by general average are:

1. The vessel if undamaged or accidentally damaged, at its full value at the end of the voyage; but if damaged by voluntary infliction for the common safety, at its full value immediately before the damage was inflicted.

2. The cargo, including the part jettisoned or damaged, at its cash value at the port of destination.

3. The freight at its full amount minus ⅓ for the wages of the crew, in a few States minus ½.

REM.—The wages of the crew are exempt from contribution, as they might otherwise become interested in opposing a necessary sacrifice. The usual practice is to deduct the *exact sum* required for seamen's wages, and to consider the remainder of the freight as a contributory interest. In the following examples, ⅓ will be the part deducted unless otherwise directed.

466. Each interest, if *fully* insured, can recover from the insurance company the *full* amount of its contribution to the general average loss; or if *partially* insured, such a part of its contribution as the amount of insurance on such contributory interest is of its full value: provided, in either case, the insurance policy covers the class of risks which produced the general average loss.

467. To apportion a general average loss.

ILLUSTRATIVE EXAMPLE.

The ship Enterprise in a voyage from New Orleans to Baltimore, loaded with cotton, encountered gales which made it necessary for the common safety to cut away spars, etc., which cost $3000 to replace; and to jettison a portion of her cargo, the value of which, as compared with that of the same grade delivered in Baltimore, was $5900. The value of the vessel after undergoing repairs was $80000; and of her cargo as follows: that consigned to A, $35000; to B, $58000; to C, which included the jettisoned cargo, $63000. The gross freight, including that on the jettison, was $10800; freight on the jettisoned cargo, $352; and fee to the adjuster, $225. How should a settlement be effected?

Solution.

$80000, value of ship after undergoing repairs.
1000, ⅓ of $3000, increase of value by superior worth of repairs.
$79000, value of the ship before damage was sustained.

Contributory interests.	
Contributory value of ship,	$79000
Net freight ($10800 less ⅓),	7200
A's invoice	35000
B's "	58000
C's "	63000
Total,	$242200

General average charges.	
Jettison	$5900
Repairs (⅔ of $3000)	2000
Freight on jettison	352
Adjuster's fee	225
Total general average loss,	$8477
Particular average charge.	
Ship (⅓ of $3000)	$1000

$8477 ÷ $242200 = .035, or 3½%, rate of general average.

Settlement.

$$\$1000 + (\overline{\$79000 + 7200} \times .035) - \$352 = \$3665, \text{ ship pays adjuster.}$$

$$\$35000 \times .035 = 1225, \text{ A " "}$$

$$58000 \times .035 = \underline{2030}, \text{ B " "}$$

$6920, total receipts by adjuster.

$$\$5900 - (\$63000 \times .035) = \$3695, \text{ C receives from adjuster.}$$

3000, repairs paid by "
225, fees due adjuster.
$6920, total disbursements by adjuster.

467

Explanation.—As the value of the ship *after* undergoing repairs was $80000, deduct therefrom the superior value of the repairs (⅓ of $3000) to find her value before the damage was sustained ($79000). Select the remaining contributory interests as directed in **465**, and find their sum, obtaining $242200 as the total contributory interest. Select the general average charges as directed in **464**, and find their sum, obtaining $8477 as the total general average charge. Divide the sum of the general average charges ($8477) by the sum of the contributory interests ($242200), obtaining 3½% as the rate of general average charge.

Add the *particular average* against the ship (⅓ of $3000 = $1000) to her proportion of the general average loss ($\overline{\$79000 \text{ ship} + \$7200 \text{ freight}} \times .035 = \$3017$), obtaining $4017 as the total charge against the ship, from which deduct the ship's interest in the general average fund (freight on jettison, $352) to find the balance due the adjuster by the ship ($3665).

Multiply A's and B's contributory interests by the rate of general average charge, obtaining $35000 × .035, or $1225 as A's and $58000 × .035, or $2030, as B's proportionate contribution. Multiply C's contributory interest ($63000) by .035, obtaining $2205 as C's proportionate contribution, which should be deducted from C's interest in the general average fund (jettisoned cargo, $5900) to find the balance due C by the adjuster ($3695).

Rule.—I. *Divide the sum of all the losses and expenses which are subject to general average* (**464**) *by the sum of all the contributory interests* (**465**) *to find the rate per cent. of general average.*

II. *Multiply each contributory interest by the rate per cent. of general average, and the products thus obtained will be the proportionate contribution of such interests.*

III. *Add ⅓ of the total cost of repairs to the proportionate contribution of the ship; and credit the proportionate contribution of each interest with so much of the general average loss as was sustained by that interest.*

NOTE.—In finding the proportionate contribution of a *single* interest, it is sometimes more convenient to employ the "Rule of Three"; taking the total general average loss and each contributory interest as right-hand terms and the total contributory interest as the left-hand term.

EXAMPLES FOR PRACTICE.

1. The steamer Caspian was overtaken by a storm, and for the common safety it was deemed necessary to throw overboard a part of her cargo worth, at the port of destination, $7500. The steamer was valued at $35000, the remaining cargo at $65500, the gross freight at $6000, of which $600 was charged to the cargo jettisoned; and the

adjuster's charges were $120. In the final settlement what should Perry & Co. receive, whose goods were valued at $8000, of which $1500 worth was jettisoned; and what should the steamer's owners pay as their net contribution?

2. The ship Conqueror, while en route from Mobile to Boston, suffered a disaster (accident) which compelled her to put into Baltimore for repairs. The disbursements of the agent in Baltimore were: custom-house fees, protest, towage, pilotage, inspection, unloading and reloading cargo and storage of unloaded cargo, $1200; and to W. E. Woodall & Co., for repairs on vessel, $900. The agent's commission for advancing the money and paying the above bills was 3%; and on cargo valued at $5000 temporarily landed to effect repairs, 1½%. The wages and provisions of the crew from the time the ship changed her course to sail for Baltimore until she was ready for sea, were estimated at $600. The value of the cargo was: A's invoice $18000, B's $25000, C's $22000, and D's $15000; the gross freight was $7500; the value of the ship just before the accident $16500; and the adjuster's fee in Boston $175. How should a settlement be effected, the expenditures and commissions of the agent in Baltimore having been settled by a draft of the captain on the ship's owners, in favor of the agent?

3. If, in Example 2, B's invoice had been insured for $15000, how much should be B's claim against the underwriters?

4. The ship Selma, from Rio Janeiro to New York, encountered heavy gales, and, for the safety of the ship and her cargo, masts and rigging were cut away which cost $3600 to replace, and cargo jettisoned valued at $6000, $4000 of which belonged to A, $1200 to B, and $800 to C. The ship, after undergoing repairs, was valued at $60000. The gross freight was $8400 (less ½ for seamen's wages in New York), of which $800 was on the cargo jettisoned. The value of the cargo landed in New York was: A's shipment $9000, B's $6500, C's $8500, and D's $5000. The adjuster's charges were $150. How should a settlement be made, the ship's owners having paid the cost of repairs, and filed the receipted bills with the adjuster?

5. If, in Ex. 4, A's shipment had been insured for $7500 and D's for $5000, how much insurance ought each to receive?

6. The ship May Queen, from Charleston to Philadelphia, encountered a storm off Cape Hatteras and was compelled to jettison a part of her cargo and put into Norfolk for repairs. The cost of

repairing the accidental damage was $450, and of repairing the voluntary damage was $3000. The total expenses of detention in Norfolk were $800. The appraised value of the ship in Philadelphia was $25000; and of the cargo jettisoned $5000, the freight on which was $200. The gross freight was $3000 (less $\frac{1}{3}$ in Philadelphia), and the adjuster's charges $130. A's invoice was $20000, of which $2000 was jettisoned; B's $15000, of which $3000 was jettisoned; and C's $12000. How should a settlement be effected if the cost of repairs and of detention was paid by the captain out of the ship's funds?

7. If, in Example 6, A's invoice had been insured for $16000, and B's for $12000, how much insurance ought each to receive?

STORAGE.

468. Storage is the placing and safe keeping of merchandise, household goods, etc., in a warehouse.

469. The **Rate of Storage** is a certain price per barrel, bale, bag, bushel, etc., charged for each storage term.

470. A **Storage Term** is the time of storage for which the rate is charged. It is usually 1 week, 10 days, 20 days, or 1 month.

REM. 1.—When goods are withdrawn before the expiration of a storage term, storage is usually charged for the *full* term. In some cities, however, when the storage term is one month, if goods are withdrawn after having been stored one or more full months, storage for any fraction of a month more than one-half is charged at the full rate; and for one-half of a month or less, at one-half the full rate.

REM. 2.—When goods are received and delivered at different times without settlement, the final settlement is often made by finding the *average storage*, and charging for the actual time the goods are stored. This is called *storage on account*. When storage is paid or computed on each delivery, it is called *cash storage*.

471. To find the average storage when there are no deliveries.

ILLUSTRATIVE EXAMPLE.

A warehouseman received the following quantities of wheat at the rate of 1 cent a bushel per month of average storage: June 15, 500 bu.; June 30, 400 bu.; July 18, 300 bu.; Aug. 5, 800 bu. What was the amount of storage due Aug. 15?

Solution.

Dates. *Days.* *Bushels.*
June 15 to Aug. 15 = 61 × 500 = 1 bushel stored for 30500 days.
" 30 " " " = 46 × 400 = 1 " " " 18400 "
July 18 " " " = 28 × 300 = 1 " " " 8400 "
Aug. 5 " " " = 10 × 800 = 1 " " " 8000 "
The total storage = 1 " " " 65300 days.
" " " = 1 " " " $\frac{65300}{30}$, or 2176⅔ months.
$.01 × 2177 = $21.77, required amount of storage.

Rule.—I. *Multiply the number of barrels, bales, etc., of each receipt by the number of days between the date of such receipt and the date of settlement.*

II. *Divide the sum of the obtained products by the number of days in the storage term; and multiply the quotient into the rate of storage.*

EXAMPLES FOR PRACTICE.

1. At 2½ cents a barrel per month of average storage, what is the storage due Dec. 8 on the following receipts: Sept. 9, 200 bbl. pork; Sept. 26, 300 bbl. beef; Oct. 16, 500 bbl. beef; Nov. 10, 250 bbl. pork; Nov. 23, 300 bbl. pork; Dec. 1, 150 bbl. beef?

2. At 1 cent a bushel per month of average storage, what is the storage due Oct. 19 on the following receipts: July 3, 300 bu.; July 12, 500 bu.; July 30, 150 bu.; Aug. 16, 400 bu.; Aug. 25, 800 bu.; Sept. 20, 500 bu.; and Oct. 5, 300 bu.?

3. At 3 cents a barrel per month of average storage, what is the storage due May 18, 1884, on the following receipts: Jan. 20, 125 bbl.; Jan. 31, 75 bbl.; Feb. 12, 140 bbl.; Feb. 29, 80 bbl.; Mar. 15, 150 bbl.: Mar. 24, 95 bbl.; Apr. 4, 475 bbl.; Apr. 9, 38 bbl.; Apr. 15, 100 bbl.; Apr. 28, 50 bbl.; May 10, 150 bbl.?

472. To find the average storage when there are deliveries.

ILLUSTRATIVE EXAMPLE.

A warehouseman received and delivered the following:

Received.		*Delivered.*	
Feb. 22, ...	200 bbl.	Mar. 2, ...	50 bbl.
Mar. 18, ...	150 "	" 8, ...	80 "
" 25, ...	100 "	Apr. 3, ...	200 "
Apr. 8, ...	150 "	" 18, ...	270 "

472

What was paid for storage at 3 cents a barrel per month of average storage, a settlement having been made Apr. 18?

Solution.

Dates.	*Days.*	*Barrels.*	
Feb. 22 to Mar. 2 =	8 ×	200 (received)	= 1 bbl. stored for 1600 days.
" "		50 (delivered)	
Mar. 2 " " 8 =	6 ×	150 (in store)	= 1 " " " 900 "
" "		80 (delivered)	
" 8 " " 18 =	10 ×	70 (in store)	= 1 " " " 700 "
" "		150 (received)	
" 18 " " 25 =	7 ×	220 (in store)	= 1 " " " 1540 "
" "		100 (received)	
" 25 " Apr. 3 =	9 ×	320 (in store)	= 1 " " " 2880 "
" "		200 (delivered)	
Apr. 3 " " 8 =	5 ×	120 (in store)	= 1 " " " 600 "
" "		150 (received)	
" 8 " " 18 =	10 ×	270 (in store)	= 1 " " " 2700 "
" "		270 (delivered)	

Total storage = 1 " " " 10920 days.

" " = $\frac{10920}{30}$, or 364 months.

$.03 × 364 = $10.92, required amount of storage.

Rule.—I. *Multiply the first receipt by the number of days between its date and the date of the next receipt or delivery.*

II. *Multiply the increased or decreased quantity then on storage by the number of days between the date of such increase or decrease to the date of the next increase or decrease; and so continue to the date of final settlement.*

III. *Divide the sum of the obtained products by the number of days in the storage term; and multiply the quotient into the rate of storage.*

Note.—If several classes of packages are stored, on which there are different rates of storage, make separate computations for each class of packages; and combine the results thus obtained to find the total storage.

EXAMPLES FOR PRACTICE.

1. At 4 cents a barrel per month of average storage, what is the storage due Aug. 15, 1883, on the following produce, received and delivered as stated?

Received.	*Delivered.*
1883. June 12, 200 bbl. potatoes.	1883. June 17, 75 bbl. potatoes.
" " 20, 150 " apples.	" " 25, 125 " "
" July 18, 60 " turnips.	" " 30, 90 " apples.
" Aug. 2, 90 " onions.	" July 5, 60 " "
	" " 25, 40 " turnips.
	" Aug. 9, 20 " "
	" " 15, 90 " onions.

2. Compute the storage on the following account to Dec. 31, 1886, at 2¼ cents a barrel per month of average storage:

Received.	*Delivered.*
1886. Aug. 17, 250 bbl. mdse.	1886. Aug. 23, 200 bbl. mdse.
" " 25, 90 " "	" Sept. 25, 240 " "
" Sept. 19, 200 " "	" Oct. 13, 300 " "
" Oct. 12, 300 " "	" Nov. 20, 150 " "
" Nov. 18, 200 " "	" Dec. 31, Balance.
" Dec. 17, 400 " "	

3. Find the pasturage, at 65 cents a head per month of average pasturage, on the following account:

Cattle received.	*Cattle withdrawn.*
1885. May 24, 250 head.	1885. June 2, 200 head.
" June 8, 60 "	" " 20, 40 "
" " 25, 90 "	" July 2, 160 "

473. To find the cash storage on merchandise.

ILLUSTRATIVE EXAMPLE.

A warehouse received and delivered merchandise as follows:

Received.	*Delivered.*
May 6, 300 bbl. pork.	May 23, 200 bbl. pork.
" 27, 250 " "	June 3, 80 " "
June 12, 180 " "	" 7, 220 " "
	" 25, 230 " "

How much must be paid for storage on the above at the rate of 5 cents per barrel for the first 10 days or part thereof, and 3 cents per barrel for each subsequent 10 days or part thereof?

473

Solution.

Date.	*Time of Storage.*	*Receipts and Deliveries.*	*Rate.*	*Storage.*
May 6,		received 300 bbl.		
“ 23	(17 da., or 2 terms),	delivered 200 “	× 8 cents	= $16.00
		Bal. of 1st receipt = 100 “		
June 3	(28 da., or 3 terms),	delivered 80 “	× 11 “	= 8.80
		Bal. of 1st receipt = 20 “		
“ 7	(32 da., or 4 terms),	Of 220, delivered 20 “	× 14 “	= 2.80
May 27,		received 250 “		
June 7	(11 da., or 2 terms),	Of 220, delivered 200 “	× 8 “	= 16.00
		Bal. of 2d receipt 50 “		
“ 25	(29 da., or 3 terms),	Of 230, delivered 50 “	× 11 “	= 5.50
“ 12,		received 180 “		
“ 25	(13 da., or 2 terms),	Of 230, delivered 180 “	× 8 “	= 14.40
			Storage due,	$63.50

Explanation.—The first delivery of 200 bbl. was made May 23, or 17 days after the date of the first receipt, constituting one term of 10 days and part of another, or practically 2 storage terms, which at the given rate of storage (5 cents for first term + 3 cents for second) produces 200 times 3 cents, or $16 storage. Of the remainder of the first receipt (100 bbl.), 80 bbl. were delivered June 3, or 28 days after the date of their receipt, constituting 2 terms of 10 days and part of another, or 3 storage terms; hence the storage thereon must be 80 times (5 + 3 + 3 =) 11 cents, or $8.80. The remainder of the first receipt (20 bbl.) was included in the delivery of 220 bbl. on June 7, and hence was stored 32 days, or 4 terms, producing 20 times (5 + 3 + 3 + 3 =) 14 cents, or $2.80 storage.

The second receipt (250 bbl.) was on May 27, and the next delivery (of 220 bbl.) on June 7, on 20 bbl. of which storage has already been computed, leaving 200 bbl. which were in store 11 days or 2 terms, producing 200 times 8 cents, or $16 storage. The remaining 50 bbl. of the second receipt were included in the delivery of 230 bbl. on June 25, after being stored 29 days or 3 terms, producing 50 times 11 cents, or $5.50 storage.

The third receipt (180 bbl.) was on June 12, all of which were delivered on June 25 (being included in the delivery of 230 bbl.) after being stored 13 days, or 2 terms, producing 180 times 8 cents, or $14.40 storage. Hence, the total storage is the sum of the storage on all the deliveries, or $63.50.

Rule.—I. *Multiply each delivery of the first receipt by the rate of storage for the number of terms which such delivery has been in store, considering any fraction of a term as an entire term.*

II. *Similarly compute the storage on each delivery of the* *second receipt; and so continue in regular order with subsequent receipts until the storage on all the deliveries has been found.*

III. *The sum of the storage on all the deliveries will be the required cash storage.*

EXAMPLES FOR PRACTICE.

1. The receipts and deliveries at a certain warehouse were:

Received.	*Delivered.*
June 25, 250 bbl.	July 3, 90 bbl.
Aug. 4, 350 "	Aug. 12, 150 "
" 14, 50 "	" 20, 170 "
Sept. 4, 70 "	" 25, 180 "
	Sept. 15, 130 "

What was the entire cash storage due Sept. 15, the rate being 5 cents per barrel for the first 10 days or part of 10 days, and 3 cents per barrel for each subsequent 10 days or part of 10 days?

2. Find the cash storage on the following account, the rate of storage being 6 cents per barrel for the first 10 days or part thereof, and 3 cents for each subsequent 10 days or part thereof:

Received.	*Delivered.*
June 20, 400 bbl.	June 27, 300 bbl.
" 25, 60 "	" 30, 40 "
July 7, 20 "	July 12, 20 "
" 16, 40 "	" 30, 160 "

3. Compute the storage on the following account, the rate being 1 cent per bushel for the first 20 days or part thereof, and ½ cent per bushel for each subsequent 20 days or part thereof:

Received.	*Delivered.*
Aug. 15, 1500 bu. wheat.	Aug. 19, 1200 bu. wheat.
" 28, 2500 " "	" 30, 2000 " "
Sept. 3, 5000 " "	Sept. 5, 4000 " "
" 16, 3000 " "	" 19, 3500 " "
" 27, 6000 " "	" 30, 5000 " "
Oct. 3, 4000 " "	Oct. 8, 6300 " "

EQUATION OF PAYMENTS.

474. **Equation of Payments** is the process of finding the date at which one payment can be made in settlement of two or more debts falling due at different dates, without loss to the debtor or creditor.

475. The **Average Date** or **Equated Time** is the date at which two or more debts can be equitably discharged by one payment.

476. The **Focal Date** is any assumed date taken as a standard to which the dates of two or more payments may be compared to find their average date.

REM.—Though any date may be assumed as a standard of comparison, it is usually most convenient to take as the *Focal Date* the *first* or the *last* of the several given dates, or the first day of the month, inclusive, in which the earliest item falls due.

477. The **Term of Credit** is the time allowed for the payment of a debt, or of goods purchased on account.

478. The **Average Term of Credit** is the average time allowed for the payment of the *sum* of two or more debts due at different dates.

479. To find the average date for the payment of two or more debts due at different dates is an application of the following

Principles:—1. *Money is worth its interest for the time it is used.*

2. *The interest or use of money paid before it is due is offset by the interest or use of an equal sum kept an equal time after it is due.*

3. *Assuming any common date of comparison, the product of the prepayment multiplied by the time of prepayment should equal the product of the after-payment multiplied by the time of after-payment.*

480. To find the average date for the payment of two or more debts contracted at the same time but due at different dates.

ILLUSTRATIVE EXAMPLE.

A wholesale merchant sold goods amounting to $1800, as follows: $400 on 3 months' credit, $600 on 5 months' credit, and $800 on 6 months' credit. What is the equated time for the payment of the whole?

Solution.

Debts.	*Terms of credit.*	*Use of $1 for*
400 ×	3 months =	1200 months.
600 ×	5 " =	3000 "
800 ×	6 " =	4800 "
1800 ×	? months =	9000 "

9000 months ÷ 1800 = 5 months.

Explanation.—Assume the entire bill ($1800) to have been paid on the day of purchase. The buyer would then have been deprived of the use, as per terms of sale, of $400 for 3 mo., or of $1 for 1200 mo.; of $600 for 5 mo., or of $1 for 3000 mo.; of $800 for 6 mo., or of $1 for 4800 mo.; or, altogether, of $1 for 1200 + 3000 + 4800, or 9000 mo. To be deprived of the use of $1 for 9000 months is equivalent to being deprived of the use of 1800 times $1, or $1800, for $\frac{1}{1800}$ of 9000 months, or 5 months. Hence, the correct or equated time of paying the entire amount should be 5 months after the date of purchase.

Rule.—I. *Multiply each item by its term of credit, and divide the sum of the products by the sum of the items to find the average term of credit.*

II. *Add the average term of credit to the date of purchase to find the average date.*

NOTE 1.—Fractions of a month should be reduced to days; and fractions of a day should be discarded if less than ½, or considered an entire day if ½ or more.

NOTE 2.—If an item contains cents, the cents may be rejected from the solution if less than 50, or considered an additional dollar if 50 or more.

EXAMPLES FOR PRACTICE.

Find the average term of credit of the following bills:

(1.)

Items.	*Terms of credit.*
$100	. . . . 2 months.
300	. . . . 4 "
200	. . . . 7 "
400	. . . . 8 "

(2.)

Items.	*Terms of credit.*
$100	. . . 1 month.
200	. . . 2 "
300	. . . 3 "
400	. . . 4 "

(3.)

Items.	*Terms of credit.*
$700	. . . . 5 months.
350	. . . . 4 "
550	. . . . 2 "
400	. . . . 5 "

Apply Note 1 in the solution of the next three examples.

(4.)

$50	. . . . 2 months.
700	. . . . 3 "
800	. . . . 4 "
500	. . . . 6 "

(5.)

$1000	. . 3 months.
2500	. . 5 "
1500	. . 4 "
4000	. . 6 "

(6.)

$800	. . . 4 months.
750	. . . 2 "
325	. . . 5 "
585	. . . 3 "

Find the equated date of payment of the following :

(7.)	(8.)
May 12, $150 on 30 days' credit.	June 20, $1200 on 90 days' credit.
" 400 " 90 " "	" 2400 " 30 " "
" 300 " 60 " "	" 1800 " 60 " "
" 500 " 30 " "	" 1500 " 90 " "

9. A wholesale merchant sold goods amounting to $3600, of which $1800 was to be paid in 6 months, $900 in 8 months, and the remainder in 9 months. What was the average term of credit ?

10. An invoice of goods amounting to $1200 was purchased May 30, 1886, on the following terms : $600 payable in 30 days, $400 in 60 days, and $200 in 90 days. What is the equated date for the payment of the entire bill ?

11. If I buy goods amounting to $1600, and agree to pay half in 3 months, and the remainder in 5 months, and afterward wish to pay the entire debt in one amount, when should the payment be made ?

12. A plantation was sold for $18000 upon the following terms : one-third in cash, one-half of the remainder in 12 months, and the balance in 18 months, without interest. What is the equated time for the payment of the entire sum ?

13. On July 15, 1885, an importer sold the following bills to a wholesale firm : No. 1, $500 on 30 days' credit ; No. 2, $792 on 60 days ; No. 3, $820 on 90 days ; No. 4, $624 on 120 days. What is the equated time for the payment of the entire purchase ?

14. On Aug. 4, 1884, an agent received a consignment of 570 barrels of apples. He sold 75 barrels on Aug. 9, 40 barrels on Aug. 18, 150 barrels on Aug. 30, 160 barrels on Sept. 8, and the remainder on Sept. 15. What is the equated date of the total sales ?

15. On June 17, 1885, an agent received a consignment which he sold for $1250, as follows : $200 on June 20, $150 on June 30, $325 on July 8, $415 on July 18, and the remainder on Aug. 3. What is the equated date of the total sales ?

16. On Sept. 3, 1886, a commission merchant received 200 barrels of flour to be sold on account. On Sept. 15, he sold 50 barrels at $6.25 per barrel ; on Sept. 25, 30 barrels at $6.40 ; on Oct. 19, 75 barrels at $6.30 ; and on Oct. 22, the remainder at $6.25. What is the equated date of the total sales ?

17. Oct. 12, 1885, a commission merchant received a consignment which he sold for $3200 as follows: $1800 on Oct. 20, $500 on Nov. 3, $600 on Nov. 18, and the remainder on Nov. 28. What is the equated date of the total sales?

18. On May 3, 1886, I bought the following bills of goods: $240 on 30 days' credit; $175 on 60 days' credit; and $350 on 90 days' credit. What is the equated date of the total purchase?

481. To find the average date for the payment of two or more debts contracted at different dates when no terms of credit are included.

ILLUSTRATIVE EXAMPLE.

J. M. Gaulden bought of Geo. W. Keller & Co., the following merchandise: Jan. 18, 1886, $720; Feb. 17, 1886, $280; March 26, 1886, $460. No credit having been allowed, what is the average date for the payment of the sum of the several bills?

First Solution.—*Product Method.*

FOCAL DATE, JAN. 0, 1886 (= DEC. 31, 1885).

Dates.	*Debts.*		*Days.* (From focal date.)		*Products.* (Equivalent use of $1 for)
1886, Jan. 18,	$720	×	18	=	12960 days.
Feb. 17,	280	×	48	=	13440 "
Mar. 26,	460	×	85	=	39100 "
	$1460				1460)65500(45—days.

Average date: 45 days after Jan. 0, 1886, or Feb. 14, 1886.

Explanation.—Since no credits were allowed, each bill was due on the day of purchase; and it is required to find a common date at which all *three* bills might have been paid without loss of interest to buyer or seller.

Assume Jan. 0, 1886 (= Dec. 31, 1885) as the required date. As the first debt ($720) was not due until Jan. 18 (18 days later than the assumed date), to pay it on Jan. 0 would have deprived the buyer of the use of $720 for 18 days, or of $1 for 720 times 18, or 12960 days; as the second debt ($280) was not due until Feb. 17 (48 days after the assumed date), to pay it on Jan. 0 would have deprived the buyer of the use of $280 for 48 days, or of $1 for 13440 days; as the third debt ($460) was not due until Mar. 26 (85 days after the assumed date), to pay it on Jan. 0 would have deprived the buyer of the use of $460 for 85 days, or of $1 for 39100 days. Hence, to pay the entire bill ($720 + $280 + 460 = $1460) on Jan. 0 would have deprived the buyer of the use of $1 for (12960 + 13440 + 39100 =) 65500 days, or of 1460 times $1 for $\frac{1}{1460}$ of 65500 days, or 44$\frac{63}{73}$ days (practically 45 days; Note

1, **480**); and the assumed date (Jan. 0, 1886) is thus found to be 45 days earlier than the correct date. Hence, etc.

Second Solution.—*Interest Method.*

FOCAL DATE, JAN. 0, 1886.—RATE OF INTEREST 12% PER ANNUM.

Mo.	*Dates.*	*Items.*	*Interest.*	
0	Jan. 18	$720	$4.32,	int. of $720 for 18 da.
1	Feb. 17 + 1	280	2.80,	" $280 " 1 mo. (of 30 da.)
			1.68,	" " " 18 da.
2	Mar. 26 — 1	460	9.20,	" $460 " 2 mo. (of 30 da. each)
			3.68,	" " " 24 da.
			.15,	" " " 1 da.
		$1460	$21.83	
			3	

Int. of $1460 for 3 da. = $1.46)$65.49(45 — days.

Average date: 45 days after Jan. 0, 1886, or Feb. 14, 1886.

Explanation.—Assume Jan. 0, 1886, as the date at which the entire amount should be paid; and to the left of the date of each item, write the number of complete calendar months between it and the assumed date. For convenience in calculating interest, suppose each of these intervening complete months to be 30-day months; and in correction, plus or minus each date with the excess over, or deficiency under 30 days, of each intervening complete month.

By paying the first bill ($720) on Jan. 0, the buyer loses the use of $720 for 18 days which, at 12% per annum, is equivalent to the loss of .006 of the debt, or $4.32; by paying the second bill ($280) on Jan. 0, the buyer loses the use of $280 for the 31-day month of January and 17 days in February (= 1 30-day month and 18 days), which, at 12% per annum, is a loss of 1% of $280 ($2.80) for 1 month, and .006 of $280 ($1.68) for 18 days; by paying the last bill ($460) on Jan. 0, the buyer loses the interest of $460 for the 31-day month of January, the 28-day month of February, and 26 days in March (= 2 30-day months and 25 days), which is 2% of $460 ($9.20) for the 2 30-day months, .008 of $460 ($3.68) for 24 days, and .000⅓ of $460 ($.15) for 1 day. Hence, by paying the sum of the three items ($1460) on Jan. 0, 1886, the buyer loses the sum of the obtained interests ($21.83).

If by paying $1460 on Jan. 0, the buyer loses interest amounting to $21.83, the proper date for paying $1460 without loss of interest must be as many days after Jan. 0 as will enable $1460 to produce $21.83 interest.

At 12%, the interest of $1460 for 1 day is .000⅓ of $1460. Hence, applying **326**, divide the given interest ($21.83) by .000⅓ of $1460, or to avoid fractions, divide 3 times the interest ($65.49) by .001 of $1460 ($1.46), obtaining practically 45 days as the time after Jan. 0 to pay the sum of the items without loss of interest to either buyer or seller.

482. Rules for finding the equated date of purchase.

Product Method.—I. *Take as the focal date, the date at which the earliest item is due, or any convenient prior date.*

II. *Multiply each item by the difference in days between the focal date and the date at which such item falls due; and divide the sum of the obtained products by the sum of the items.*

III. *The quotient will denote the number of days to count forward from the focal date to find the equated date.*

NOTE 1.—The date at which the *latest* item is due, or any convenient subsequent date, may be taken as the focal date. The quotient of dividing the sum of the products by the sum of the items will then denote the number of days to count *backward* from the focal date to find the equated date; because, in this case, the sum of the products will denote the number of days during which the buyer has withheld $1 *beyond* the equated date.

NOTE 2.—The most convenient focal date for computing time is 0 of the month at which the earliest item is due, an expression which is equivalent to the last day of the preceding month.

Interest Method.—I. *Select the focal date as directed in the preceding Rule.*

II. *Find the interest of each item for the time between the focal date and the date at which such item falls due; and divide three times the sum of the obtained interests by .001 of the sum of the items.*

III. *The quotient will denote the number of days to count forward from the focal date to find the equated date.*

NOTE 3.—The focal date may be selected as directed in Note 1. The final quotient will then denote the number of days to count backward from the focal date to find the equated date.

NOTE 4.—While interest may be computed at any rate per annum, 12% will be found the most convenient. At 12%, the interest for 30 days or 1 30-day month is 1% of the principal (obtained by moving the decimal point of the principal *two* places to the left); for 2 30-day months take twice the preceding result; for 5 30-day months take 5 times the preceding result, etc. For 3 days the interest is .001 of the principal (obtained by moving the decimal point of the principal *three* places to the left); for twice 3 days take twice the preceding result; for 4 times 3 days take 4 times the preceding result, etc. For 1 day the interest is .000⅓ of the principal (obtained by moving the decimal point of the principal *three* places to the left and dividing the result by 3).

NOTE 5.—Since, at 12% per annum, the interest is 1% *per month of 30 days*, if the time be expressed in months and days, change the calendar months to *30-day months* by increasing the time 1 day for each included 31-day month, or by *decreasing* the time 2 days (1 day for leap year) if February is included.

EXAMPLES FOR PRACTICE.

Find the equated date of purchase of the following bills:

(1.)

1886. June 5, $500, no credit.
July 20, 300, “
Aug. 15, 200, “

(2.)

1886. Jan. 3, $250, no credit.
Feb. 18, 375, “
Apr. 4, 530, “

(3.)

1884. Mar. 9, $735, no credit.
“ 31, 840, “
Apr. 8, 750, “
May 15, 475, “

(4.)

1885. Aug. 12, $690, no credit.
Sept. 4, 325, “
Oct. 21, 512, “
Dec. 8, 450, “

5. I bought of Simmons & Bro., Pensacola, the following invoices of merchandise:

Jan. 3, 1885, invoice amounting to $750.
“ 15, “ “ “ “ 320.
Feb. 9, “ “ “ “ 195.
Mar. 11, “ “ “ “ 275.
Apr. 18, “ “ “ “ 525.

What was the amount of my bill, and the average date of purchase?

6. J. P. Jordan & Co.’s sales to T. A. Smith are as follows:

Mar. 19, 1886, a bill of $275.
“ 31, “ “ “ 340.
Apr. 25, “ “ “ 520.
May 15, “ “ “ 150.
June 12, “ “ “ 260.

What is the average date of purchase?

7. A bought of B the following merchandise:

Feb. 5, 1885, a bill amounting to $420.
Mar. 13, “ “ “ “ 762.
Apr. 14, “ “ “ “ 578.
May 21, “ “ “ “ 284.
June 6, “ “ “ “ 512.

What is the average date of purchase?

8. A sold goods to B as follows: May 30, $525.75; June 15, $325.40; June 28, $275; July 7, $214.30; July 20, $390.15; Aug. 12, $125.45; and Sept. 10, $263.20. What is the equated date of total sales?

9. W. J. Robb & Co. sold to L. W. Bristol the following invoices of tobacco: July 7, 3840 lb. @ 27¢; July 19, 2640 lb. @ 26¼¢; Aug. 4, 3752 lb. @ 28¾¢; Aug. 15, 1464 lb. @ 29¢; and Sept. 3, 3890 lb. @ 28¼¢. What was the average date of sale?

483. To find the average date for the payment of two or more debts contracted at different dates if all the debts have the same term of credit.

ILLUSTRATIVE EXAMPLE.

L. T. Hooker bought of John Hodge & Co. the following:

Feb. 5, 1885, merchandise amounting to $274 on 30 days' credit.
Mar. 19, " " " " 382 " " "
Apr. 9, " " " " 260 " " "
May 13, " " " " 185 " " "
June 3, " " " " 90 " " "

What is the equated date for the payment of the sum of the several bills?

Solution.—*Interest Method.*

FOCAL DATE, FEB. 0, 1885.—RATE 12% PER ANNUM.

Mo.	*Dates.*	*Items.*	*Interest.*	
0	Feb. 5	$274	$.46,	interest of $274 for 5 da. (⅙ of $2.74).
1	Mar. 19-2	382	3.82, 1.91, .25,	" " $382 " 1 mo. (of 30 da.). " " " " 15 da. (½ of $3.82). " " " " 2 da. (1/15 of $3.82).
2	Apr. 9-1	260	5.20, .52, .17,	" " $260 " 2 mo. (of 30 da. each). " " " " 6 da. (⅕ of $2.60). " " " " 2 da. (⅓ of $.52).
3	May 13-1	185	5.55, .74,	" " $185 " 3 mo. (of 30 da. each). " " " " 12 da. ($.185 × 4)
4	June 3	90	3.60, .09,	" " $90 " 4 mo. (of 30 da. each). " " " " 3 da. (.001 of $90).
		$1191	$22.31 3	

Interest of $1191 for 3 da. = $1.191) $66.930 (56 + days { time after focal date if there had been no term of credit.

56 da. + 30 da. = 86 da., time after focal date, including the term of credit.

Feb. 0, 1885, + 86 da. = Apr. 27, 1885, required equated date.

483

Explanation.—Proceed as fully explained in Second Solution of Ill. Ex., **481**, as if no terms of credit had been given, obtaining 56 + days, to which add the uniform credit of 30 days, obtaining 86 days after Feb. 0, 1885, or Apr. 27, 1885, as the required equated date.

Rem.—When the time in days after the focal date has been found, it is puerile to *separately* deduct the days in each of the succeeding months. It is as easy, and much more expeditious, to deduct the days of two, three, or even four succeeding months in *one operation*. Thus, in the preceding example, 86 — 59 (days in Feb. and Mar.) = 27, days in the following month of April.

Rule.—*Proceed, by either the Product or Interest method,* **482,** *to find the time as if no term of credit had been given, and add the uniform credit to the result.*

EXAMPLES FOR PRACTICE.

1. S. H. Hanson sold to a retail merchant merchandise as follows:

1884. Feb. 6, an invoice amounting to $673.40.
" Mar. 24, " " " " 462.60.
" Apr. 18, " " " " 327.90.
" June 24, " " " " 128.10.

If 60 days' credit was allowed on each item, what was the average date for the payment of the entire amount?

When a payment contains cents, apply Note 2, **480**.

2. A wholesale merchant sold to a customer the following:

1885. Sept. 4, an invoice of merchandise amounting to $624.20.
" Oct. 12, " " " " " " 920.80.
" Nov. 26, " " " " " " 440.72.
" Dec. 18, " " " " " " 364.28.

A credit of 90 days being allowed on each item, what was the equated date for the payment of the amount of the several bills?

3. M. A. Davenport bought of Johnson & Co.:

Mar. 12, 1886, Merchandise amounting to $742.83.
April 4, " " " " 926.17.
May 24, " " " " 432.25.
June 17, " " " " 327.75.

What is the equated date for payment of the amount of the several bills, a credit of 30 days being allowed on each item? **483**

4. J. W. Miller bought of Freeman & Co.:

May 30, 1886, Merchandise amounting to $345.30.
June 19, " " " " 576.75.
Aug. 3, " " " " 453.40.

A credit of 90 days having been allowed on each item, what was the equated date for paying the entire amount?

5. I bought the following bills of goods on 3 months' credit: Aug. 18, 1885, $430.80; Sept. 8, 1885, $215.65; Sept. 25, 1885, $300; and on Oct. 15, 1885, $591.20. What was the equated date of total purchase?

484. To find the average date for the payment of two or more debts contracted at different dates, and having different terms of credit.

ILLUSTRATIVE EXAMPLE.

PHILADELPHIA, *Sept. 1, 1886.*

E. G. CHURCH,
1886. *To* R. S. SCOTT & Co., *Dr.*

May 15	To merchandise per inv., @ 30 da.	$500	00		
June 9	" " " " " 60 da.	350	00		
July 18	" " " " "	250	00		
Aug. 5	" " " " " 30 da.	470	00	$1570	00

First Solution.—*Product Method.*

FOCAL DATE, MAY 0, 1886 (= APR. 30, 1886).

	Dates of Sale.	*Credits.*	*Time.*	*Items.*	*Products.*
	May 15	+ 30 da.	= 45 da.	× 500	= 22500 da.
31 +	June 9	+ 60 "	= 100 "	× 350	= 35000 "
61 +	July 18		= 79 "	× 250	= 19750 "
92 +	Aug. 5	+ 30 "	= 127 "	× 470	= 59690 "
				1570)	136940 (87 + da.

Average date: 87 days after May 0, 1886, or July 26, 1886.

Explanation.—The above solution is similar to that by the product method, **481**, except that each item is multiplied by the number of days between the focal date (May 0=Apr. 30) and the date at which such item falls due, obtained by adding the term of credit (if any) to the number of days between the focal date and the date of sale.

484

Second Solution.—*Interest Method.*

FOCAL DATE, MAY 0, 1886. RATE PER ANNUM, 12%.

Mo.	*Dates of Sale.*	*Items.*		*Credits.*	*Interest.*	
0	May 15	$500	@	30 da.	$5.00,	int. of $500 for 30 da. (1% of $500).
					2.50,	" " " " 15 " (½ of $5.00).
1	June 9+1	350	@	60 "	10.50,	" " $350 " 3 mo. ($3.50 × 3).
					1.17,	" " " " 10 da. (⅓ of $3.50).
2	July 18+1	250			5.00,	" " $250 " 2 mo. ($2.50 × 2).
					1.50,	" " " " 18 da. ($ .25 × 6).
					.08,	" " " " 1 " (⅓ of $ .25).
3	Aug. 5+2	470	@	30 "	18.80,	" " $470 " 4 mo. ($4.70 × 4).
					.94,	" " " " 6 da. ($ .47 × 2).
					.16,	" " " " 1 " (⅓ of $ .47).
		$1570			$45.65	
					3	

Interest of $1570 for 3 da. = $1.57) $136.95 (87 + da.

Average date: 87 days after May 0, 1886, or July 26, 1886.

Explanation.—The above solution is like that of the interest method, **481**, except that the interest is found on each item from the focal date to its due date, obtained by adding the term of credit to the date of purchase, considering each 30 days of the terms of credit as an additional 30-da. month.

Rule.—*Add the term of credit to the time from the focal date to the date of the item; then proceed by either the product or interest Rule,* **482.**

NOTE 1.—If a term of credit is given in months, calendar months are meant; and they should be reduced to 30-day months by Note 5, **482.**

NOTE 2.—In obtaining the due date of notes, drafts, etc., add 3 days for grace to their nominal time of payment.

EXAMPLES FOR PRACTICE.

Find the equated date of payment.

(1.)

1886. Mar. 3, $300 @ 60 da.
Apr. 15, 475 @ 30 "
May 5, 520 @ 30 "

(2.)

1885. Aug. 7, $625 @ 90 da.
Sept. 3, 550 @ 30 "
Oct. 12, 225 @ 60 "

(3.)

1885. Sept. 20, $750 @ 60 da.
" 28, 825 @ 90 "
Oct. 15, 575 @ 30 "
Nov. 1, 150 @ 30 "

(4.)

1885. Jan. 18, $380 @ 45 da.
Feb. 3, 720 @ 60 "
Mar. 9, 260 @ 30 "
Apr. 20, 440 @ 60 "

484

(5.)

1886. Mar. 21, $825.30 @ 3 mo.
Apr. 3, 512.75 @ 1 "
" 29, 250.90 @ 2 "
May 8, 175.25 @ 3 "
" 19, 520.00 @ 2 "

(6.)

1886. May 2, $150.35 @ 1 mo.
June 4, 275.58 @ 3 "
" 30, 182.15 @ 4 "
Aug. 1, 325.75 @ 2 "
Sept. 15, 415.10 @ 3 "

7. W. H. Ransom & Co. bought merchandise, as follows:

Mar. 15, 1884,	a bill of	$324	on 60 days'	credit.
May 14,	" "	676	" 30 "	"
June 12,	" "	480	" 3 months'	"
July 15,	" "	225	" 90 days'	"
Aug. 4,	" "	422	" 3 months'	"

What is the equated date of payment?

8. Average the following statement of account:

NEW YORK, *Oct. 1, 1885.*

W. W. TREVOR & Co.,

In account with H. A. FAIRMAN. *Dr.*

1885.	Mar. 25,	To Mdse. per bill rendered,	$340.72	@ 30 da.
"	Apr. 14,	" " "	671.34	@ 60 "
"	May 14,	" " "	374.62	@ 90 "
"	June 2,	" " "	517.27	@ 30 "
"	" 24,	" " "	243.25	@ 3 mo.
"	July 18,	" " "	721.80	@ 30 da.

9. On Jan. 25, 1885, a grocer sold goods amounting to $1340, one-half to be paid in 90 days, one-fourth in 4 months, and the remainder in 5 months. What was the equated date of payment?

10. On March 24, 1886, M. A. Davenport bought of J. R. Compton merchandise amounting to $1644, one-fourth payable in 60 days, one-fourth in 3 months, and the remainder in 4 months. At what date could the entire debt be equitably paid?

11. E. I. Collier sold E. A. Ellicott & Co., Feb. 4, 1885, 200 bbl. flour at $6 per bbl., on 30 days' credit; Mar. 12, 1885, 23 bbl. sugar, averaging 240 lb. per bbl., at 7½¢ per pound, on 60 days' credit; Apr. 16, 1885, 320 lb. N. Y. butter, at 22½¢ per pound, on 3 months' credit; Mar. 22, 1885, 640 doz. eggs at 12½¢ per dozen. When should E. A. Ellicott & Co.'s entire indebtedness to E. I. Collier be paid by average?

12. F. H. Smith & Son sold H. W. Shaw the following bills of merchandise: Mar. 3, 1884, $467.20 @ 30 days; Mar. 25, 1884,

$562.75 @ 60 days; Apr. 12, 1884, $210.58 @ 4 months; May 3, 1884, $535.10 @ 30 days; and May 15, 1884, $285 @ 90 days. What is the equated date for the payment of the sum of the several bills?

AVERAGING ACCOUNTS.

485. The **Average of an Account** is the average or equated time for the payment of the balance of an account which contains both debit and credit items.

REM.—In Equation of Payments, only *one* side of an account is considered, the items being either all debits or all credits; but when an account involves both debit and credit items, it is necessary to consider *both* sides of the account in ascertaining the equated date to pay the balance due. The former is sometimes called *Simple Average*, and the latter *Compound Average.*

486. Accounts are averaged upon the following principles:

1. *Goods delivered and money paid a certain time* BEFORE *they are due should equitably be* CREDITED *with interest for that time.*

2. *Goods delivered and money paid a certain time* AFTER *they are due should equitably be* DEBITED *with interest for that time.*

3. *The equated time for paying the balance of an account is that date at which the credit interest of Prin. 1 will offset the debit interest of Prin. 2.*

REM.—If the balance of an account is paid before the equated date, it is customary to deduct bank discount on the balance from the date of its payment to the equated date; and if paid after the equated date, to add interest to the balance from the equated date to the date of its payment. If an interest-bearing note is given in settlement of an account, the equated date is taken as the date of the note.

487. To find the equated date for paying the balance of a debt, if credited with one or more partial payments before the debt is due.

ILLUSTRATIVE EXAMPLE.

W. H. Crampton bought April 8, 1884, of A. D. Frank & Co., an invoice of merchandise amounting to $640 on 4 months' credit. On May 19, Mr. Crampton paid $200, and on June 2, $240. What is the equated date for paying the balance?

Explanation.—Apr. 8 + 4 mo. = Aug. 8, the due date. If $200 were paid May 19 (81 days before the due date), the loss to Crampton would be the use of $200 for 81 days, or of $1 for 16200 days. If $240 were paid June 2 (67 days before the due date), the loss to Crampton would be the use of $240 for 67 days, or of $1 for 16080 days: both payments involving a loss to Crampton of the use of $1 for 32280 days. Hence, to restore this loss to Crampton, he should be permitted to use the unpaid balance ($640 − $440 = $200) for as many days after the due date (Aug. 8) as will make the unpaid balance ($200) equivalent to the use of $1 for 32280 days; that is, for $\frac{1}{200}$ of 32280 days, or 161 + days.

Solution.

$$\begin{array}{ll} 200 \times 81 \text{ da.} = & 16200 \text{ da.} \\ \underline{240} \times 67 \text{ ``} = & \underline{16080} \text{ ``} \\ 440 & 32280 \text{ ``} \end{array}$$

$640 − $440 = $200, balance due.

32280 da. ÷ 200 = 161 + da., extension.

Apr. 8 + 4 mo. = Aug. 8, due date.

Aug. 8 + 161 da. = Jan. 16, 1885.

Rule.—*Multiply each payment by the number of days between the date of such payment and the due date, and divide the sum of the obtained products by the unpaid balance. The quotient will denote the number of days to count forward from the due date to find the equated date.*

NOTE.—While it is customary as well as equitable to proportionately extend the term of credit on the balance of a debt if a part thereof has been prepaid, the law does not enforce such extension if the creditor objects.

EXAMPLES FOR PRACTICE.

1. If, on March 3, I purchase $800 worth of merchandise on 90 days' credit, and on April 15 make a partial payment of $400, what is the equated date for paying the balance?

2. On May 5, 1883, James Atwater sold to H. P. Underhill an invoice of goods amounting to $1500, on 1 month's credit. On May 10, H. P. Underhill paid $300, and on May 20, $570. On the date of the second payment, H. P. Underhill gave an interest-bearing note, properly dated, for the unpaid balance. What was the date of the note?

3. On Aug. 15, 1884, C. F. Dickinson sold an invoice of merchandise for $1200 on 90 days' credit, and on Sept. 20 he received from the purchaser $400. When in equity should the balance be paid?

4. I sold on Sept. 5, 1884, a bill of goods for $875 on 3 months' credit. When in equity should I receive the balance if on Oct. 25, 1884, the purchaser made a partial payment of $375?

5. On Dec. 8, 1883, a manufacturer sold an invoice of goods to a merchant for $1800, on 3 months' credit; and on Jan. 15, 1884, the

487

manufacturer bought from the merchant a bill of goods for $2500 on which he was allowed no credit. What is the equated date for the merchant to receive the balance due from the manufacturer?

6. On May 13, 1884, J. W. Horn sold a bill of goods to Kendall & Co., amounting to $350, on 2 months' credit; and on June 1 Kendall & Co. paid on account $250. What is the equated date for J. W. Horn to receive the balance due him ?

7. On July 19, 1886, E. H. Foley bought goods to the amount of $750 on 60 days' credit; on Aug. 3, he made a payment thereon of $230, and on Sept. 5, a second payment of $350. What is the equated date for Foley to pay the remainder of the purchase money ?

8. On Oct. 15, 1883, B bought merchandise of A, amounting to $675, on 3 months' credit; on Nov. 1, B paid A $225 on account, and on Dec. 12, B paid A $340 additional, and gave an interest-bearing note for the remainder. What was the date of the note ?

9. On Jan. 12, 1884, J. Fagan & Son sold Ferguson Bros. & Co. a bill of merchandise amounting to $1825, on 3 months' credit; and on Feb. 11, 1884, J. Fagan & Son borrowed of Ferguson Bros. & Co. $2500. What should be the date of an interest-bearing note to be given by J. Fagan & Son in settlement of the balance due Ferguson Bros. & Co.?

10. A has three of B's notes: one of $375, dated Aug. 21, 1884, and payable in 3 months; one of $530, dated Sept. 3, 1884, and payable in 60 days; and one of $425, dated Oct. 15, 1884, and payable in 4 months. On Nov. 1, 1884, B paid A $600, canceled his three notes, and in exchange gave a single note for the remainder. What was the date of the latter note ?

488.—To find the equated date for paying the balance of any account.

ILLUSTRATIVE EXAMPLES.

1. Average the following ledger account :

Dr. O. F. WILLIAMS. *Cr.*

1886.				*1886.*			
Oct. 6	To Mdse.,	$350	00	Nov. 8	By Cash,	$300	00
Nov. 15	" "	450	00	" 25	" "	200	00
Dec. 3	" "	280	00	Dec. 12	" Sundries,	500	00
" 22	" "	520	00				

First Solution.—*Product Method.*

FOCAL DATE, OCT. 0, 1886.

Dates.	*Time.*	*Items.*	*Products.*
Oct. 6	= 6 da.	× 350	= 2100 da.
31 + Nov. 15	= 46 “	× 450	= 20700 “
61 + Dec. 3	= 64 “	× 280	= 17920 “
61 + “ 22	= 83 “	× 520	= 43160 “
		1600	83880 “
		1000	59400 “
		600)	24480 (41—days.

Dates.	*Time.*	*Items.*	*Products.*
31 + Nov. 8	= 39 da.	× 300	= 11700 da.
31 + “ 25	= 56 “	× 200	= 11200 “
61 + Dec. 12	= 73 “	× 500	= 36500 “
		1000	59400 “

Average date : 41 days after Oct. 0, 1886, or Nov. 10, 1886.

Explanation.—Equating the debit side of the account by Rule for Product Method, **482**, it is seen that if the entire bill ($1600) had been paid on Oct. 0, Williams would have been deprived of the use of $1 for 83880 days ; and similarly equating the credit side, it is seen that Williams really had the use of $1 of his debits for 59400 days after the focal date : leaving a net deprivation of the use of $1 for 83880—59400, or 24480 days. Hence, the average date for Williams to pay the balance of his account ($1600—$1000), or $600, must be as many days after the focal date as will make the use of his balance ($600) equivalent to the use of $1 for 24480 days ; that is, 41 days after Oct. 0, or Nov. 10, 1886.

Second Solution.—*Interest Method.*

FOCAL DATE, OCT. 0, 1886.—RATE PER ANNUM 12%.

Mo.	*Dates.*	*Items.*	*Interest.*
0	Oct. 6	$350	$0.70 (6 da.).
1	Nov. 15 + 1	450	4.50 (1 mo.). 2.25 (15 da.). .15 (1 da.).
2	Dec. 3 + 1	280	5.60 (2 mo.). .28 (3 da.). .09 (1 da.).
2	“ 22 + 1	520	10.40 (2 mo.) 3.64 (21 da.). .35 (2 da.).
		$1600	$27.96
		1000	19.80
		$600	$8.16
			3
		Int. of $600 for 3 da. = $.60)	$24.48 (41—days.

Mo.	*Dates.*	*Items.*	*Interest.*
1	Nov. 8 + 1	$300	$3.00 (1 mo.). .90 (9 da.).
1	“ 25 + 1	200	2.00 (1 mo.). 1.60 (24 da.). .13 (2 da.).
2	Dec. 12 + 1	500	10.00 (2 mo.). 2.00 (12 da.). .17 (1 da.).
		$1000	$19.80

Average date : 41 days after Oct. 0, 1886, or Nov. 10, 1886.

488

Explanation.—Equate the debit side by Rule for Interest Method, **482**, finding that if the entire bill ($1600) had been paid on Oct. 0, Williams would have lost interest amounting to $27.96, at 12% per annum. Then similarly equate the credit side, finding that Williams had the benefit of $1000 of his debits after Oct. 0, until it produced $19.80 interest at 12%: leaving, on the basis of a settlement on Oct. 0, a net loss to Williams of $27.96 — $19.80, or $8.16 of interest. Hence, in equity, Williams should retain the use of the balance of his account ($600) as long after the focal date as will enable this balance to produce $8.16 interest; that is, 41 days after Oct. 0, 1886, or Nov. 10, 1886.

2. Average the following ledger account:

Dr. S. F. Lee, *in account with* C. E. Hill. *Cr.*

1885.				*1885.*			
Aug. 15	To Mdse.,	$300	00	Nov. 1	By Cash,	$500	00
Sept. 8	" "	500	00	" 20	" "	200	00
Oct. 20	" "	840	00	Dec. 3	" "	250	00

First Solution.—*Product Method.*

FOCAL DATE, AUG. 0, 1885.

Dates.	*Time.*	*Items.*	*Products.*	*Dates.*	*Time.*	*Items.*	*Products.*
Aug. 15	=15 da. ×	300=	4500 da.	92+Nov. 1	= 93 da. ×	500=	46500 da
31+Sept. 8	=39 " ×	500=	19500 "	92+ " 20	=112 " ×	200=	22400 "
61+Oct. 20	=81 " ×	840=	68040 "	122+Dec. 3	=125 " ×	250=	31250 "
		1640	92040 "			950	100150 "
		950					92040 "
							8110 "

690) 8110 (12—days.

Average date: 12 days before Aug. 0, 1885, or July 19, 1885.

Explanation.—Equating each side as in Ill. Ex. 1, it is found that, on the basis of a settlement on the focal date (Aug. 0, 1885) the debit side exhibits a loss to Lee of the use of $1 for 92040 days; but the credit side shows that Lee had the use of $950 of his debits after Aug. 0, until it was equivalent to the use of $1 for 100150 days. Hence, on the basis of a settlement on Aug. 0, Lee had the use of $1 for 100150 — 92040, or 8110 days more than he was entitled to, and Lee should therefore pay the balance of his account ($690) as many days before the focal date as will make the use of this balance to Hill equivalent to the use of $1 for 8110 da., that is, 12 days before Aug. 0, or July 19, 1885.

Second Solution.—*Interest Method.*

FOCAL DATE, AUG. 0, 1885.—RATE PER ANNUM, 12%.

No.	*Dates.*	*Items.*	*Interest.*	*No.*	*Dates.*	*Items.*	*Interest.*
0	Aug. 15	$300	$1.50 (15 da.).	3	Nov. 1 + 2	$500	$15.00 (3 mo.). .50 (3 da.).
1	Sept. 8 + 1	500	5.00 (1 mo.). 1.50 (9 da.).	3	" 20 + 2	200	6.00 (3 mo.). 1.40 (21 da.). .07 (1 da.).
2	Oct. 20 + 1	840	16.80 (2 mo.). 5.88 (21 da.).	4	Dec. 3 + 2	250	10.00 (4 mo.). .42 (5 da.).
		$1640	$30.68			$950	$33.39
		950					30.68
		$690					$2.71
							3

Interest of $690 for 3 days = $.69) $8.13 (12—

Average date : 12 days before Aug. 0, 1885, or July 19, 1885.

Explanation.—If the entire bill ($1640) had been paid Aug. 0, Lee would have lost interest amounting to $30.68, at 12% per annum; but he had the use of $950 of his debits after Aug. 0, until it produced $33.39 interest, that is, $33.39 − $30.68, or $2.71 more than enough to offset his loss. Hence, Lee should pay the balance of his account ($690) as many days *before* Aug. 0 as will enable it by Aug. 0 to accrue $2.71, and thus offset the interest balance against him. At 12%, it will require $690 practically 12 days to accrue $2.71 interest; it should therefore be paid 12 days before Aug. 0, or July 19.

429. Rules for averaging any account containing both debit and credit items.

Product Method.—I. *Assume as a convenient focal date, the last day of the calendar month preceding the date of the earliest item of the account; and multiply each item of the account by the difference in days between this focal date and the date at which such item falls due.*

II. *Separately add the obtained products of each side of the account, and divide the difference between the sum of the debit products and credit products by the balance of the account.*

III. *The quotient will denote the number of days between the focal date and equated date, which should be counted forward from the focal date (added to it) if the product balance and balance of account are on the same side of the account; or be* *counted backward from the focal date (subtracted from it) if the product balance and balance of account are on opposite sides.*

NOTE 1.—The date of the latest item, or any convenient subsequent date, may be taken as the focal date. The quotient, as obtained by the above rule, will then denote the number of days to be counted backward if both balances are on the same side of the account, or to be counted forward if on opposite sides—the reverse of what is required by the rule.

NOTE 2.—For fractions of a day, apply Note 1, **480**; and for fractions of a dollar, apply Note 2, **480**.

Interest Method.—I. *Select the focal date as directed in the preceding Rule; and find the interest of each item of the account for the time between the focal date and its due date.*

II. *Find the difference between the total debit interest and total credit interest, and divide three times this difference by one thousandth of the balance of the account. The quotient will denote the time in days between the focal date and equated date, which should be counted forward from the focal date if the interest balance and balance of account are on the same side, or counted backward from the focal date if the interest balance and balance of account are on opposite sides.*

NOTE 3.—The focal date may be selected by Note 1, in which case the process of counting backward or forward must be in agreement with said note.

EXAMPLES FOR PRACTICE.

Find the equated date for paying the balances of the following:

Dr. (No. 1.) J. A. HARRISON. *Cr.*

1886.				*1886.*			
May 12	To Mdse.,	$575	00	June 8	By Cash,	$150	00

Dr. (No. 2.) T. C. BRUFF & CO. *Cr.*

1885.				*1885.*			
Oct. 18	To Mdse.,	$848	00	Nov. 5	By Cash,	$180	00

Dr. (No. 3.) E. BENSEL & CO. *Cr.*

1886.				*1886.*			
Mar. 20	To Mdse.,	$695	00	May 8	By Cash,	$840	00

489

Dr. (No. 4.) G. A. HAMILTON. *Cr.*

1886.				*1886.*			
Jan. 9	To Mdse.,	$1390	00	Mar. 6	By Cash,	$250	00

Dr. (No. 5.) W. E. KENDALL & SON. *Cr.*

1886.				*1886.*			
June 25	To Mdse.,	$724	00	July 10	By Cash,	$518	00

Dr. (No. 6.) JOS. LATIMER. *Cr.*

1885.				*1885.*			
Feb. 2	To Mdse.,	$623	00	Apr. 15	By Cash,	$450	00

Dr. (No. 7.) O. FOREMAN & BRO. *Cr.*

1886.				*1886.*			
Apr. 5	To Mdse.,	$415	00	May 4	By Cash,	$275	00
May 21	" "	327	00	" 15	" "	112	00

Dr. (No. 8.) J. J. JANNEY. *Cr.*

1885.				*1886.*			
Nov. 8	To Mdse.,	$380	00	Jan. 3	By Cash,	$219	00
Dec. 2	" "	516	00	Feb. 25	" "	410	00

Dr. (No. 9.) JOHN B. HUNTON. *Cr.*

1885.				*1885.*			
June 15	To Mdse.,	$375	00	July 3	By Cash,	$150	00
July 8	" "	250	00	Aug. 12	" "	230	00
Sept. 23	" "	190	00	Oct. 9	" "	210	00

Dr. (No. 10.) R. W. MERCHANT. *Cr.*

1886.				*1886.*			
Aug. 4	To Mdse.,	$815	00	Aug. 15	By Cash,	$500	00
Sept. 30	" "	350	00	Sept. 6	" Mdse.,	250	00
Oct. 8	" "	480	00	Oct. 18	" Cash,	390	00

Dr. (No. 11.) J. H. BUTLER. **Cr.**

1884.				*1884.*			
Apr. 19	To Mdse.,	$175	18	May 8	By Mdse.,	$450	00
May 5	" "	325	25	June 2	" "	351	40
" 30	" "	536	30	" 17	" "	258	90
June 10	" "	275	40				

Dr. (No. 12.) L. J. IVES. **Cr.**

1885.				*1885.*			
Sept. 18	To Mdse.,	$925	12	Sept. 30	By Cash,	$325	00
" 25	" "	381	75	Oct. 15	" Sundries,	600	00
Oct. 3	" "	426	60	Nov. 20	" Cash,	500	00
" 19	" "	241	15				

Dr. (No. 13.) JNO. W. GRAVES. **Cr.**

1886.				*1886.*			
Jan. 21	To Mdse.,	$125	00	Feb. 6	By Cash,	$100	00
Feb. 15	" "	284	75	Mar. 18	" "	220	00
Apr. 7	" "	190	45	Apr. 3	" "	130	00
May 27	" "	325	65	" 23	" "	200	00

Dr. (No. 14.) E. C. BERKELEY. **Cr.**

1884.				*1884.*			
July 27	To Mdse.,	$358	30	Aug. 1	By Cash,	$350	00
Aug. 17	" "	215	75	Sept. 20	" "	275	00
Sept. 3	" "	426	40	Oct. 15	" "	250	00
" 19	" "	372	95				

Dr. (No. 15.) L. L. OSMOND. **Cr.**

1883.				*1883.*			
Feb. 15	To Mdse.,	$325	15	Apr. 3	By Cash,	$500	00
Mar. 9	" "	324	58	May 19	" "	375	00
" 30	" "	175	25	" 31	" "	250	00
Apr. 8	" "	215	45	June 8	" "	340	00
May 21	" "	348	30	July 15	" "	210	00
June 15	" "	273	25	Aug. 20	" "	125	00

Dr. (No. 16.) C. E. NICOL. *Cr.*

1884.				*1884.*			
May 12	To Mdse., 30 da.,	$700	00	June 18	By Cash,	$200	00
June 5	" " 60 "	450	00	Aug. 20	" "	400	00
July 8	" " 90 "	825	00	Sept. 3	" Note, 30 da.,	300	00
Aug. 6	" " Net	340	00				

Dr. (No. 17.) W. H. DEVON. *Cr.*

1884.				*1884.*			
June 15	To Mdse. @ 30 da.	$285	00	July 1	By Cash,	$200	00
July 18	" " " 10 "	340	00	Aug. 10	" "	300	00
Aug. 4	" " " 60 "	782	00	Sept. 1	" Note, 60 da.,	1000	00
" 17	" " " 90 "	945	00	" 10	" Cash,	400	00

Dr. (No. 18.) W. A. HEITMUELLER. *Cr.*

1884.				*1884.*			
Sept. 5	To Mdse @ 60 da.	$376	00	Oct. 9	By Cash,	$350	00
Oct. 9	" " " 30 "	428	00	Nov. 10	" "	275	00
Nov. 14	" " " 60 "	768	00	Dec. 10	" "	360	00
Dec. 7	" " " 10 "	470	00	*1885.*			
				Jan. 1	" "	200	00

Dr. (No. 19). W. H. PATRICK. *Cr.*

1884.				*1884.*			
Jan. 4	To Mdse. @ 10 da.	$550	00	Jan. 7	By Cash,	$360	00
" 5	" " " 30 "	620	00	Feb. 15	" "	290	00
" 7	" " " 60 "	128	00	Mar. 9	" "	550	00
" 10	" " Net	470	00	Apr. 18	" "	200	00
Feb. 15	" " " 90 da.	875	00	May 5	" "	125	00

Dr. (No. 20.) S. F. LACKLAND. *Cr.*

1884.				*1884.*			
Apr. 18	To Mdse., 3 mo.	$1375	00	July 2	By Note, 3 mo.	$1500	00
May 14	" " 60 da.	2725	00	Sept. 29	" Cash,	1250	00
June 16	" " 30 "	966	00	*1885.*			
July 7	" " 10 "	720	00	Jan. 15	" "	890	00
				" 20	" "	1200	00

Dr. (No. 21.) E. H. NORMAN. *Cr.*

1885.				*1885.*			
Feb. 4	To Mdse., 90 da.	$1284	64	Mar. 12	By Note, 60 da.,	$1298	12
Mar. 12	" " " "	1263	28	June 17	" Cash,	2480	00
April 8	" " " "	1480	37	July 24	" Note, 3 mo.,	2648	88
May 12	" " " "	2474	88	" 31	" Cash,	1000	00
June 8	" " " "	1820	83				

Dr. (No. 22.) M. E. WHITNEY. *Cr.*

1885.				*1885.*			
Apr. 13	To Mdse., 30 da.	$648	78	May 24	By Cash,	$542	36
May 23	" " 60 "	212	22	Aug. 24	" "	367	27
June 10	" " 30 "	284	17	Sept. 4	" "	763	74
July 28	" " 90 "	374	83	" 20	" "	650	63

Dr. (No. 23.) E. G. TAYLOR & Co. *Cr.*

1884.				*1884.*			
Sept. 3	To Mdse., 60 da.	$963	50	Oct. 1	By Cash,	$1000	00
" 9	" " 30 "	432	82	Nov. 4	" "	500	00
Oct. 13	" " 4 mo.	496	37	" 15	" Dft., 30 da.,	260	53
" 17	" " 10 da.	786	39	Dec. 17	" Cash,	1000	00
Nov. 3	" " 3 mo.	512	40				
" 24	" Cash,	1000	00	*1885.*			
Dec. 4	" Mdse., 90 da.	826	52	Jan. 5	By Acceptance, 60 d.	1250	00
" 21	" " Net.	973	86	Feb. 15	" Cash,	500	00
				Mar. 3	" Note, 3 mo.,	1000	00
1885.				Apr. 15	" Cash,	200	00
Jan. 15	To Acceptance, 3 mo.	1000	00	May 2	" "	1000	00
Feb. 3	" Mdse., 4 mo.	821	82				
Mar. 7	" " 30 da.	723	67				

AVERAGING ACCOUNTS SALES.

490. When a consignment arrives, the consignee pays the freight charges and other necessary items of expense, and frequently makes cash advances on the consignment, charging interest on all advances or retaining the net proceeds sufficiently long to offset the accrued interest.

Rem.—Except as to finding the date for the commission, the method of averaging accounts sales is similar to that of averaging other accounts involving both debit and credit items.

491. To find the equated date for paying the net proceeds of an account sales.

ILLUSTRATIVE EXAMPLE.

Find the equated date for paying the net proceeds of the following account sales of 255 bbl. of pork, received from E. Meredith & Co., Chicago, to be sold on their account and risk:

1886. April 12,	Sold 75 bbl.	@ 12.00,	Cash,	$900.00	
May 9,	" 80 "	@ 13.00,	"	1040.00	
June 17,	" 60 "	@ 13.50,	"	810.00	
Aug. 28,	" 40 "	@ 12.50,	"	500.00	$3250.00

Charges:

April 3,	To cash for freight and cartage,	$252.00	
" 3,	" cooperage	12.00	
" 3,	" advertising and insurance .	15.00	
June 20,	" cash advanced consignor .	300.00	
Aug. 28,	" storage	14.00	
	Commission at 4% (on $3250)	130.00	$723.00
	Net proceeds, ...		$2527.00

Explanation.—First find the equated date of sales by Interest Method, **482**, and take it (May 28) as the date of the total sales ($3250) and of the commission ($130). Then average the account by Interest Method, **489**, obtaining May 31, 1886, as the equated date for paying the net proceeds ($2527).

Solution—*Focal date, Apr.* 0, 1886.

	Date of Sales.	*Sales.*	*Interest.*
0	Apr. 12	$900	$3.60
1	May 9	1040	10.40 3.12
2	June 17 + 1	810	16.20 4.86
4	Aug. 28 + 2	500	25.00
		$3250	$63.18

3

3.25)$189.54(58 + da.

1625

2704

2600

Apr. 0 + 58 da = May 28, equated date of commission.

Mo.	*Dates.*	*Charges.*	*Interest.*
0	Apr. 3	252 . .	
	" 3	12 . .	
	" 3	15 . .	
		$279 . .	$ .28 (3 da.).
2 .	June 20+1	300 . .	6.00 (2 mo.). 2.10 (21 da.).
4 .	Aug. 28+2	14 . .	.70 (5 mo.).
1 .	May 28	130 . .	1.30 (1 mo.). 1.17 (27 da.). .04 (1 da.).
		$723	$11.59

Mo.	*Dates.*	*Sales.*	*Interest.*
1 .	May 28	$3250 . .	$32.50 (1 mo.). 29.25 (27 da.). 1.08 (1 da.).
		$3250	$62.83
		723	11.59
		$2527	$51.24
			3

$2.527) $153.720 (61.

Apr. 0+61 da. = May 31, equated date for paying the net proceeds ($2527).

Rule.—I. *If the sales have been effected at different dates, find the equated date of sales by* **482**, *and take the result as the date of commission and of total sales.*

II. *Write the total sales as a credit item, and the several charges as debit items; and find the equated date for paying the net proceeds or balance of the account by* **489**.

EXAMPLES FOR PRACTICE.

1. Find the equated date for paying the net proceeds of the following account sales:

BALTIMORE, MD., *May 1, 1884.*

Account Sales of Flour,

Sold for account of JNO. G. HIXON & CO.,

By J. J. DAVIES & SON.

1884.					
Jan. 18	150 bbl. to C. M. Bennett @ $6, cash	***	**		
Feb. 5	160 " " D. E. Pine @ $7, "	****	**		
Mar. 19	90 " " R. C. Wheeler @ $6, "	***	**		
Apr. 27	200 " " R. M. Weir @ $6^{11}, "	****	**	****	**
	Charges:				
Jan. 6	Freight and Drayage	$175	00		
" 6	Advertising and Insurance	25	00		
" 30	Advance on Consignment	600	00		
Feb. 18	Cooperage	16	70		
	Commission 2%.	***	**	****	**
	Net Proceeds per Average.			****	**

481

Arrange the following in the form of accounts sales, as in the preceding example, and find the net proceeds of each account, with the equated date of its payment:

2. J. B. Perrie, of Baltimore, sold for account and risk of J. E. Huntt, of Washington, the following: 1884—Mar. 15, 30 bbl. mess beef at $11.25; Apr. 2, 75 bbl. mess beef at $10.95; Apr. 18, 50 bbl. mess beef at $11.15; Apr. 30, 25 bbl. mess beef at $11.05. The charges are: Mar. 8, freight, $50; Mar. 9, cartage, $12; Apr. 30, storage, $9; commission, 2%.

3. On Oct. 15, 1883, C. J. Joyce, of New Orleans, sold for account of S. R. Reveira, of Rio Janeiro, 350 bags, 46257 lb., coffee at 11⅛¢; 119 bags, 15712 lb., at 11¾¢; 220 bags, 29043 lb., at 11⅞¢. The charges were: Oct. 10, drayage, $75; Oct. 13, advertising, $3.50; Oct. 15, sundries, $12.75; commission, 1¼%.

4. W. H. Rogers & Co., of Philadelphia, sold for account of R. O. Phelps, of Boston, the following consignment of Young Hyson tea: 1886—Mar. 3, 20 half-chests, 1210 lb., at 65¢; Apr. 12, 44 half-chests, 2640 lb., at 72¢; May 5, 130 half-chests, 7635 lb., at 68¢. The charges are: Feb. 24, freight, $25; Mar. 1, cartage, $2.25; Mar. 20, cash advanced on consignment, $250; commission, 3%.

5. W. Tippett, of New York, sold for account of R. Moss, of Chicago, the following: 1886—May 12, 9280 lb. bacon at 6⅛¢; May 25, 15370 lb. bacon at 6⅝¢; and June 18, 25396 lb. bacon at 6⅞¢. The charges were: May 15, cash advanced Moss, $500; May 2, drayage, $18; commission, 1⅛%.

CASH BALANCE.

492. The **Cash Balance** is the sum due on an account current at a specified date.

Rem. 1.—An *Account Current* is a statement of the gross debits and credits of each business transaction, within a certain period, between two houses; and may be rendered annually, semi-annually, quarterly, or monthly.

Rem. 2.—Interest is usually, though not invariably, charged on the balance of an account; that being regulated by a previous understanding between the parties thereto, or by the usage of each particular business.

Rem. 3.—To charge interest on periodical balances and carry the amount to a new account, is virtually compounding interest. Hence, the oftener accounts are closed, the more unfavorable it will prove to the debtor. Some merchants, however, who render accounts current oftener than once a year, do not carry the interest balance to the main column of the account until the end of the year.

493. **To find the cash balance of an account at a given date.**

ILLUSTRATIVE EXAMPLE.

Find the cash balance, Sept. 30, 1886, on the following account, including interest at 6%:

Dr. WM. CALLEN, *in % current with* H. A. RICE. *Cr.*

1886.				*1886.*			
Feb. 5	To Mdse., 30 da.,	$284	00	Apr. 30	By Cash,	$200	00
Mar. 7	" " " "	370	00	May 18	" "	300	00
Apr. 12	" " " "	590	00	Aug. 28	" "	120	00
May 24	" Cash,	840	00	Sept. 15	" "	124	00

First Solution.—*Interest Method—Assumed Rate 12%.*

Due dates. *1886.*	*Items.*	*Time.* mo.	da.	*Interest.*	*Credit dates.* *1886.*	*Items.*	*Time.* mo.	da.	*Interest.*
Mar. 7,	$284	6	27	$17.04 2.56	Apr. 30,	$200	5	3	$10.00 .20
Apr. 6,	370	5	27	18.50 3.33	May 18,	300	4	15	12.00 1.50
May 12,	590	4	21	23.60 4.13	Aug. 28,	120	1	3	1.20 .12
May 24,	840	4	9	33.60 2.52	Sept. 15,	124	0	15	.62
						$744			$25.64
	$2084			$105.28					
	744			25.64					
	1840			2) 79.64, interest balance, at 12%.					
	39.82			$39.82, " " " 6%.					
	$1879.82, cash balance.								

Explanation.—First add the term of credit to the date of each credit item, to find its due date; then compute interest on each item from its due date to the date of settlement (Sept. 30) at any convenient rate, say 12% as in the solution, obtaining $105.28 as the total debit interest, and $25.64 as the total credit interest. Subtract the less from the greater, obtaining $79.64 as the interest balance at 12% per annum; then divide by 2, obtaining $39.82 as the correct interest balance at 6%. Since both the interest balance ($39.82) and the balance of items ($1840) are on the same side, that is, owed by the same party (Wm. Callen), their sum ($1879.82) must be the cash balance due by Callen on Sept. 30, 1886.

If the interest balance had been on the opposite or credit side, it would have been in favor of Callen; and the cash balance against him would have been the *difference* between the balance of the items and the interest balance.

Second Solution.—*Product Method.*

Due dates. 1886.	*Items.*		*Days.*		*Products.*	*Credit dates.* 1886.	*Items.*		*Days.*		*Products.*
Mar. 7,	$284	×	207	=	58788	Apr. 30,	$200	×	153	=	30600
Apr. 6,	370	×	177	=	65490	May 18,	300	×	135	=	40500
May 12,	590	×	141	=	83190	Aug. 28,	120	×	33	=	3960
May 24,	840	×	129	=	108360	Sept. 15,	124	×	15	=	1860
	$2084				315828		744				76920
	744				76920						
Bal. of items=	$1840			6)	238908						
	39.82				39.818, balance of interest.						
	$1879.82 = Cash balance.										

Explanation.—Multiplying each item by the number of days between its due date and the date of settlement (Sept. 30), it is found that the total debit interest is equivalent to the interest of $1 for 315828 days, and the total credit interest to the interest of $1 for 76920 days. The balance of interest against Callen is therefore equivalent to the interest of $1 for 315828 — 76920, or 238908 days. At 6% per annum, the interest of $1 is ⅙ as many mills as there are days (**316**); hence, divide 238908 by 6, obtaining 39818 mills, or $39.818. Continue as already explained in the first solution.

Interest Rule.—I. *Compute the interest of each item from its due date to the date of settlement. The difference between the total debit interest and total credit interest will be the balance of interest.*

II. *To find the cash balance, add the balance of interest to the balance of items if both are on the same side; or subtract if on opposite sides.*

Product Rule.—I. *Multiply each item by the number of days between its due date and the date of settlement. The difference between the total debit product and total credit product will be the balance of products.*

II. *Divide the balance of products by 6 to find the balance of interest in mills, at 6% per annum; then find the cash balance as by II, Interest Method.*

NOTE 1.—If the due date of an item extends beyond the date of settlement, either subtract the interest for the intervening time from the interest on its own side or add it to the interest on the opposite side. The latter is the usual practice.

NOTE 2.—The cash balance can also be obtained by averaging the account

(**489**) to find the equated date for the payment of the balance, and computing interest on the balance for the time between the equated date and the date of settlement; adding the interest to the balance of the items, if the equated date is earlier than the date of settlement, or subtracting if the equated date is later.

EXAMPLES FOR PRACTICE.

1. Find the cash balance of the following account on Sept. 12, 1885, at 6% interest:

Dr. ROBERT TYLER *in account with* JOHN A. HILL. *Cr.*

1885.				*1885.*			
June 3	To Mdse.,	$900	00	June 15	By Cash,	$500	00
" 25	"	800	00	July 8	"	600	00
July 20	"	750	00	Aug. 12	"	850	00
Aug. 2	"	290	00				

2. Find the cash balance of the following account on July 1, 1886, at 6% interest:

Dr. W. R. WIRT *in account with* T. S. STONE. *Cr.*

1886.				*1886.*			
Jan. 1	To Balance,	$325	00	Feb. 1	By Cash,	$325	00
Feb. 8	" Mdse.,	450	00	Mar. 1	"	400	00
Mar. 20	" "	386	00	May 13	"	250	00
May 2	" "	530	00	June 5	"	300	00

3. Find the cash balance of the following account on Dec. 1, 1886, at 6% interest:

Dr. J. R. POPE *in account with* W. J. BLAIR. *Cr.*

1886.				*1886.*			
Feb. 5	To Mdse., 30 da.,	$284	00	Apr. 30	By Cash,	$200	00
Mar. 7	" 60 "	370	00	May 18	" "	300	00
Apr. 12	" 30 "	590	00	Aug. 28	" Note, 30 da.,	120	00
May 24	" Cash,	840	00	Sept. 15	" Cash,	124	00
June 1	" Mdse., 60 da.,	350	00	" 25	" "	350	00
" 20	" " 30 "	425	00	Oct. 3	" Note, 60 da.,	500	00
July 12	" " 10 "	860	00	" 18	" Cash,	250	00
" 25	" Cash,	175	00	Nov. 5	" Note, 30 da.	300	00
Aug. 3	" Mdse., 30 da.,	328	00	" 20	" Cash,	200	00
Sept. 8	" " 60 "	290	00				

4. The following account was settled on Aug. 15, 1884. Find the amount paid, interest 6%.

Dr. C. F. HAINES *in account with* A. S. POTTER. *Cr.*

1884.				*1884.*			
Jan. 18	To Mdse., 30 da.,	$450	00	Feb. 2	By Cash,	$300	00
Feb. 10	" " 10 "	872	00	Mar. 13	" "	1000	00
" 16	" " 60 "	920	00	Apr. 5	" "	200	00
Mar. 3	" Cash,	500	00	May 7	" "	500	00
Apr. 5	" Mdse., 30 "	638	00	June 19	" "	400	00
May 29	" " Net	494	00				

5. Find the amount due on the following account current on Sept. 1, 1885, including interest at 7%:

Dr. E. A. ELLICOTT *in % current with* R. S. BRISTOL. *Cr.*

		Days.	*Int.*	*Amts.*				*Days.*	*Int.*	*Amts.*	
1885.						*1885.*					
Jan. 1	Balance.			$1421	53	Jan. 10	Cash,			$1000	00
" 5	Mdse., 30 da.			187	60	Feb. 15	"			675	00
" 17	" 60 "			260	40	Mar. 7	Note, 90 da.			500	00
Feb. 9	Cash,			500	00	Apr. 15	Cash,			300	00
" 15	Mdse., Net,			786	50	May 20	"			260	00
Mar. 7	" 30 da.			176	25	June 24	"			200	00
Apr. 3	" 30 "			217	60	" 25	"			360	00
May 19	Bill, J. C. S.			75	85						
June 14	Mdse., 10 da.			126	84						
" 23	" 30 "			185	90						

PARTNERSHIP.

494. A **Partnership** is an association of two or more persons who combine their capital, skill, labor, or all of them, for the purpose of conducting business; each person sharing the gains or losses in a certain proportion as stipulated in their agreement.

REM. 1.—Such an association is called a *firm*, a *house*, or a *company*, and derives its name from the parties of whom it is composed; as J. E. Scott & Co., Wood & Fisher, etc. Each individual of the association is called a *partner*.

REM. 2.—An **Active Partner** is one who is publicly known as such, sharing in all the profits or losses of the business. A **Silent Partner** is one who, though actually a partner, is not publicly known as such, his name being concealed to avoid liability to the creditors of the firm. If, however, it should become known

494

Dr

380.00

Cr

2400.00

lly liable with
s one who fur-
lf liable for that
hed and legally
s. **A Nominal**
ho assumes the
artnership.

rty invested in

perty, including

oligations due by

or resources over
to any partner's

e the creditors

an its assets will

realizes a gain
ucy is called their

of the money
n.

ice between the

age sum in con-
both, have been

of a firm over

tal losses over

nent, setting
he amount of
ship, the pro-
proportion of

1404
137

9828
4212
1404

6)19.23.48
3.2058

31
13
18
28
31
31
30

137

profit or loss to each partner, and such other particulars as may be deemed necessary.

REM.—A partner may receive a regular salary in addition to his proportionate share of the profits for taking charge of the books, or for some other special duty. A salary to a partner is an indebtedness of the *firm*, which is placed to the credit of the salaried partner at the close of each month, at the close of the year, at its average due date, etc., according to the partnership contract. In the following examples, a salary is supposed to be charged at the close of the year.

501. Computations in Partnership are made in accordance with the following formulas:

(*1.*) *Present worth = resources — liabilities.*

(*2.*) *Present insolvency = liabilities — resources.*

$$(3.)\ \textit{Gain} = \begin{cases} \textit{greater capital at closing} - \textit{capital at commencing.} \\ \textit{insolvency at commencing} + \textit{capital at closing.} \\ \textit{greater insolvency at commencing} - \textit{insolvency at closing.} \end{cases}$$

$$(4.)\ \textit{Loss} = \begin{cases} \textit{greater capital at commencing} - \textit{capital at closing.} \\ \textit{capital at commencing} + \textit{insolvency at closing.} \\ \textit{greater insolvency at closing} - \textit{insolvency at commencing.} \end{cases}$$

$$(5.)\ \textit{Capital at commencing} = \begin{cases} \textit{greater capital at closing} - \textit{gain.} \\ \textit{less capital at closing} + \textit{loss.} \\ \textit{loss} - \textit{insolvency at closing.} \end{cases}$$

$$(6.)\ \textit{Capital at closing} = \begin{cases} \textit{less capital at commencing} + \textit{gain.} \\ \textit{greater capital at commencing} - \textit{loss.} \\ \textit{gain} - \textit{insolvency at commencing.} \end{cases}$$

$$(7.)\ \textit{Insolvency at commencing} = \begin{cases} \textit{gain} - \textit{capital at closing.} \\ \textit{less insolvency at closing} + \textit{gain.} \\ \textit{greater insolvency at closing} - \textit{loss.} \end{cases}$$

$$(8.)\ \textit{Insolvency at closing} = \begin{cases} \textit{loss greater than capital} - \textit{capital.} \\ \textit{less insolvency at commencing} + \textit{loss.} \\ \textit{greater insolvency at commencing} - \textit{gain.} \end{cases}$$

EXAMPLES FOR PRACTICE.

What is the gain or loss in 1885?

1. Capital Jan. 1, '85, $4150.75; capital Jan. 1, '86, $8175.09.
2. Insolvency Jan. 1, '85, $2375.80; capital Jan. 1, '86, $2846.30.
3. Insolv. Jan. 1, '85, $12420.38; insolv. Jan. 1, '86, $5407.25.
4. Capital Jan. 1, '85, $7250.19; capital Jan. 1, '86, $3251.28.
5. Capital Jan. 1, '85, $5260.80; insolvency Jan. 1, '86, $1380.30.
6. Insolv. Jan. 1, '85, $3243.25; insolv. Jan. 1, '86, $5124.50.

What is the capital or insolvency on Jan. 1, 1885?

7. Gain in 1885, $1825.50; capital Jan. 1, '86, $4760.50.
8. Loss in 1885, $3257.25; capital Jan. 1, '86, $5172.90.
9. Loss in 1885, $8254.20; insolvency Jan. 1, '86, $2125.
10. Gain in 1885, $5139.25; capital Jan. 1, '86, $4150.25.
11. Gain in 1885, $2730.45; insolvency Jan. 1, '86, $9140.25.
12. Loss in 1885, $2172.80; insolvency Jan. 1, '86, $5282.30.

What is the capital or insolvency on Jan. 1, 1886?

13. Capital Jan. 1, '85, $9284.16; gain in '85, $4264.15.
14. Capital Jan. 1, '85 $18275.32; loss in '85, $2769.40.
15. Insolvency Jan. 1, '85, $391.26; gain in '85, $2825.50.
16. Capital Jan. 1, '85, $1126.40; loss in '85, $3285.16.
17. Insolvency Jan. 1, '85, $1926.42; loss in '85, $1815.20.
18. Insolvency Jan. 1, '85, $1538.25; gain in '85, $1282.95.

19. The firm of Sonn and Jones commenced business with the following resources: Cash $13500, real estate $12000, merchandise $17000. They owed on personal accounts $7500, and on bills payable $10000. What was their net capital?

20. In the above example what was the net gain or loss at the end of the first year, if they then had in stock merchandise $21000, due them on personal accounts $13522, bills receivable $9527, real estate $17000; and they owed on personal accounts $14712, and on notes $12356.50?

21. The capital of a firm at commencing business, Jan. 1, 1885, consisted of cash $1000, merchandise $1200, bills receivable $900. At the end of that year they had on hand, cash $800, merchandise $1500, bills receivable $500, and owed on personal accounts $2500, and on outstanding notes $1800. What was the condition of the firm at the end of the year?

22. If, in Ex. 21, the firm had been permitted to continue in business at the end of 1885, and on Jan. 1, 1887, their assets were, cash $750.21, bills receivable, $1352, personal accounts $2126.74, merchandise $1722; and their liabilities were, personal accounts $1982, bills payable $1562; what was the condition of the firm on Jan. 1, 1887? What was the net gain or loss during 1886?

23. A firm commenced business as follows: Assets—cash $7500, merchandise on hand $4500, personal accounts due the firm $3200, bills receivable $875. Liabilities—due by the firm on personal ac-

counts $2500, bills payable $3600. At the end of the first year they had $8000 in cash, $5600 in merchandise, $4100 due the firm on personal accounts; and they owed $4000 on personal accounts, and $900 on outstanding notes. What was the net gain or loss of the firm?

24. On Jan. 1, 1885, J. R. Mills entered into copartnership with D. E. Snow, the former investing $2800 and the latter $2200. At the end of the year the firm had merchandise on hand worth $3282, cash in bank $4625, bills receivable $3500; and owed J. A. Watson $1200, and bills payable $1827. What was the net gain of the firm?

25. J. B. Perrie and H. H. Pfeiffer engaged in partnership, Perrie investing $9000 and Pfeiffer $7000. The firm's resources at closing consisted of cash $7095, real estate $7220, amount due by J. D. Gwynn $1680, merchandise $1340. The firm owes W. E. Gibbons $1249, and S. Blandford $585. What was the net loss of the firm?

26. J. B. Will and A. Meyers formed a copartnership, the former investing 5/8 of the capital, and the latter 3/8, and agreed to share the gains and losses in proportion to their investments. At the date of settlement the resources of the firm consisted of cash $7240, bills receivable $1420, real estate $11600, merchandise $9320; and their liabilities were bills payable $6200, due Benton & Co. $2720, due on bond and mortgage $2250. If the net gain was $2410, how much did each partner invest in the business?

27. Wells and McComas lost in the dry goods business $4342. Their resources are cash $1875, real estate $12500, merchandise $4700, personal accounts $2840, bills receivable $2400; and their liabilities are bills payable $3560, mortgage on real estate $8000, interest on mortgage $480, personal accounts $4625. If Wells invested 3/8 of the capital and McComas 5/8, what was the investment of each partner?

28. W. H. Devon and J. M. Green are partners in the coal business. At commencing, the firm was insolvent, Devon's proportion of the insolvency being 5/8 and Green's 3/8. At the close of the year, the net gain was found to be $11758; the firm's resources being, cash $4200, merchandise $13800, horses $800, wagons $400, carts $650, harness $325; and their liabilities, bills payable $3500, Jones & Co. $4800, Green & Bro. $3250, Anderson & Smith $4435, J. B. Ross $8432. What was each partner's insolvency at commencing?

29. S. R. Waite, H. Foley, and M. R. Wiley are partners in a firm. *At commencing* the firm was insolvent, Waite's insolvency being 2/5 of *the whole,* Foley's 2/5, and Wiley's 1/5. They agreed to share the gains

501

and losses equally. At closing, their books showed a net gain of $10779, their resources being, bills receivable $2417, cash $1560, merchandise $3320, real estate $4490, J. Nolan's account $6850, Miller & Co.'s account $2231; and their liabilities, notes and acceptances outstanding $1260, due to T. E. Taylor $3240, to H. Carter $3816, to W. E. Norman $2192, to J. E. Lowe $4581. What was each partner's insolvency at commencing? What was each partner's capital at closing?

502. To divide the gain or loss among partners in proportion to their investments, if contributed for equal periods.

ILLUSTRATIVE EXAMPLE.

A and B engaged in business, A investing $8500 and B $6000. The net profit for the first year was $2500, which was divided between them in proportion to their respective investments. What was each partner's share of the gain?

Solution.

$8500 + $6000 = $14500, total capital.

$$\begin{array}{r|l} & \overset{5}{\$\cancel{2500}} \\ \underset{29}{\$\cancel{14500}} & \$8500 \end{array} \qquad\qquad \begin{array}{r|l} & \overset{5}{\$\cancel{2500}} \\ \underset{29}{\$\cancel{14500}} & \$6000 \end{array}$$

($5 × 8500) + 29 = $1465.52–, A's profit. ($5 × 6000) + 29 = $1034.48+, B's profit.

Rule.—*Find each partner's gain or loss by the Rule of Three,* **443,** *taking the total profit or loss and each partner's investment as right-hand terms, and the total investment as the left-hand term.*

1. J. W. Langworthy, J. C. Shimer, and C. F. Dickinson formed a partnership, and invested respectively $8400, $9600, $10800. The net profit for the first year was $7200, which was divided in proportion to the partners' investments. What was each partner's share of the gain?

2. Three partners invested respectively $2900, $3400, and $4200 in business, and agreed to share the gains or losses in proportion to their investments. If the net profit is $3150, how much should be each partner's share?

3. Hodges and Creel's profit in a speculation was $2274. Hodges
502

invested $3200 and Creel $2485. In proportion to investment, how much of the gain should each partner receive?

4. J. E. Herrell and J. W. Bell are partners in the grain business. Herrell invested 8000 bushels of wheat, invoiced at 90 cents per bushel, and Bell's investment embraced personal accounts previously due him amounting to $500, bills receivable $800, cash $700, and a warehouse valued at $4500. Their profits for the year were $4425. In proportion to their investments, how much of the gain should each receive?

5. W. Russell and M. Florence are contractors, and engaged to build a bridge for a corporation for $40250. Russell furnished the material at a cost of $18240, and Florence paid mechanics' and laborers' wages to the amount of $15760, no other outlay being made by either partner. If the gain was divided in proportion to investment, how much of the $40250 should each receive?

6. D. P. Barnette and R. A. Brown engaged in business with a joint capital of $13100. What was the investment of each, if Barnette's gain was $1800 and Brown's $3440?

7. J. K. Reid and L. J. Haines engaged in business, the former investing $18000 and the latter $16000; and agreed to share the gains and losses in proportion to investment. At the end of the year their resources were $58000 and their liabilities $9000. If Reid was to receive a salary of $1500 per annum for taking charge of the books, and nothing was withdrawn by either during the year, what was the interest of each partner at the end of the year?

8. C. A. Joyce and S. Arnold are partners, the former investing $8000 and the latter $6000. The profits during the year were $2100, which they agreed to share in proportion to their respective investments. How much of the profit should each receive?

503. To divide the gain or loss among partners in proportion to their investments, if contributed for unequal periods.

ILLUSTRATIVE EXAMPLE.

J. W. Bond and N. A. Wright engaged in business on Jan 1, 1886, each investing $6000. On March 1, Wright made an additional investment of $3000, and Bond withdrew from the firm $1500; on July 1, Bond invested $2900, and Wright withdrew $3000. If the profits during the year, $4620, were to be divided between the partners in proportion to their investments, what was the share of each?

503

Solution.

From Jan. 1 to March 1 = 2 mo. Hence, \$6000 × 2 = \$12000 for 1 mo.
Withdrawal, 1500
" March 1 to July 1 = 4 mo. Hence, \$4500 × 4 = 18000 " " "
Investment, 2900
" July 1 to Jan. 1 = 6 mo. Hence, \$7400 × 6 = 44400 " " "
Bond's investment = \$74400 " " "

\$74400 ÷ 12 = \$6200, Bond's average investment through the year.

From Jan. 1 to March 1 = 2 mo. Hence, \$6000 × 2 = \$12000 for 1 mo.
Investment, 3000
" March 1 to July 1 = 4 mo. Hence, \$9000 × 4 = 36000 " " "
Withdrawal, 3000
" July 1 to Jan. 1 = 6 mo. Hence, \$6000 × 6 = 36000 " " "
Wright's investment = \$84000 " " "

\$84000 ÷ 12 = \$7000, Wright's average investment through the year.

\$6200 + \$7000 = \$13200, average investment of the firm for the year.

$$\begin{array}{r|l} & 35 \\ 1 & \\ \cancel{132} & \$\cancel{4620} \\ \$\cancel{13200} & \$\cancel{6200} \\ & 62 \end{array} \qquad\Bigg\|\qquad \begin{array}{r|l} 1 & 35 \\ \cancel{132} & \$\cancel{4620} \\ \$\cancel{13200} & \$\cancel{7000} \\ & 70 \end{array}$$

\$35 × 62 = \$2170, Bond's gain. ‖ \$35 × 70 = \$2450, Wright's gain.

Explanation.—Bond's capital on Jan. 1 (\$6000) remained unchanged to March 1, or for 2 months; and \$6000 invested for 2 months equals 2 times \$6000, or \$12000, invested for 1 month. On March 1, Bond withdrew \$1500, leaving \$4500 in the business, which remained unchanged to July 1, or for 4 months, and \$4500 invested for 4 months equals 4 times \$4500, or \$18000 invested for 1 month. On July 1, Bond invested \$2900, making his investment \$7400, which remained unchanged to the end of the year, or for 6 months, and \$7400 invested for 6 months equals 6 times \$7400, or \$44400 invested for 1 month. Hence, Bond's total investment equals \$12000 + \$18000 + \$44400, or \$74400 invested for 1 month, or an average investment of $\frac{1}{12}$ of \$74400, or \$6200 for the year.

Similarly, Wright's average investment for the year is found to be \$7000.

Divide the gains for the year (\$4620) between the partners in proportion to their average investments by Rule, **502**.

Rule.—I. *Multiply each partner's original investment by the months or days between the date of such investment and the date of his next withdrawal or investment.*

II. *Multiply the increased or decreased investment then in*

503

the business by the number of months or days between the date of such increase or decrease and the date of the next increase or decrease; and so continue to the date of adjusting the gains or losses.

III. *Divide the sum of the products thus obtained by the number of months or days in the period of adjustment to find each partner's average investment.*

IV. *Divide the gains or losses among the partners in proportion to their average investments by Rule,* **502**.

Note.—To avoid fractions, the gains or losses may be adjusted by the Rule of Three by taking the total gain or loss and each partner's average investment per month or day as the right-hand terms, and the average capital per month or day as the left-hand term.

Examples for Practice.

1. A, B, and C formed a copartnership, and invested respectively $9600, $8400, and $7200. At the end of 4 months, A invested $2000 additional, B $1400, and C $800. If the net gains during the year were $12800, and all gains and losses were to be divided in proportion to their average investments, what was each partner's share of the profits?

2. A and B are partners in business, A investing $3000 for 2 months, $9000 for 3 months, and $8000 for 5 months; B investing $7000 for 4 months and $6000 for 8 months. The partners are to share the gains and losses in proportion to their average investments, and settlements are to be made annually. If the net gain is $7280, what was each partner's share?

3. Wilson, Holmes, and Farr engaged in business on Jan. 1, 1886, Wilson investing $8000, Holmes $5000, and Farr $6000. On May 1, Wilson put in $2000 additional; on July 1, he withdrew $3000; and on Oct. 1, he put in $1500. On Feb. 1, Holmes withdrew $1000; on Apr. 1, he put in $4000; on Aug. 1, he withdrew $500; and on Sept. 1, he put in $3000. On July 1, Farr put in $2500 additional. At the end of the year the net gain was found to be $5000. What was each partner's share of the gain if it was divided in proportion to their average investments?

4. Harvey and Roberts engage in business Jan. 1, 1885, Harvey investing $5000, and Roberts $8000. On May 1, Harvey invested

[illegible]03

$2000 additional; and on June 1, Roberts withdrew $1000. On Aug. 1, they took in Williams as a third partner, who invested $7000. On Jan. 1, 1886, their net gain was $5000. What was the share of each partner?

5. A and B entered into partnership and agreed to share the gains and losses in proportion to their average investments. A furnished 4/9 of the capital, and B 5/9. Five months later, A withdrew 3/4 and B 4/9 of their respective investments. If the net gain during the year was $9250, what was each partner's share?

6. On Jan. 1, S. H. Merritt and J. L. Reeves formed a copartnership for one year, each investing $8000. On Apr. 1, Merritt withdrew $3200, and Reeves $1600; on Sept. 1, each withdrew $2400; and on Nov. 1, Merritt withdrew $900, and Reeves $700. At the end of the year, the partnership was dissolved with a capital of $6000. What proportion of the capital should each partner then receive?

504. To adjust the partners' accounts when the proportion of profit or loss is fixed, and interest is allowed on the excess and charged on the deficiency of each partner's required investment.

ILLUSTRATIVE EXAMPLE.

On Jan. 1, 1883, E. J. Ayres, R. W. Malley, and J. A. Manning entered into copartnership. It was agreed that Ayres should contribute 4/9 of the capital and receive 4/9 of the gains; that Malley should contribute 3/9 of the capital and receive 3/9 of the gains; and that Manning should contribute 2/9 of the capital and receive 2/9 of the gains. Interest at the rate of 10% per annum was to be allowed on the excess, and charged on the deficiency of each partner's required investment. A settlement was effected at the end of the year, at which time the net gain was found to be $3600. Find Ayres's and Malley's net interest, and Manning's insolvency on Jan. 1, 1884, if the following is a statement of each partner's account:

Dr. E. J. AYRES. *Cr.*

1883—Apr. 23, Withdrawal,	$1500	1883—Jan. 1, Investment,	$16000
" June 16, "	800	" Mar. 18, "	2400
" Aug. 17, "	900	" Oct. 20, "	3000
Total withdrawal,	$3200	Total investment,	$21400

504

Dr.	R. W. MALLEY.		*Cr.*
1883—July 28, Withdrawal,	$600	1883—Jan. 1, Investment,	$12000
" Dec. 4, "	800	" " 21, "	1800
		" May 17, "	600
Total withdrawal,	$1400	Total investment,	$14400

Dr.	J. A. MANNING.		*Cr.*
1883—Mar. 30, Withdrawal,	$6000	1883—Jan. 1, Investment,	$6000
" Sept. 5, "	4000	" Aug. 3, "	600
Total withdrawal,	$10000	Total investment,	$6600

Solution.

In estimating time in partnership settlements Jan. 1 is considered Jan. 0 (Note 2, 482).

Int. of $16000 from Jan. 1, '83, to Jan. 1, '84, at 10%	$1600.00	
" 2400 " Mar. 18, " " " " (288 da.)	192.00	
" 3000 " Oct. 20, " " " " (72 da.)	60.00	
Int. of Ayres's total investment, ...		$1852.00
Int. of $1500 from Apr. 23, '83, to Jan. 1, '84, at 10% (252 da.)	$105.00	
" 800 " June 16, " " " " (198 da.)	44.00	
" 900 " Aug. 17, " " " " (136 da.)	34.00	
Int. of Ayres's total withdrawal, ...		$183.00
" " average investment, . . .		$1669.00

Int. of $12000 from Jan. 1, '83, to Jan. 1, '84, at 10%	$1200.00	
" 1800 " Jan. 21, " " " " (344 da.)	172.00	
" 600 " May 17, " " " " (228 da.)	38.00	
Int. of Malley's total investment, ...		$1410.00
Int. of $300 from July 28, '83, to Jan. 1, '84, at 10% (156 da.)	$26.00	
" 800 " Dec. 4, " " " " (27 da.)	6.00	
Int. of Malley's total withdrawal, ...		$32.00
" " average investment, . . .		$1378.00

Int. of $6000 from Jan. 1, '83, to Jan. 1, '84, at 10%	$600.00	
" 600 " Aug. 3, " " " " (150 da.)	25.00	
Int. of Manning's total investment, . . .		$625.00
Int. of $6000 from Mar. 30, '83, to Jan. 1, '84, at 10% (276 da.)	$460.00	
" 4000 " Sept. 5, " " " " (117 da.)	130.00	
Int. of Manning's total withdrawal, . . .		$590.00
" " average investment, . .		$35.00

504

$1669 + $1878 + $35 = $3082, interest of firm's average capital.
$\frac{4}{9}$ of $3082 = $1369.78, interest of Ayres's required investment.
$\frac{3}{9}$ of $3082 = $1027.33, " Malley's " "
$\frac{2}{9}$ of $3082 = $684.89, " Manning's " "

$21400 − $3200 = $18200.00, Ayres's net investment, as per his account.
$\frac{4}{9}$ of $3600 = 1600.00, " proportion of the total profits.
$1669 − $1369.78 = 299.22, int. of Ayres's excess over required investment.
$20099.22, amount due Ayres, Jan. 1, 1884.

$14400 − $1400 = $13000.00, Malley's net investment, as per his account.
$\frac{3}{9}$ of $3600 = 1200.00, " proportion of the total profits.
$1878 − $1027.33 = 850.67, int. of Malley's excess over required investment.
$14550.67, amount due Malley, Jan. 1, 1884.

$10000 − $6600 = $3400.00, Manning's indebtedness, as per his account.
$684.89 − $35 = 649.89, int. of Manning's deficiency of required investment.
$4049.89, Manning's total indebtedness.
$\frac{2}{9}$ of $3600 = 800.00, " proportion of the total profits.
$3249.89, " net indebtedness, Jan. 1, 1884.

Explanation.—Separately compute the interest on the investments and withdrawals of each partner, and obtain the interest balance of each account, which will also be the interest of each partner's average investment. Then add the interest of Ayres's average investment ($1669), of Malley's ($1878), and of Manning's ($35), to find the interest of their total average investment, or of the firm's average capital ($3082). Since Ayres agreed to contribute $\frac{4}{9}$ of the capital of the firm, $\frac{4}{9}$ of the interest of the average capital of the firm, or $1369.78, must be the interest of Ayres's required investment; but the interest of Ayres's actual investment ($1669) is $299.22 greater than the interest of his required investment ($1369.78). Therefore the interest of this excess of investment ($299.22) should be placed to his credit as per agreement, which ($299.22) with his share of the profits ($1600) should be added to the balance of his account ($18200), to find the sum due him by the firm ($20099.22).

The sum due Malley is similarly obtained. Manning's indebtedness, as per account, is $1400; besides which he owed the remaining partners $649.89 as interest on the deficiency of his required investment. His total indebtedness would therefore have been $4049.89, were it not for his share of the gain ($800), which reduced his indebtedness to $3249.89.

Rule.—I. *Compute the interest on the investments and withdrawals of each partner from the time they were made to the date of settlement, and subtract the debit interest from the credit interest of each partner's account to find his net interest, or the interest of his net average investment* (see *a*, Note).

II. ***Add the balances of interest thus found to obtain the interest on the firm's net average capital*** (see *b*, Note) ***; and take such proportions of this result as each partner agreed to contribute to the capital of the firm.***

III. ***If the interest of any partner's net average investment exceeds the interest of his required investment, the difference will be the interest due to such partner; if less, the difference will be the interest due from such partner*** (*c*, Note).

IV. ***Find the capital or indebtedness of each partner at closing by Form. 6 or 8,*** **501.**

NOTE.—(*a*) Should any partner's total debit interest be greater than his total credit interest, the difference will be the interest of such partner's net average indebtedness to the firm; (*b*) which should be subtracted from the sum of the interest balances to the credit of the remaining partners, to find the interest of the firm's net average capital. (*c*) The interest of any partner's net average indebtedness to the firm should be added to the interest of his required investment to find the total interest due from such partner.

EXAMPLES FOR PRACTICE.

1. J. R. Cross and J. J. Davies engaged in business on Jan. 1, 1884, and agreed to invest equally and share equally in gains and losses. Interest at 8% was to be allowed on the excess, and charged on the deficiency of each partner's required investment. The following is a statement of each partner's account:

Dr. J. R. CROSS. *Cr.*

1884—Apr. 3, Withdrawal,	$3000	1884—Jan. 1, Investment,	$6000
" June 7, "	1500	" Mar. 14, "	4000
" Oct. 15, "	800	" July 3, "	3000
		" Sept. 18, "	5000

Dr. J. J. DAVIES. *Cr.*

1884—Mar. 8, Withdrawal,	$2000	1884—Jan. 1, Investment,	$4000
" Aug. 15, "	1000	" May 12, "	5000
" Sept. 30, "	1200	" July 16, "	2000
		" Oct. 5, "	1500

If the net gains during the year were $5800, what was the condition of each partner's account on Jan. 1, 1885?

504

2. H. Gould, J. Hunter, and S. W. Nichols formed a copartnership on Jan. 1, 1883, and agreed to invest equal sums, and share equally in gains and losses; interest at 9% per annum being allowed on the excess, and charged on the deficiency of each partner's required investment. H. Gould invested $8000 on Jan. 1, $6400 on Apr. 15, $4000 on May 20, $1600 on June 24; and withdrew $1200 on May 5, $1800 on Aug. 13, and $2400 on Nov. 16. J. Hunter invested $7000 on Jan. 1, $9000 on July 24, $1600 on Oct. 12; and withdrew $6000 on Mar. 26, $8000 on May 5, $5500 on June 14, and $1200 on Sept. 2. S. W. Nichols invested $7500 on Jan. 1, $6000 on Jan. 25, $4000 on Feb. 14, $2800 on May 25; and withdrew $2000 on Apr. 27, $3600 on June 4, and $800 on Aug. 2. If the net gain during the year was $6842.19, what was H. Gould's interest in the business on Jan. 1, 1884? How much was J. Hunter indebted to the remaining partners? How much was due S. W. Nichols from J. Hunter?

3. On Jan. 1, 1882, T. Herndon, W. Newman, and B. P. Dulin engaged in business, Herndon agreeing to furnish ½ of the capital and receive ½ of the gains; Newman and Dulin each to furnish ¼ of the capital and receive ¼ of the gains; and interest at 10% per annum was to be allowed on the excess, and charged on the deficiency of each partner's required investment. On Jan. 1, 1882, Herndon invested $8000; on Aug. 12, $6000; and on July 3, withdrew $3000. On Jan. 1, 1882, Newman invested $5000; and on July 3, withdrew $800. On Jan. 1, 1882, Dulin invested $5000; on Mar. 15, $2000; on June 18, $1500; on Sept. 1, $800; on May 1, he withdrew $1200; and on Sept. 9, $500. The net gain during the year was $5300. What was each partner's interest in the firm on Jan. 1, 1883?

4. L. A. Horn and J. R. McLane engaged in business on Jan. 1, 1884, each agreeing to furnish ½ the capital of the firm and to share ½ the gains and losses, McLane to receive a salary of $800 per annum for taking charge of the books; and interest at 8% per annum to be allowed on the excess, and charged on the deficiency of either partner's required investment. On Jan. 1, 1884, Horn invested $9000; on Mar. 1, $5000; on Aug. 3, $3000. On Jan. 1, 1884, McLane invested $8000; on July 20, $3000; on May 15, he withdrew $500, and on Oct. 8, $300. The net gains during the year were $6200. What was each partner's interest in the firm on Jan. 1, 1885? What would it have been if Horn had withdrawn $18000 on Dec. 1, 1884?

5. G. W. Hixon, R. Merritt, and W. Pickett entered into copart-

504

nership to invest equally, and share equally in gains or losses, and agreed to allow interest on each partner's excess, or charge interest on each partner's deficiency of required investment. At the date of adjustment, Hixon's account showed an excess of withdrawals of $900; of debit interest of $314.62. Merritt's excess of debit interest was $214.42; but his investments exceeded his withdrawals by $1200. Pickett's excess of debit interest was $312.80, and he overdrew his investment by $1400. If there was a net loss of $1800, what was the condition of each partner's account at the date of adjustment?

6. J. P. Smith, J. A. Harrison, and W. Yeatman formed a copartnership, Smith agreeing to furnish $\frac{6}{15}$ of the capital, Harrison $\frac{5}{15}$, and Yeatman $\frac{4}{15}$. At the end of the first year a settlement was effected, the books showing that Smith's account was overdrawn by $1500, Harrison's by $1200, and Yeatman's by $960. There was a net gain of $6000, which was shared in proportion to each partner's required investment. Interest was allowed on the excess, and charged on the deficiency of each partner's required investment. Smith's excess of debit interest was $124.62, Harrison's $120.81, and Yeatman's $98.07. How much was due each partner at the date of settlement?

ADDITIONAL EXAMPLES IN PARTNERSHIP.

7. Whitney and Norman are partners in the knitting business, Whitney investing $14000 and Norman $18000. Whitney took charge of the books and received for his services $1000 a year; and Norman superintended the business at a salary of $1200 a year. At the yearly settlement, after all the other expenses were paid, there remained but $2040 cash, or $160 less than the amount of the partners' salaries. What proportion of the cash should each partner receive if the gains and losses were to be equally divided?

8. S. R. White and J. Moore jointly purchased the steamer Excelsior for $90000. White paid $\frac{2}{3}$ of this amount, and Moore paid $\frac{1}{3}$. During the season White paid from his private funds, for wages, fuel, etc., $960, and drew out $300. Moore paid steamer's expenses, amounting to $680, and drew out $150. The partners realized a net gain of $8400, and then sold the steamer for $92000, receiving $46000 in cash, and a note for $46000, which White accepted

at 5% discount, to apply on his account. The gains and losses were divided equally. How much was due each partner at closing?

9. M. A. Dyer and C. E. Carr are partners in the furniture business. Dyer invested $3000 of the capital, and Carr invested $2000. Carr paid mechanics' wages during the year amounting to $2200. Dyer drew out $250, and bought material at a cost of $1720. The partners agreed to share the gains or losses equally. There was a net gain of $1860. Dyer purchased Carr's interest in the concern by paying him cash in full. How much should Dyer pay Carr?

10. The following statement was handed to an accountant for adjustment by a firm of two partners, who shared the gains and losses equally. A's total investments, $13246, and total withdrawals, $7381. B's total investments, $12863, and total withdrawals, $5369. Cash on hand, $3125.25; merchandise on hand per inventory, $4222.84; notes on hand, $375; personal accounts due firm per list, $1784.67; store fixtures, $1596; house and lot valued at $4500; horses and wagons, $520. The firm owed on notes outstanding $2575, and interest on same $172.49. C & Co's account against the firm was $170.52; F. E. H's, $140.19. There was a mortgage of $2000 on house and lot, and $65.18 interest due on same. The total purchases of merchandise amounted to $6128.07, and the total sales, $62324.33. The total expenses of the business were $2148.94. Upon examination the accountant observed that such a statement necessarily involved an error, and asked for the books, invoices, etc., of the concern. He found that all the accounts were correct, except the sales account. Was there a net gain or net loss in the business? What was each partner's net capital after adjustment, according to the correct statement of the accountant? What was the correct amount of sales?

11. Hale and Greer entered into copartnership equally insolvent. The resources of the firm at closing are: cash, $2440.60; bills receivable, $1644; personal accounts, $4234.73. The liabilities were: bills payable, $2149; due W. R. Small, $3241.67; due E. T. Joyce, $1340.33. The firm paid a private debt of $260 for Hale, and Greer paid sundry expenses for the firm, amounting to $375. There was a net gain of $2136.72. What was each partner's insolvency at commencing? What was each partner's net capital at closing?

12. Scott, Gordon & Hall are partners in business, and on date of settlement the books of the concern show that Scott drew out for private use $590, that the firm assumed a private debt of $930 for Gor-

don, and that Hall's account was overdrawn by $125. On the date of the last settlement each partner had an equal sum standing to his credit. The resources at closing were: cash, $8294; personal accounts, $1560; bills receivable, $4400; merchandise, $18220. The liabilities at closing were: bills payable, $7278; personal accounts due, $1365; interest on notes unpaid, $190. What was each partner's net capital at closing, there being a net gain of $2100?

13. The total liabilities of a firm, including partners' investments, were $18794.16, and the total resources, except the amount of cash on hand, were $12414.20. There was a net gain in the business of $1240.60. The cash book and amount of cash were locked up in the safe. If all the other accounts were correct, how much cash should be in the safe, or what balance of cash should the cash book exhibit?

14. George N. Mackenzie and A. L. Sonn are carpenters, and formed a copartnership to do job work. The gains and losses were to be divided equally. Each partner kept an account of the labor he performed, the collections he made, and the expenses he paid. Mackenzie's labor amounted, during the season, to $3264.70, and he paid expenses for the business amounting to $274.19. Sonn paid expenses amounting to $311.65, and his labor amounted to $2982.28. Mackenzie collected $2360.82, and Sonn collected $1874.27. At the date of settlement how should the partners adjust their accounts?

15. A firm engaged an agent to buy and sell merchandise at an annual salary of $600. During the year the firm delivered to the agent goods amounting to $30264, and advanced $1865 cash. The agent at different times purchased goods amounting to $18673, and the total sales made by the agent were $40234. At the date of settlement the agent had on hand goods amounting at cost price to $12316. What was the net gain or net loss? How much did the agent owe the firm?

16. R. Coe, W. May, A. Lewis, and C. Hill formed a copartnership. It was agreed that Coe was to receive ⅓ of the gains, May ¼, Lewis ⅕, and Hill ⅙. The remainder of the gains was to be placed to the reserve profit account, to be used for the contingencies of the business. It was agreed that each partner should invest $12000, on condition of receiving the above proportion of the profits, but Coe invested only $10000, and May $9000. There was a net gain of $18240. In consequence of the failure of both Coe and May to invest their required $12000, how should the gain be divided among all the partners?

APPENDIX.

THE STOCK EXCHANGE.

505. A **Stock Exchange** is an association of brokers for the purpose of buying and selling stocks, bonds, and other securities that may be listed upon the books of the exchange.

Rem. 1.—Stock Exchanges are established in all the larger cities; and each exchange is governed by its own rules and regulations.

Rem. 2.—At the regular sessions of the stock exchange, all the stocks are called in a certain order from a list; and as each kind of stock is called, bids or offers are made by "buyers" or "sellers."

Rem. 3.—Stocks are bought and sold "cash," "regular," "buyer 3, 30, or 60," or "seller 3, 30, or 60." Stocks are sold "cash" when they are to be delivered on the day of sale; and are sold "regular" when they are to be delivered at or before a certain hour on the day following the sale. If not otherwise specified, "regular" sale is always understood.

Rem. 4.—If stock is sold "seller 3," "seller 10," "seller 30," etc., the seller can, at his option, deliver it at any time within the days mentioned, but is not obliged to deliver it until the last of the 3, 10, or 30 days specified in the contract.

Rem. 5.—If stock is sold "buyer 20," "buyer 30," "buyer 60," etc., the buyer, at his option, can demand the stock at any time within the days mentioned, but cannot be compelled to receive and pay for it until the last of the 20, 30, or 60 days specified in the contract.

Rem. 6.—When the term of option is over 3 days, the buyer is required to pay interest at the legal rate, unless sold "flat"; and the seller to give one day's notice, unless sold "without notice." All dividends declared and paid on stock within the time specified in the contract, belong to the purchaser.

506. Stock privileges, other than "options," are classified as "Puts," "Calls," "Spreads," and "Straddles."

Rem.—Unlike options, persons securing these privileges are not obliged to take or deliver the stock named in the contract; and will simply lose the price paid for the privilege if not to their advantage to use it.

507. A **Put** is a contract which secures to the holder the privilege of delivering to the maker of the "put" a certain number of shares of a designated stock, at a specified price, at any time within the continuance of the privilege.

Rem.—"Puts" are purchased when stock is expected to decline to an extent greater than the cost of the privilege.

PUT CONTRACT.

New York, *Aug. 8, 1886.*

For value received, the Bearer may deliver to me Two Hundred Shares of the stock of the New York Central Rail Road Co. at one hundred and two per cent. of its par value, at any time within sixty days from this date.

The undersigned is entitled to all regular or extra dividends for which transfer books close during said time.

Expires 2 P.M. Russell Sage.

508. A **Call** is a contract which secures to the holder the privilege of buying from the maker a certain number of shares of a designated stock, at a specified price, at any time within the term specified in the privilege.

Rem.—A "Call" is the reverse of a "Put," and is therefore purchased when stock is expected to advance to an extent greater than the cost of the privilege.

CALL CONTRACT.

New York, *June 25, 1886.*

For value received, the Bearer may call on me for Three Hundred Shares of the Stock of the Western Union Telegraph Co., at Seventy-three per cent. of its par value, at any time within thirty days from this date.

The bearer is entitled to all regular or extra dividends declared during the continuance of this contract.

Expires 2 P.M. Cyrus W. Field.

509. A **Spread** is a contract which secures to the holder the privilege of either buying or selling a certain number of shares of a designated stock, at a specified price, within a limited time, without the obligation of taking or delivering it.

Rem. 1.—The holder of a "spread" has the advantage of a *double* privilege, or that of a "put" and a "call" combined. Hence, to realize a profit on "calling" or "putting" stock to the maker of a spread, the price of the stock must either advance or decline to an extent greater than the cost of the privilege.

Rem. 2.—In an active market, profits on a "put," "call," or "spread" are secured by selling the contract at a premium, even before the advance or decline should reach the price named in the contract.

510. A **Straddle** is a contract which secures to the holder the privilege of buying or selling, within a limited time, a certain number of shares of a designated stock, not only at the price mentioned in the contract, but also at the market price of the stocks at the date on which the privilege was purchased.

Rem.—Any fluctuation in the price of stocks which will more than cover the cost of the privilege, will inure to the advantage of the holder of a "straddle."

511. **Commission** is the charge made for buying or selling stocks, bonds, or other securities. It is usually $\frac{1}{8}\%$ on the par value of the stock bought or sold.

Rem. 1.—The cost of Puts or Calls is 1% of par and a commission of $\frac{1}{16}\%$ of par, in addition to the regular commission of $\frac{1}{8}\%$ of par for buying or selling the stock specified if the privilege of "calling" or "delivering" it is exercised.

Rem. 2.—The contract price of stock in a Call is usually from 2% to 5% of par greater than the market price at the date of contract; and the contract price of a Put is from 2% to 5% of par below the market price.

512. A **Margin** is a deposit of cash or its equivalent in collaterals, made with a broker by one who wishes to speculate in a certain kind of stock, to secure the broker against loss while "carrying" such stock.

Rem.—The usual margin required is 10% of the par value of the stock purchased. As the capital needed to operate in stocks is furnished by the brokers, they charge legal interest on the sum which they expend in purchases, and allow interest on all margins deposited.

513. A **Bull** is an operator who has bought stock in expectation of a rise. The operator is then said to be "long" of stock.

Rem.—Bulls are charged with interest on the stocks which they carry, and hence desire to advance the market price to enable themselves to sell out or "unload" at a profit.

514. A **Bear** is an operator who has sold stock, which he does not own, for future delivery. He is then said to be "short" of stock.

Rem.—Operators who are short of stock pay no interest. They desire to depress the market price before their contracts expire, that they may secure the stock at the depreciated price and sell it at their contract price, the difference minus the charges being their profit.

515. A **Corner** is a term applied to the buying up, by a few operators, of the greater portion of the stock of a corporation, for the purpose of creating a deficiency in the market and a consequent advance in its market price to the point at which they desire to sell.

516. The following are the principal words, abbreviations, and phrases used at the Stock Exchange other than those already explained:

1. b 10; buyer's option to purchase within 10 days.

2. b 30 w n; buyer's option to purchase within 30 days, without notice.

3. s 60; seller's option at 60 days.

4. s 30 w n; seller's option at 30 days, without notice.

5. b c; "between calls"; stock transaction after the first call and before the second call of the Stock Exchange.

6. a c; "after calls"; stock transaction after the second call.

7. Ex. div.; "without dividend"; seller retaining the privilege to collect a recently declared dividend on the stock sold.

8. Ex. coup.; "without coupons"; seller having the privilege of detaching maturing interest coupons before delivery of the bonds quoted.

9. b f; "buyer flat"; purchaser waiving interest on options.

10. b 20 f; buyer's option at 20 days, without interest.

11. collaterals; securities deposited as a pledge for money borrowed.

12. hypothecating; to put up collaterals in pledge.

13. cover; to fill an order for stock which had been sold "short."

14. unload; to sell stock which has been held for a rise.

15. 1st m bds, 2d m bds; first mortgage bonds, second mortgage bonds.

16. r bds, g bds; registered bonds, bonds payable in gold.

17. pref; preferred stock (**379**).

18. conv. "convertible," bonds convertible into stock.

19. Pool or *Syndicate;* combination of persons who unite their capital and operations to secure and maintain control of certain stock and manipulate it for mutual profit.

20. Difference; variation between the contract price of stock and its market price. Instead of an actual delivery or receipt of the stock specified in a contract, brokers usually pay the "difference" if against them, or receive the "difference" if in their favor.

21. Point; special, important information respecting the future of certain stocks, supposed to have been acquired from a reliable source.

22. Wash, washed sale; a pretended sale by one broker to another, to give inactive stock the appearance of activity, and thus stimulate outsiders to deal in it.

23. Stop order; an order to buy or sell stock when it fluctuates to certain limits.

24. Limited order; an order to buy or sell stock at a specified price.

For definitions of other terms, see **Stocks and Bonds, 371** to **382,** *inclusive.*

EXAMPLES FOR PRACTICE.

517.—1. I sell through a broker 100 shares of Chicago and North Western R. R. stock at 116⅞, and 50 shares of Central Pacific R. R. stock at 42⅛. What is the sum due me from the broker, brokerage ⅛%?

Solution.

Cr.	100 sh. Chi. and N. W. stock @ 116⅞	$11687.50	
	50 " Central Pacific " 42⅛	2106.25	
		$13793.75	
	⅛% Commission	18.75	
	Bal. due		$13775.00

2. I bought 200 shares of N. Y. C. R. R. stock at 103⅜, and 150 shares Mich. Cent. R. R. stock at 74¼. What will be the amount required to pay for the same, brokerage ⅛%?

Solution.

Dr.	200 sh. N. Y. Central stock at 103⅜	$20675	
	150 " Mich. Central " 74¼	11137.50	
	⅛% Commission	43.75	
			$31856.25

3. On June 1, I purchased through a broker 100 shares of Lake Shore R. R. stock at 83⅞, and deposited $1000 as margin. On June 19, the stock was sold at 86⅜. What was my profit, allowing ⅛% for brokerage, and 6% interest on purchase and margin?

Solution.

Cr.	June 1,	Margin		$1000.	
	" 19,	Interest for 18 da. on $1000		3.	
	" "	100 sh. L. S. stock at 86⅜	$8637.50		
		Less ⅛% brokerage	12.50	8625.	$9628.
Dr.	June 1,	100 sh. L. S. stock at 83⅞		$8387.50	
		⅛% brokerage		12.50	
	" 19,	Interest, 18 da. on $8400		25.20	8425.20
		To Credit Bal.			$1202.80
		Margin			1000.
		Profit			$202.80

4. On July 1, a broker purchased for account of J. S. Smith 200 shares Louisville and Nashville R. R. stock at 43⅝, 30 days later sold 100 shares at 41⅛, and on Aug. 18, sold the remainder at 39¾. What was the loss, interest 6%, margin $2000, and brokerage ⅛% ?

Solution.

Cr.	July 1, Margin		$2000.	
	" 31, 100 sh. L. S. stock at 41⅛	$4112.50		
	Less ⅛% brokerage	12.50	4100.	
	Aug. 18, 100 sh. L. & N. stock at 39¾	$3975.		
	Less ⅛% brokerage	12.50	3962.50	
	" " Interest, 48 da. on $2000	$16.		
	" " " 18 " " 4100	12.30	28.30	$10090.80
Dr.	July 1, 200 sh. L. & N. stock at 43⅝		$8725.	
	Brokerage ⅛%		25.	
	Aug. 18, Interest, 48 da. on $8750		70.	8820.
	To Credit Bal.			$1270.80
	Loss ($2000 — $1270.80) =	729.20		

5. A speculator being "bearish" on Western Union Telegraph stock, gave an order to his broker to sell 200 shares "short." The order was executed on June 12 at 69⅝, and $2000 deposited as margin. On June 27, the sale was covered at 66⅜. What was the speculator's profit, brokerage ⅛%, interest 6%?

Solution.

Cr.	June 12, 200 shares W. U. at 69⅝	$13925.	
	Less ⅛% brokerage	25.	$13900.
	" " Margin	$2000.	
	" 27, Interest, 15 da. on $2000	5.	2005.
			$15905.
Dr.	" " 200 sh. W. U. at 66⅜	$13275.	
	⅛% brokerage	25.	13300.
	To Credit Bal.		$2605.
	Less Margin		2000.
	Profit		$605.

Note.—Interest is not charged on "short" sales, but a premium is frequently charged when a particular stock becomes scarce and not readily borrowed. The broker who borrows deposits the market price of the stock with the party who lends it; and in case of an advancing market, the lender is privileged to call for additional margin. In calling or delivering borrowed stocks, it is customary to give one day's notice.

6. On May 19 I purchased 300 shares Reading R. R. stock at 22⅜, b 60, and sold the same on June 13 at 27⅝, s 30. A settlement was made on June 28. What was the sum due from my broker, margin $1500, interest 6%, and commissions ⅛%?

Solution.

Cr.	May 19, Margin		$1500.	
	June 28, Interest, 40 da. on $1500		10.	
	" " " 15 da. on (27⅝ × 300) =		20.72	
	" " (27⅝ − 22⅜) × 300		1575.	
			$3105.72	
	Less ⅛% brokerage on 300 sh.		37.50	$3068.22
Dr.	May 19, 300 sh. at 22⅜	$6712.50		
	" " ⅛% brokerage	37.50	37.50	
	June 28, Interest, 40 da. on $6750		45.	82.50
	Credit Bal.			$2985.72

$2985.72 − $1500 = $1485.72, profit.

7. A speculator paid $200 for a "put" at 30 days from Apr. 12, on 200 shares of Fort Wayne, at 95¾, and on May 3, delivered the stock at 91⅞. What was his profit, the charges for settling being ⅛% commission on stock, and $\frac{1}{16}$% commission on put?

Solution.

Gain (95¾ − 91⅞) × 200		$775.
Cost of Put	$200.	
$\frac{1}{16}$% Com. on Put	12.50	
⅛% Com. for settling	25.	237.50
Net gain,		$537.50

NOTE.—No interest is allowed on "puts" or "calls"; and the charge for settling is ⅛% on the par value of the actual number of shares. In most of these transactions, the difference is simply paid over, instead of an actual delivery of stock.

8. I paid $300 for a "call" for 30 days from July 10, on 300 shares of New Jersey Central, at 80⅜, and on Aug. 5, called the stock at 85⅛. What was my profit, allowing ⅛% commission for settling and $\frac{1}{16}$% commission on the "call"?

Solution.

Gain (85⅛—80⅜) × 300 =		$1425.
Cost of Call	$300.	
$\frac{1}{16}$% Com. on Call	18.75	
⅛% Com. for settling	37.50	356.25
Net gain,		$1068.75

9. Find the balance due and gain on the following:

Dr. W. E. KENDALL.

1886.						
May	2	To 100 sh. Penn. R.R. @ 50¼	$*****	**		
"	"	" 300 " W. U. Tel. @ 102	*****	**		
"	"	" 400 " Lake Shore @ 98¾	*****	**		
"	"	" ⅛% Com. 800 sh.	***	**	$******	**
"	29	" ** da. interest at 6%			***	**
					$******	**

Cr.

1886.						
May	2	By Cash (Margin)	$8000	00		
"	29	** da. interest at 6%	**	**	$****	**
"	"	" Sale 100 sh. P. R. R. @ 52⅞	****	**		
"	"	" " 300 " W. U. Tel. @ 105¼	*****	**		
"	"	" " 400 " Lake Shore @ 99¼	*****	**		
			$******	**		
"	"	Less ⅛% Commission	***	**	*****	**
					$******	**

10. Bought a "call" on 200 shares Pacific Mail at 40¼, March 20, on which there was a cash dividend declared March 25, of 5%. Called *the stock,* April 17, and sold my stock at 45⅛. What was my profit?

11. What is the balance and loss on the following account?

Dr. SMITH & MCDOUGAL.

1885.					
Nov. 12	To 150 sh. Can. Southern @ 47⅛	$*****	**		
" "	" 200 " Union Pacific @ 94⅛	*****	**		
" "	" 500 " Mich. Central @ 90	*****	**		
" "	" ⅛% Commission	***	**	$******	**
" 30	" ** da. interest at 6%			***	**
				$******	**

Cr.

1885.					
Nov. 12	By Cash (Margin)	$8500	00		
" 30	" ** da. interest @ 6%	**	**	$*****	**
" "	" Sale 150 sh. Can. S. @ 45¾	****	**		
" "	" " 200 " Un. Pac. @ 93⅛	*****	**		
" "	" " 500 " Mich. Cent. @ 87¼	*****	**		
		$******	**		
" "	Less ⅛% Commission	***	**	*****	**
				$******	**

12. My broker bought for me, December 19, 1885, 300 shares Wabash & Pacific, pref., at 56¾, and sold the same, January 3, 1886, at 60⅜. What was my profit?

13. Deposited, August 19, with my broker, $6000 for the purchase of 200 shares Rock Island at 105¼, 200 shares N. Y. Central and Hudson at 102⅛, and 200 shares Union Pacific at 93⅝. On October 5, Rock Island was sold at 107⅛, N. Y. Central and Hudson at 104, and the Union Pacific at 92⅛. There was a cash dividend of 4% declared, September 12, on Rock Island. What was my gain?

14. On April 8, a broker purchased, on his own account, 200 shares Michigan Central at 90¼, s. 30, on which a dividend of 4% was declared April 15. He received the stock May 2, paid interest on the same, and was allowed interest on the dividend. What sum did he pay in settlement?

15. A broker sold on his own account, September 8, 300 shares

Union Pacific at 94⅛, s. 30. On September 30, a 5% dividend was paid. He delivered the stock October 8. How much did he receive in settlement ?

16. A speculator purchased, through his broker, a "spread" on 100 shares of Michigan Central, with instructions to use the privilege as a margin. The broker secured the privilege at 88 for the "put" and 90 for the "call." When the stock advanced to 91, the broker sold it "short" and placed the sales to the credit of the speculator. Michigan Central afterwards declined to 86, and the broker bought in the 100 shares at that price, so as to cover the "*short*" sale at a profit. When the stock declined still further, the broker bought 100 shares of same at 84 and afterwards sold it at 85¾. Upon a still further decline, another 100 shares of same were purchased at 84, and again sold at 87. Before the contract expired, the stock again declined to 86, when it was "put" at 88 and the account closed. What profit did the speculator realize ?

17. On May 12, I purchased through my broker 100 shares of railroad stock at 107⅛, and on June 3 a stock dividend of 60% was paid. I then sold my entire stock at 105⅛. How much did I gain by the transaction, allowing the usual commissions ?

18. On April 12, 1886, I purchased through my broker a "straddle," and paid 3⅛% premium for the privilege. The following is a copy of the contract :

NEW YORK, *April 12, 1886.*

The bearer may put or deliver to me 100 shares of the Capital Stock of the Western Union Telegraph at 102⅜ per cent. par value, any time within 30 days from date.

Or the bearer may, at his option, call on me for 100 shares of the above named stock at 102⅜, any time within 30 days.

All dividends paid during that time go with the stock.

WM. ALLEN MILLER.

The stock was quoted on May 10, 1886, at 107¼, at which price and date my broker sold the same and "called" it for delivery. What was my net profit, allowing the usual commissions ?

19. May 18, 1885, I bought 300 shares Canada Southern at 69¾, and July 11, 1885, sold the same at 68⅛. There were paid, meanwhile, two cash dividends of 5% each and a stock dividend of 60%. I sold *the new stock*, or script, at 54⅜. What was my net profit, allowing the usual commissions on stock and script, interest on purchase, but no interest on cash dividends ?

20. A broker having purchased more stocks than he could carry for his customers was obliged to "*make a turn,*" *i. e.,* to sell for *cash* and buy back *regular*. He consequently sold 400 shares New Jersey Central at 82 and 400 shares Delaware and Hudson Canal at 74. He bought back 400 shares New Jersey Central, regular, at 82⅞, and 400 shares Delaware and Hudson Canal at 73½, regular. What did the "*turn*" cost the broker ?

RAPID ADDITION.

518. To add by grouping figures into convenient sets.

ILLUSTRATIVE EXAMPLE.

Explanation.—To add rapidly, it is necessary to acquire facility in grouping the figures of a column into convenient sets, whose sum can be determined at a glance. The greater number of skilled accountants add but one column at a time; yet, by combining two or more figures whenever possible, and adding these combinations instead of the figures of which they are composed, results are obtained so quickly as to excite the surprise of those unacquainted with the process. Thus: to add the columns herewith appended, group into convenient sets (which are here enclosed in braces to be better understood by the pupil, but which in practice should be discarded) and add the numbers represented by these sets; as follows, 7+8+9+8+9+8+7+8+10= 74 units, or 7 tens and 4 units. Carry the 7 tens to the tens' column, obtaining 7+9+8+8+8+9+10+7+8+9 = 88 tens, or 8 hundreds and 8 tens. Carry the 8 hundreds to the hundreds' column, obtaining 8+9+8+9+8+8+9+10+9+8+10 = 96 hundreds, making the sum of the three columns equal to 9634.

Operation.

3	10	6	9	2	10
2		3		5	
1		8		3	
4		5	7	6	8
3	8	2		2	
5		1	10	7	
9		4		4	8
7	10	5		1	
3		3	9	3	
9		4		5	9
6	8	2		4	
2		8		6	8
8		3	8	2	
3	9	5		3	9
2		1	8	5	
4		2		1	
8		5		8	
6	9	2	9	3	7
3		7		4	
96		3		4	

518

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
3	7	5	41	31	85	253	516
8	3	2	58	27	54	138	273
6	4	1	34	62	48	847	832
5	8	8	72	16	32	413	127
2	5	2	67	75	76	272	983
7	3	7	25	83	27	765	275
1	2	1	13	28	63	994	764
5	7	6	92	54	84	283	353
3	2	2	84	12	12	812	626
8	1	8	38	47	41	647	189
6	4	1	12	83	38	733	841
2	8	4	79	35	29	528	237
1	3	5	23	21	75	264	723

(9.)	(10.)	(11.)	(12.)
224639865	3878945	72813456	538193716
442314143	2127624	29738268	445362437
343823234	5622346	46382759	758276354
827231421	8246393	58349271	394583725
214482178	2933528	43627183	876345678
145358723	7285342	29148364	691827364
528442837	3424637	75314627	253458926
393214253	4379464	28392673	514875343
724283645	5232521	84736829	482692736
272828432	3743255	41538294	854829176
489273229	6122732	32839267	382453826
162791318	1898326	75482739	623432714
323185986	2313652	34265873	255783468
873923882	3635218	82895852	987375384
418146179	5252867	24376437	142839267
152439652	9718343	65843745	483948945
237397254	2383734	46278426	827658372
689281543	6522425	58745954	586573486
341813437	3274612	93426374	712843943
734252154	7432536	35786453	347891658
125658415	8256541	87438265	439247386

518

RAPID SUBTRACTION.

519. To find the remainder in one operation, when a minuend is to be diminished by the sum of two or more subtrahends.

ILLUSTRATIVE EXAMPLE.

A merchant had bought 7135 bushels of corn, of which he sold 1204 bushels to A, 934 bushels to B, 796 bushels to C, and 1439 bushels to D. How many bushels had he remaining?

Explanation.—Write the minuend (7135) and draw a line underneath, then write the several subtrahends, placing units under units, tens under tens, etc. Then add the units' orders of the subtrahends, producing 9+6+4+4=23. The additional units required to make the right-hand figure of this result (23) correspond with the units' order of the minuend being 2 (23+2=25), 2 will be the units' order of the final remainder. Carry 2 tens to the sum of the tens' order of the subtrahends, and add, producing 2+3+9+3+0=17 tens. The additional tens required to make the right-hand figure of this sum (17) correspond with the tens' order of the minuend (3), being 6 (17+6=23), 6 will be the tens' order of the final remainder. Proceeding in this manner, it will be found that the sum of the hundreds' column (24) lacks 7 (24+7=31) of corresponding with the hundreds' order of the minuend (1); and the sum of the thousands' column (5) lacks 2 (5+2=7) of corresponding with the thousands' order of the minuend (7). Therefore 7 is the hundreds' order, and 2 the thousands' order of the final remainder, and 2762 bushels of corn is the total remainder.

Solution.

```
7135
————
1204
 934
 796
1439
————
2762
```

Find the balances of the following ledger accounts:

(1.)		(2.)		(3.)	
Dr.	*Cr.*	*Dr.*	*Cr.*	*Dr.*	*Cr.*
$7143.28	$1456.30	$3127.48	$7124.58	$2783.40	$4121.38
273.82	283.95	286.32	8296.25	251.31	5938.24
9549.37	2736.72	750.45	432.30	1586.90	726.35
4378.62	45.34	1430.15	50.80	732.35	4829.60
529.15	383.27	8274.20	1437.15	4829.17	2748.42
75.32	987.36	326.75	2137.80	1238.25	3214.50
918.24	2478.31	2183.92	450.16	2874.32	291.32
46.38	(Bal.)	7189.35	(Bal.)	(Bal.)	450.38
$22914.18	$22914.18	$23568.62	$23568.62	$22320.19	$22320.19

RAPID MULTIPLICATION.

520. To multiply when the multiplier is 11, and the multiplicand consists of two figures whose sum is less than 10.

ILLUSTRATIVE EXAMPLE.

Multiply $35 by 11.

Explanation.—3 + 5 = 8; and 8 placed between 3 and 5 produces $385, the product. Hence,

Rule.—*Between the two figures of the multiplicand write their sum.*

NOTE.—When the sum of the two orders of the multiplicand is more than 10, add the excess to the left-hand order.

EXAMPLES FOR PRACTICE.

Multiply		Multiply	
1. $52 by 11.	4. $53 by 11.	7. $16 by 11.	10. $45 by 11.
2. $72 by 11.	5. 25 rods by 11.	8. $81 by 11.	11. $34 by 11.
3. $36 by 11.	6. 18 miles by 11.	9. $27 by 11.	12. $62 by 11.

521. To multiply any number by 11.

ILLUSTRATIVE EXAMPLE.

Multiply 4762 pounds by 11.

Explanation.—Take the units' figure of the multiplicand (2) as the units' figure of the product; and for the remaining figures of the product take the units' figure (2) plus the tens' figure (6), obtaining 8; then the tens' figure (6) plus the hundreds' figure (7), obtaining 13; write 3, and carry 1 to the hundreds' figure (7) plus the thousands' figure (4), obtaining 12; write 2, and carry 1 to the thousands' figure (4), obtaining 5; making the total product 52382 pounds. Hence,

Rule.—*To the left of the units' order, write successively the units plus the tens, the tens plus the hundreds, the hundreds plus the thousands, etc., carrying as usual, and terminating at the left with the highest order, or the highest order plus the carrying figure.*

EXAMPLES FOR PRACTICE.

Multiply
1. $5136 by 11.
2. $72638 by 11.
3. $514832 by 11.

Multiply
4. $37281.05 by 11.
5. $41268.45 by 11.
6. $3275.75 by 11.

Multiply
7. $48263.25 by 11.
8. $94873.30 by 11.
9. $15238.65 by 11.

522. To multiply when the multiplier consists of two figures and its units' order is 1.

ILLUSTRATIVE EXAMPLE.

Multiply $387 by 41.

Explanation.—Take the units' figure of the multiplicand (7) as the units' figure of the product, and for the remaining figures of the product take 4 times the units' figure (7), plus the tens' figure (8), obtaining 36. Write 6, and carry 3 to 4 times the tens' figure (8), plus the hundreds' figure (3), obtaining 38. Write 8, and carry 3 to 4 times the hundreds' figure (3), obtaining 15; making the complete product $15867. Hence,

Rule.—*Take the units' order of the multiplicand as the units' order of the product; and multiply the tens' order of the multiplier into each order of the multiplicand, adding the next higher order of the multiplicand to the successive products thus obtained, and carrying as in ordinary multiplication.*

EXAMPLES FOR PRACTICE.

Multiply
1. $2735.50 by 51.
2. $1824.75 by 81.
3. $5104.30 by 31.

Multiply
4. $6238.65 by 21.
5. $7415.40 by 61.
6. $8291.25 by 91.

Multiply
7. $2134.15 by 71.
8. $5827.80 by 41.
9. $2584.93 by 11.

523. To multiply when the multiplier consists of two figures and its tens' order is 1.

ILLUSTRATIVE EXAMPLE.

Multiply $873 by 19.

Explanation.—Multiply the units' figure of the multiplicand (3) by the units' figure of the multiplier (9), obtaining 27. Write 7, and carry 2 to 9 times the tens' figure of the multiplicand (7), increased by its units' figure (3), obtaining 68. Write

8, and carry 6 to 9 times the hundreds' figures of the multiplicand (8), increased by its tens' figure (7), obtaining 85. Write 5, and carry 8 to the hundreds' figure of the multiplicand (8), obtaining 16; making the complete product $16587. Hence,

Rule.—*Multiply the units' order of the multiplier into each order of the multiplicand, adding the next lower order of the multiplicand to the successive products thus obtained, carrying as in ordinary multiplication, and finally adding the last carrying figure to the left-hand order of the multiplicand.*

EXAMPLES FOR PRACTICE.

Multiply

1. $7125.45 by 16.
2. $815.30 by 13.
3. $758.42 by 14.
4. $925.35 by 18.

Multiply

5. $2580.95 by 18.
6. $1497.60 by 15.
7. $917.36 by 16.
8. $532.16 by 19.

Multiply

9. $9125.25 by 17.
10. $8174.85 by 19.
11. $415.18 by 18.
12. $214.46 by 15.

524. **To multiply when an aliquot part or a multiple of the multiplier will form a more convenient multiplier.**

ILLUSTRATIVE EXAMPLE

Multiply $75 by 56, and $628 by 25.

First Solution.

8 times $75 = $600
$\frac{1}{8}$ of 56 = 7
$4200

Second Solution.

$\frac{1}{4}$ of $628 = $157
4 times 25 = 100
$15700

Explanation.—Both of the above solutions are practical applications of the principle that to multiply one factor and divide the other by the *same* number does not affect the value of their product.

Rule.—*Multiply one factor and divide the other by such a number as will simplify the multiplication.*

EXAMPLES FOR PRACTICE.

Multiply

1. $35 by 15.
2. $42 by 18.
3. $24 by 16.
4. $18 by 45.

Multiply

5. $72 by 75.
6. $36 by 25.
7. $52 by 250
8. $42 by 350

Multiply

9. $150 by 48.
10. $64 by 25.
11. $35 by 18.
12. $28 by 450

Multiply

13. $36 by $33\frac{1}{3}$.
14. $72 by $16\frac{2}{3}$.
15. $24 by $13\frac{1}{3}$.
16. $96 by $12\frac{1}{2}$.

524

525. To multiply when one part of the multiplier is an aliquot of another part.

ILLUSTRATIVE EXAMPLE.

Multiply $2574 by 186, and $5462 by 856.

Explanation.—In the first solution, since one part of the multiplier (18) is *three times* the remaining part (6), first multiply $2574 by 6, obtaining $15444; then take three times 15444 as the product by 3 times 6, or 18 tens, obtaining 46332 tens; and add the two results.

In the second solution, since one part of the multiplier (56) is *seven times* the remaining part (8), first multiply $5462 by 8 hundreds, obtaining 43696 hundreds; then take seven times 43696 as the product by 7 times 8, or 56 units, obtaining 305872 units; and add the two results. Hence,

First Solution.

$2574
186
15444
46332
$478764

Second Solution.

$5462
856
43696
305872
$4675472

Rule.—*Multiply by the selected order of the multiplier; and take as many times the obtained result as the remaining orders of the multiplier are times the order used. Add the partial products.*

EXAMPLES FOR PRACTICE.

Multiply
1. $385.85 by 642.
2. $742.50 by 328.
3. $82615 by 729.

Multiply
4. $4265.25 by 255.
5. $9541.12 by 546.
6. $172.48 by 763.

Multiply
7. $2153.28 by 18624.
8. $530.75 by 315.
9. $618.40 by 16412.

526. To multiply when the multiplier is a little more or a little less than 100, 1000, 10000, etc.

ILLUSTRATIVE EXAMPLE.

Multiply $385.20 by 998.

Explanation.—Multiply $385.20 by 1000; the resulting product (385200) will be 1000—998, or 2 times $385.20 too great. Hence deduct 2 times $385.20, or $770.40 from $385200, obtaining the true product, $384429.60.

Solution.

$385200
770.40
$384429.60. *Ans.*

Rule.—*Multiply by the nearest number of 100's, 1000's, 10000's, etc.; and add to the result as many times the multiplicand as the true multiplier is units more than the assumed multiplier; or subtract from the result as many times the multiplicand as the true multiplier is units less than the assumed multiplier.*

EXAMPLES FOR PRACTICE.

Multiply

1. $425 by 999.
2. $865 by 98.
3. $735.25 by 998.

Multiply

4. $258.30 by 104.
5. $827.58 by 1003.
6. $516 by 1.02.

Multiply

7. $989 by 992.
8. $99 by 97.
9. $1005 by 1002.

527. To multiply any number of two or three figures by itself.

ILLUSTRATIVE EXAMPLE.

Multiply 48 by itself.

Explanation.—8 × 8 = 64 units. Write 4 and carry 6.
(4 + 4) × 8 = 64 + 6 to carry = 70. Write 0 and carry 7.
(Or, $\overline{8+8} \times 4 = 64 + 6$ to carry = 70.)
4 × 4 = 16 + 7 to carry = 23. Write 23.

Solution.

$$\begin{array}{r} 48 \\ \underline{48} \\ 2304 \end{array}$$

Rule.—*First, multiply the units' order by itself; then twice the tens' order by the units' order (or twice the units' order by the tens' order, if more convenient); then the tens' order by itself, carrying as usual.*

EXAMPLES FOR PRACTICE.

Multiply

1. $36 by 36.
2. $42 by 42.
3. $56 by 56.

Multiply

4. $85 by 85.
5. $74 by 74.
6. $93 by 93.

Multiply

7. $82 by 82.
8. $64 by 64.
9. $38 by 38.

Multiply

10. $19 by 19.
11. $113 by 113.
12. $122 by 122.

528. To find the product of two numbers when each consists of two or three orders, and their units' orders are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $74 by 94.

Explanation.—The solution of this example depends upon the same principle as that of Ill. Ex. **527**. Hence, 4 × 4 = 16 units. Write 6, and carry 1. (7 + 9) × 4 = 64 + 1 to carry = 65. Write 5, and carry 6. 7 × 9 = 63 + 6 to carry = 69.

Solution.

$74
94
$6956

Rule.—*First find the product of the units' orders; then the*

73.
85.
y 88.
by 76.

nsists of

of
ng
s,

Solution.

$83
89
$7387

Multiply

10. $62 by 65.
11. $84 by 83.
12. $52 by 55.

hree figures and ending

XAMPLE.

Solution.
25

25, obtaining $5625, the answer. Hence,

Rule.—*Multiply by the nearest number of 100's, 1000's, 10000's, etc.; and add to the result as many times the multiplicand as the true multiplier is units more than the assumed multiplier; or subtract from the result as many times the multiplicand as the true multiplier is units less than the assumed multiplier.*

EXAMPLES FOR PRACTICE.

Multiply
1. $425 by 999.
2. $865 by 98.
3. $735.25 by 998.

Multiply
4. $258.30 by 104.
5. $827.58 by 1003.
6. $516 by 1.02.

Multiply
7. $989 by 992.
8. $99 by 97.
9. $1005 by 1002.

527. To multiply any number of two or three figures by itself.

ILLUSTRATIVE EXAMPLE.

Multiply 48 by itself.

Explanation.—$8\times8=64$ units. Write 4 and carry 6.
$(4+4)\times8=64+6$ to carry$=70$. Write 0 and carry 7.
(Or, $\overline{8+8}\times4=64+6$ to carry$=70$.)
$4\times4=16+7$ to carry$=23$. Write 23.

Solution.

$$\begin{array}{r}48\\ \underline{48}\\ 2304\end{array}$$

Rule.—*First, multiply the units' order by itself; then twice the tens' order by the units' order (or twice the units' order by the tens' order, if more convenient); then the tens' order by itself, carrying as usual.*

EXAMPLES FOR PRACTICE.

Multiply
1. $36 by 36.
2. $42 by 42.
3. $56 by 56.

Multiply
4. $85 by 85.
5. $74 by 74.
6. $93 by 93.

Multiply
7. $82 by 82.
8. $64 by 64.
9. $38 by 38.

Multiply
10. $19 by 19.
11. $113 by 113.
12. $122 by 122.

528. To find the product of two numbers when each consists of two or three orders, and their units' orders are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $74 by 94.

Explanation.—The solution of this example depends upon the same principle as that of Ill. Ex. **527**. Hence, 4×4 = 16 units. Write 6, and carry 1. (7+9)×4=64+1 to carry=65. Write 5, and carry 6. 7×9=63+6 to carry=69.

Solution.

$74
 94
$6956

Rule.—*First find the product of the units' orders; then the product of the sum of the tens' order into the common units' order; then the product of the tens' orders, carrying as usual.*

EXAMPLES FOR PRACTICE.

Multiply

1. $42 by 72.
2. $38 by 58.
3. $61 by 21.
4. $34 by 64.

Multiply

5. $93 by 43.
6. $74 by 24.
7. $85 by 35.
8. $52 by 72.

Multiply

9. $51 by 31.
10. $92 by 52.
11. $132 by 92.
12. $135 by 75.

Multiply

13. $153 by 73.
14. $145 by 85.
15. $128 by 88.
16. $156 by 76.

529. To find the product of two numbers when each consists of two or three orders, and their tens' orders are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $83 by 89.

Explanation.—The solution of this example is a modification of that of Ill. Ex. **527, 528**. The orders which are *unlike* being added together, and then multiplied by the common order. Thus, 3×9=27; (9+3)×8+2 to carry=98; (8×8)+9 to carry=73.

Solution.

$83
 89
$7387

EXAMPLES FOR PRACTICE.

Multiply

1. $58 by 53.
2. $75 by 79.
3. $32 by 34.

Multiply

4. $81 by 85.
5. $64 by 67.
6. $25 by 23.

Multiply

7. $18 by 19.
8. $48 by 41.
9. $92 by 93.

Multiply

10. $62 by 65.
11. $84 by 83.
12. $52 by 55.

530. To multiply any number of two or three figures and ending in 5, by itself.

ILLUSTRATIVE EXAMPLE.

Multiply $75 by 75.

Solution.—7 × (7 + 1) = 56; annex 25, obtaining $5625, the answer. Hence,

Rule.—*Multiply the tens' order by itself increased by 1, and annex 25.*

EXAMPLES FOR PRACTICE.

Multiply
1. \$25 by 25.
2. \$65 by 65.

Multiply
3. \$45 by 45.
4. \$85 by 85.

Multiply
5. \$35 by 35.
6. \$75 by 75.

Multiply
7. \$55 by 55.
8. \$115 by 115.

531. To find the product of two numbers whose tens' orders are alike, and the sum of their units' orders is 10.

ILLUSTRATIVE EXAMPLE.

Multiply \$83 by 87, and \$114 by 116.

Solutions.

$$\begin{array}{r} \$83 \\ 87 \\ \hline \$7221 \end{array} \qquad \begin{array}{r} \$114 \\ 116 \\ \hline \$13224 \end{array}$$

Explanation.—Both solutions are modifications of **527**, and the answers are obtained as follows:

$(8+1) \times 8 = 72$, annex $3 \times 7 = 21$, obtaining \$7221.

$(11 + 1) \times 11 = 132$, annex $4 \times 6 = 24$, obtaining \$13224.

Rule.—*Multiply one of the tens' orders plus 1 by the other tens' order, and annex the product of the units' orders.*

EXAMPLES FOR PRACTICE.

Multiply
1. \$82 by 88.
2. \$73 by 77.
3. \$54 by 56.

Multiply
4. \$96 by 94.
5. \$23 by 27.
6. \$41 by 49.

Multiply
7. \$25 by 25.
8. \$36 by 34.
9. 103 by 107.

Multiply
10. \$17 by 13.
11. \$98 by 92.
12. \$112 by 118.

532. To find the product of two numbers whose tens' orders differ by 1, and the sum of their units' orders is 10.

ILLUSTRATIVE EXAMPLE.

Multiply \$73 by 67.

Solution.

$$\begin{array}{r} \$73 \\ 67 \\ \hline \$4891 \end{array} \qquad \begin{array}{rcl} 7^0 \times 7^0 & = & 49^{00} \\ 3 \;\times 3 & = & 9 \\ \hline & & 4891 \end{array}$$

Explanation.—Disregard the less number, and multiply the tens' order (7) of the greater number by itself, obtaining 49 (hundreds). Next multiply the units' order (3) of the greater number by itself, obtaining 9. Subtract 9 from 4900, obtaining \$4891, the required product.

531—532

Rule.—*Multiply the tens' order of the greater number by itself; and from the product (considered as hundreds), subtract the product of the units' order of the greater number multiplied by itself.*

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply	Multiply
1. $42 by 58.	4. $72 by 68.	7. $84 by 96.	10. $124 by 116.
2. $75 by 65.	5. $63 by 77.	8. $53 by 47.	11. $102 by 118.
3. $39 by 41.	6. $75 by 65.	9. $81 by 99.	12. $106 by 94.

533. To find the product of two mixed numbers when their terminal common fractions are alike.

ILLUSTRATIVE EXAMPLE.

Multiply \$8½ by 4½, and \$9¼ by 7¼.

First Solution.

$$\begin{array}{rl} \$8\tfrac{1}{2} & \\ 4\tfrac{1}{2} & \\ \hline 32 & = 8 \times 4 \\ 6 & = (8 + 4) \times \tfrac{1}{2} \\ \tfrac{1}{4} & = \tfrac{1}{2} \times \tfrac{1}{2} \\ \hline \$38\tfrac{1}{4} & \end{array}$$

Second Solution.

$$\begin{array}{rl} \$9\tfrac{1}{4} & \\ 7\tfrac{1}{4} & \\ \hline 63 & = 9 \times 7 \\ 4 & = (9 + 7) \times \tfrac{1}{4} \\ \tfrac{1}{16} & = \tfrac{1}{4} \times \tfrac{1}{4} \\ \hline \$67\tfrac{1}{16} & \end{array}$$

Rule.—*Find the product of the integers; then the product of the sum of the two integers into the common terminal fraction; then the product of the two fractions. Add the three results.*

NOTE.—When both integers are alike, and their terminal common fractions are ½, annex the product of the terminal common fractions to the product of one integer plus 1 multiplied by the other integer. Thus \$8½ × 8½ = (9 × 8) + (½ × ½), or \$72¼. This NOTE will not hold good with any other fraction than ½.

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply
1. \$4¼ by 8¼.	4. \$5¼ by 7¼.	7. \$5½ by 5½.
2. \$6⅓ by 8⅓.	5. \$3⅓ by 9⅓.	8. \$7½ by 7½.
3. \$5⅕ by 3⅕.	6. \$7⅕ by 5⅕.	9. \$8½ by 8½.

533

534. To find the product of two mixed numbers whose integers are alike and whose terminal common fractions have, or may be conveniently reduced to, a common denominator.

ILLUSTRATIVE EXAMPLE.

Multiply \$9⅛ by 9⅞, and \$8¾ by 8⅝.

First Solution.

$$\begin{array}{l} \$9\tfrac{1}{8} \\ \underline{9\tfrac{7}{8}} \\ 90 \;\; = (9 + \tfrac{1}{8} + \tfrac{7}{8}) \times 9 \\ \underline{\tfrac{7}{64}} = \tfrac{1}{8} \times \tfrac{7}{8} \\ \$90\tfrac{7}{64} \end{array}$$

Second Solution.

$$\begin{array}{l} \$8\tfrac{6}{8} \\ \underline{8\tfrac{5}{8}} \\ 75 \;\; = (8 + \tfrac{6}{8} + \tfrac{5}{8}) \times 8 \\ \underline{\tfrac{15}{32}} = \tfrac{6}{8} \times \tfrac{5}{8} \\ \$75\tfrac{15}{32} \end{array}$$

Rem.—The above solutions are self-explanatory.

Rule.—*To the integer of the multiplicand, add the sum of the terminal common fractions, and multiply the result by the integer of the multiplier. To the product thus obtained, annex the product of the fractions.*

EXAMPLES FOR PRACTICE.

Multiply

1. \$9⅓ by 9⅔.
2. \$10⅜ by 10⅝.
3. \$12⅚ by 12⅙.
4. \$18½ by 18⅓.

Multiply

5. \$.03⅛ by 3⅜.
6. \$60⅜ by 60¾.
7. \$.40⅜ by 40¼.
8. \$25⅞ by 25⅜.

Multiply

9. \$.05¼ by 5½.
10. \$.08¾ by 8⅓.
11. \$6⅝ by .06⅜.
12. \$12⅓ by 12⅚.

535. To multiply when the multiplier lacks 1 fractional unit of being an integer.

ILLUSTRATIVE EXAMPLE.

What is the cost of 64 yards of silk at \$3⅞ per yard ?

Explanation.—64 times \$4, or \$256, *would be the* cost at \$4 per yard; and at \$⅛ *per yard* less than \$4, the cost must be 64 *times* \$⅛, *or* \$8 less than \$256.

Solution.

$$\begin{array}{l} 64 \times \$4 = \$256,\ \text{cost at } \$4. \\ 64 \times \$\tfrac{1}{8} = \underline{8}\ \text{“ “ } \$\tfrac{1}{8}. \\ \phantom{64 \times \$\tfrac{1}{8} = }\$248\ \text{“ “ } \$3\tfrac{7}{8}. \end{array}$$

Rule.—*Multiply the multiplicand by the integral part of the multiplier plus 1; and diminish the result by such a part of the multiplicand as the fraction lacks of being a whole.*

EXAMPLES FOR PRACTICE.

Multiply

1. $48 by $5\frac{3}{4}$.
2. $75 by $10\frac{2}{3}$.
3. $136 by $6\frac{5}{6}$.
4. $250 by $3\frac{7}{8}$.
5. $522 by $4\frac{5}{8}$.
6. $672 by $8\frac{6}{7}$.

Multiply

7. $180 by $7\frac{9}{10}$.
8. $720 by $2\frac{11}{12}$.
9. $342 by $9\frac{5}{6}$.
10. $440 by $6\frac{10}{11}$.
11. $288 by $5\frac{7}{8}$.
12. $723 by $9\frac{2}{3}$.

536. To divide when a multiple of the divisor will form a more convenient divisor.

ILLUSTRATIVE EXAMPLE.

Divide $1840 by 25, and $6375 by $7\frac{1}{2}$.

First Solution.

$1840
4
$73.60

Second Solution.

$7\frac{1}{2} + \frac{1}{3}$ of $7\frac{1}{2} = 10$
$6375 + $\frac{1}{3}$ of $6375 = $8500
$8500 ÷ 10 = $850

Explanation.—Instead of using the inconvenient divisor 25, multiply both divisor and dividend by such a number (4) as will produce the more convenient divisor 100. Then divide the new dividend ($7360) by the new divisor (100), obtaining $73.60, the answer.

In the second solution, since the true divisor ($7\frac{1}{2}$) increased by $\frac{1}{3}$ of itself ($2\frac{1}{2}$) will form a more convenient divisor (10), similarly increase the dividend ($6375) by $\frac{1}{3}$ of itself ($2125), obtaining $8500. Then divide the new dividend ($8500) by the new divisor (10), obtaining $850.

Rule.—I. *Multiply or divide both divisor and dividend by such a number as will simplify the division. Or,*

II. *Add to, or subtract from, the given divisor such a part of itself as will produce a more convenient divisor. Similarly increase or diminish the dividend. Then divide as usual.*

EXAMPLES FOR PRACTICE.

Divide

1. $5.80 by 25.
2. $7.50 by $16\frac{2}{3}$.
3. $450 by $12\frac{1}{2}$.

Divide

4. $875 by $250.
5. $640 by 125.
6. $4.35 by $33\frac{1}{3}$.

Divide

7. $1527 by 150.
8. $918 by 15 (deduct $\frac{1}{3}$).
9. $582 by $7\frac{1}{2}$ (add $\frac{1}{3}$).

536

MARKING GOODS.

537. In marking goods, various devices are adopted to prevent the cost and selling price mark from becoming known to outsiders.

Rem. 1.—The device may consist of any word or phrase containing ten different letters or characters, each representing a figure. Thus, take the following word:

b	l	a	c	k	s	m	i	t	h
1	2	3	4	5	6	7	8	9	0

If the selling price is \$5.20, and it is required to write the cost (\$4.25) by the use of this key-word, the proper mark would be $\frac{\$5.20}{\text{clk}}$.

Rem. 2.—When both cost and selling price are thus expressed, it is customary to employ two key-words, one for the selling price and the other for the cost price. Thus if "blacksmith" is the key-word for the cost price (\$4.25) and "republican" for the selling price (\$5.20), the proper mark would be $\frac{\text{ben}}{\text{clk}}$.

Rem. 3.—An extra letter, called a *repeater*, is used to prevent successive repetitions of the same letter or character, and thus afford a clue to the key-word. Hence, taking y as the repeater, and "blacksmith" as the key-word, \$5.88 would be written k i y.

Rem. 4.—In trade discounts, the rate of discount on the long price is sometimes indicated by the use of a key-word.

Rem. 5.—The following afford examples of key-words: blackhorse, cash profit, hard moneys, importance, now be sharp, gambolines, Charleston, Cumberland, Hagerstown.

Rem. 6.—Instead of letters, arbitrary characters may be used, as follows:

⊿ ÷ E □ ⧄ ⊠ ○ ⊖ ⊗ ⛛ ⟐

EXAMPLES FOR PRACTICE.

Using the key "importance" for the cost price, "blackhorse" for the selling price, and "d" as the repeater, mark the following:

	Cost.		*Gain.*		*Cost.*		*Loss.*
1.	\$2.40	per yard......	25 %	6.	\$2.50	per pair......	20%
2.	16¢	" "	12½%	7.	45¢	" "	40%
3.	20¢	" "	30 %	8.	\$4.40	" dozen.....	5%
4.	27¢	" "	33⅓%	9.	20¢	" "	10%
5.	42¢	" "	16⅔%	10.	45¢	" gross.....	50%

RAPID PROCESS OF MARKING GOODS.

538. Retail merchants purchase many articles by the dozen, such as hats, caps, boots, shoes, and notions of various kinds. When goods are thus bought, it is useful to be able to determine instantly the selling price per unit to afford a given per cent. of profit.

539. If the given cost is per dozen, to find the selling price per unit at 20% profit.

ILLUSTRATIVE EXAMPLE.

If shirts are bought at $15 per dozen, at what price should each shirt be sold to afford a gain of 20% ?

Explanation.—At 20% profit, the selling price per dozen is 120% of the cost per dozen; and the selling price per unit must be $\frac{1}{12}$ of 120%, or 10% of the cost per dozen. Hence, 10%, or $\frac{1}{10}$ of $15, must be the selling price per shirt to afford 20% gain.

Solution.

$\frac{1}{10}$ of $15 = $1.50.

Rule.—*Divide the cost per dozen by 10. The result will be the selling price per unit at 20% profit.*

EXAMPLES FOR PRACTICE.

1. A merchant bought at auction 6 dozen pairs of boots at $42.50 per dozen pairs. How much must he charge per pair to gain 20% ?

2. Bought caps at auction for $10 per dozen. To gain 20% on each cap, what must be my selling price ?

3. By selling an article at $1.25 I gain 20%. What was the cost per dozen ?

4. A second-hand dealer bought 12 chairs at auction, for which he paid $12.50, and sold them at 20% profit. What did he charge for each chair ?

5. By selling shirts at $1.75 each, I gained 20%. What was the cost per dozen ?

6. If hats cost $18 per dozen, what must be the selling price of each to yield 20% profit ?

7. If chairs are bought at $9.20 per dozen, what must be the selling price per set of 6 to make 20% profit ?

540. If the given cost is per dozen, to find the selling price per unit at any % profit.

ILLUSTRATIVE EXAMPLE.

If gloves cost $20 per dozen pairs, what must be the selling price per pair to yield 50% profit?

Explanation.—First take $\frac{1}{10}$ of the cost per dozen, obtaining $2 as the selling price per pair at 20% profit, or at 120% of cost. Since the required selling price is 150% of cost, or one-fourth greater than 120%, increase $2 by one-fourth of itself ($.50), obtaining $2.50 as the required selling price.

Solution.

120% of cost = $2.00
30 " " " = .50
150% of cost = $2.50

Rule.—*By Rule,* **539**, *find the selling price per unit at 20% profit; and increase or diminish the result by such a part of itself as the true rate of selling price is greater or less than 120%.*

NOTE.—As a help in marking goods bought by the dozen, we add the following

Table.

To make 12½% profit, deduct $\frac{1}{16}$ from the result obtained by Rule, **539**.
" 16⅔% " " $\frac{1}{36}$ " " " " "
" 25% " add $\frac{1}{24}$ to " " " " "
" 30% " " $\frac{1}{12}$ " " " " " "
" 33⅓% " " $\frac{1}{9}$ " " " " " "
" 50% " " $\frac{1}{4}$ " " " " " "

EXAMPLES FOR PRACTICE.

1. If I buy one dozen vests for $28, at what price must I retail them to make 50% profit?

2. At what price must I retail handkerchiefs costing $2.70 per dozen to realize 33⅓% gain?

3. I sold hats costing $8.40 per dozen at 30% profit. At what price were they retailed?

4. If one dozen school-books cost $4.80, at what price must they be retailed to gain 25%?

5. If goods cost $7.20 per dozen, what should be the selling price per unit to yield a profit of 16⅔%?

6. If chairs are bought at $32 per dozen, what must be the selling price per set of 3 to yield 12½% profit?

ANSWERS.

DIRECTIONS.—The column of bold figures in the margin denotes the page; the bold figures at the beginning of each section, the number of the article; the small, bold figures within the curves, the number of the example; and the figures following the curves, the answer. The heavy dash denotes the beginning of the page whose number is given in the margin.

Art. 24.—(1) $18. (2) $25. (3) $42. (4) $36. (5) $358. (6) $270. **13**
(7) $3181. (8) $4169. (9) $32104. (10) $43120. (11) $37684. (12) $358982.
(13) $38909442. (14) $72. (15) $64. (16) $59. (17) $756. (18) $671.
(19) $719. (20) $8147. (21) $7805.——(22) $17887. (23) $9818. (24) 9865 **14**
ft. (25) 25775 lb. (26) 39540 bu. (27) 23332 yd. (28) 17313 gal. (29)
81383 lb. (30) $1482240. (31) 243062 mi. (32) $4757. (33) $20242. (34)
6369 lb. (35) 116572 inhabitants. (36) $9784. (37) 1881. (38) $22068.
(39) $6171. (40) 104411 ft.——(41) $2096. (42) $1527. (43) $2552. (44) **15**
$2363. (45) $2362. (46) $15866. (47) $13511. (48) $15340. (49) $11032.
(50) $2832. (51) $82328. (52) 17843 Am.; 7140 Ger.; 6288 Ir.; 1152 Fr.;
1715 It.; 6889 others; 1st ward, 5723; 2d, 7601; 3d, 7306; 4th, 9428; 5th,
6716; 6th, 4253; total, 41027. (53) 229804 bu. corn; 300493 bu. wheat;
71086 bu. oats; 104171 bu. barley; 51716 bu. rye; Mon., 99685 bu.; Tues.,
88759 bu.; Wed., 132664 bu.; Thurs., 96195 bu.; Fri., 168496 bu.; Sat.,
171471 bu.; total, 757270 bu.

28.—(1) 3317 bu. (2) $30889. (3) $305476. (4) 235356 ft. (5) $2281563. **17**
(6) $29471991. (7) $5814559. (8) 75453 lb. (9) $35172. (10) $26609354.
(11) $69112289. (12) $20866666. (13) $23996093. (14) $391809722. (15)
$731109214. (16) $327699727. (17) $180404924. (18) $192730098. (19)
$205979921. (20) $499649401. (21) 140653930 bu. (22) $352. (23) 94 mi.
(24) $13968. (25) $298125.——(26) 1732. (27) 63 yr. (28) $6825. (29) $8150. **18**
(30) 42994 bu. (31) 2890 mi.

29.—(1) $1733. (2) $532. (3) 12926 bu.——(4) 3391 mi. (5) 3856 bu. **19**
(6) $1861. (7) 10553 gal. (8) $2993. (9) $7775. (10) 16555 men. (11) 48
yr. (12) $45585. (13) $2150. (14) $286. (15) $2249. (16) $575.——(17) **20**
$53202. (18) $240. (19) 11741 boxes peaches; 4460 boxes apples; 6203
boxes pears; 24838 melons. (20) 1384 pages.

33.—(1) 193770 bu. (2) 73461072 lb. (3) 31074828 gal. (4) 8615308 yr. **22**
(5) 4130706279 bu. (6) 1807524021 yd. (7) 7656327045 lb. (8) 10414201612
ft. (9) $32524203336. (10) $661080409648. (11) $787883833055. (12)
$520070080340.

35.—(1) 448519036 lb. (2) 443877900 bu. (3) 136357425 yd. (4) 863724945 **23**
yr. (5) 272899704804 gal. (6) 439557310584 ft. (7) 2420708114 yd. (8)
8265262505 oz. (9) $1911949469170. (10) $2581559578455. (11) $6737750842442.
(12) $14369238437484. (13) $33459723687728. (14) $373888452465246.
(15) $3550228656054. (16) $8023461082764. (17) $73390. (18) 6808

23 bu. (19) 161315 lb. (20) $372. (21) 28 mi. (22) 16125 bu. (23) 164052
24 lb. (24) $5587764. (25) $2650. (26) $900.——(27) 1050 trees. (28) $2428.
37.—(1) 363384 lb. (2) 2970080 yr. (3) 3735466 bu. (4) 3823296 gal.
(5) 683550 men. (6) $12122553432. (7) $4409322804. (8) $4788319788.
(9) $9040788144. (10) $1239184170.
25 *39.*—(1) 53600 yr. (2) 987560000 bu. (3) 6297000 gal. (4) 375000 acres.
(5) 21375000 men. (6) 2784600 ft. (7) $711000000. (8) $5966000000. (9)
$3715740000. (10) $204132500000. (11) $7280000000. (12) $37772000.
27 *43.*—(1) 958544 lb. (2) 847992 times. (3) 1854768 times. (4) $99455404.
(5) 80868 times. (6) 3536410 ft. (7) 6422774 bu. (8) 64360400 gal. (9)
74046 times. (10) 46725615 times. (11) $850017080. (12) 25875025 times.
(13) 523413626 times. (14) $2001297153. (15) 696834667 times. (16)
4319767 times. (17) $3122323001. (18) 1640120006 times. (19) $59011-
151502. (20) 104695654 times.
28 *46.*—(1) 11429042[illegible] times. (2) 781081 gal. (3) 171889[illegible] times. (4)
26972967 yr. (5) 55995[illegible] times. (6) 24163572[illegible] times. (7) 244363[illegible] times.
(8) 40484[illegible] times. (9) 1986[illegible] times. (10) 882439[illegible] times. (11)
168529[illegible] times. (12) 176 yr. (13) 78499 yd. (14) 22102[illegible] times.
(15) 6498[illegible] times. (16) 247 mi. (17) $8761848. (18) 19686[illegible] times.
29 (19) 1286[illegible] times. (20) $81205008.——(21) $85. (22) 84 horses. (23)
98 boxes. (24) 875. (25) 276 trees. (26) 23 watches. (27) 78 bu. (28)
275 miles.
30 *48.*—(1) 2900[illegible] lb. (2) 25495[illegible] gal. (3) 7198[illegible] times. (4) 171916[illegible]
times. (5) 42759[illegible] times. (6) 46274[illegible] bbl. (7) 146826[illegible] yr. (8)
141381[illegible] times. (9) 7429[illegible] times. (10) $1101[illegible]. (11) $22257[illegible].
(12) 2032[illegible] times. (13) $4096[illegible]. (14) $916[illegible]. (15) 1438[illegible] times.
(16) $455[illegible].
31 *49.*—(1) 897[illegible] gal. (2) 689[illegible] times. (3) 48753[illegible] times. (4)
19[illegible] yd. (5) $689[illegible]. (6) 537[illegible] times. (7) 82736[illegible] times. (8)
591[illegible] bbl.
32 *50.*—(1) 13119[illegible] lb. (2) 305[illegible] bbl. (3) 6475[illegible] gal. (4) 30[illegible]
yd. (5) 22[illegible] bu. (6) 61[illegible] times. (7) 6571[illegible] times. (8) 2208[illegible]
times. (9) 1421[illegible] times. (10) 27[illegible] times.
33 *53.*—(1) $9774.87. (2) $18528.72. (3) $2277.34. (4) $2068.83. (5)
$4492. (6) $8.48.
34 *54.*—(1) $150.75. (2) $3092.35. (3) $4825.60. (4) $174.15. (5).
$3657.65. (6) $1062.95. (7) $139.68. (8) $206.70. (9) $5.91. (10) $1.99
(11) $.47. (12) $188.08.
35 *55.*—(1) $176175. (2) $244416.20. (3) $91986.20. (4) $1676980.26. (5)
$27030.66. (6) $2868860.50. (7) $8776.35. (8) $2648779.25. (9) $53014.08.
(10) $1029011.76. (11) $292.59. (12) $70.05.
36 *56.*—(1) $98.75. (2) $59.41. (3) $478.81. (4) $5.18. (5) $.76. (6)
$543.90. (7) $3291.25. (8) $685.50. (9) $219.75. (10) $.75.
57.—(1) 571 times. (2) 32718 times. (3) 41800 times. (4) 9200 times.
(5) 58 times. (6) 482 times. (7) 4517800 times. (8) 197500 times. (9) 75
times. (10) 67 times.

63.—(1) \$1752.50. (2) \$10319.40. (3) \$243.75. (4) 270860 ft. (5) \$706.43. **38**
(6) 6881 bu. (7) \$1467.70. (8) 846 head. (9) \$2475. (10) \$1185.52. (11) **39**
\$4.15. (12) \$42.25. (13) 106 lb. (14) \$507.50. (15) 41 bbl. (16) 175 acres.
(17) \$537.20.—(18) 325 bu. (19) \$357.32. (20) A \$1368.75; B \$2372.50; **40**
C \$3923.75. (21) 15¢. (22) 4 times. (23) \$6.07. (24) 36 days. (25) 39
days. (26) 85¢. (27) 70 bbl.—(28) 76500 men. (29) 487763 inhabitants. **41**
(30) 72 mi. (31) 360 mi. (32) 42 yd. (33) 87°. (34) 15 days. (35) \$7.68.

70.—(1) 2 lb. × 2 × 2 × 5. (2) 3 gal. × 3 × 5. (3) 2 ft. × 3 × 3 × 7. (4) 2 yd. **43**
× 2 × 3 × 11. (5) 2 bbl. × 3 × 5 × 7. (6) 2 da. × 137. (7) 2 yr. × 2 × 2 × 7 × 7. (8)
3 men × 5 × 5 × 7. (9) 3 tons × 3 × 5 × 11 × 13. (10) 2 bbl. × 2 × 2 × 7 × 11 × 17.
(11) 2 yr. × 2 × 2 × 3 × 3 × 11 × 23. (12) 2 bu. × 2 × 3 × 3 × 7 × 53. (13) 2 oz. ×
2 × 2 × 2 × 3 × 3 × 3 × 3 × 67. (14) 2 yr. × 3 × 3 × 3 × 3 × 7 × 41. (15) 2 hr. × 2 ×
2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 19. (16) 2 yd. × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 × 7. (17)
\$2 × 2 × 3 × 3 × 5 × 5 × 11 × 11. (18) 2 ft. × 3 × 3 × 5 × 5 × 5 × 5 × 5. (19) \$5 × 5 ×
11 × 11 × 11 × 11. (20) \$2 × 2 × 3 × 3 × 3 × 3 × 3 × 3 × 7 × 11 × 19.

74.—(1) 18 bu. (2) 8 yr. (3) 12 yd. (4) 6 ft. (5) 12 lb. (6) 14 oz. **44**
(7) \$6. (8) 21 bbl. (9) 12 ft. (10) 5 bu. (11) \$17. (12) \$27. (13) 12
yd. (14) 35 oz.

75.—(1) 86 yr. (2) \$5. (3) 43 lb. (4) 42 yd. (5) 51 bu. (6) 19 bbl. **45**
(7) 719 ft. (8) 2357 doz. (9) 60 tons. (10) 131 gal. (11) \$19. (12) \$17.

80.—(1) 504 bu. (2) 60 yd. (3) 540 gal. (4) 504 lb. (5) 315 hr. (6) **47**
120 doz. (7) 756 yr. (8) 68 ft. (9) 7392 oz. (10) \$11025. (11) 8190 ft.
(12) 9828 lb. (13) \$4200. (14) \$1512. (15) \$2520. (16) \$1800. (17) 105
mi. (18) 1001 gal. (19) \$191520. (20) \$2520. (21) 24 ft.

83.—(1) \$21. (2) 40 times. (3) 35 bbl. (4) 8 times. (5) 20 yd. (6) **48**
5 horses. (7) 7 bbl.—(8) 300 bu. (9) 94 chickens. (10) 12¢. (11) 3 yd. **49**
(12) 75¢. (13) 40 bu. (14) 158 doz. (15) 156 pieces.

97.—(1) $\frac{35}{4}$ lb. (2) $\frac{182}{5}$ bu. (3) \$$\frac{114}{5}$. (4) $\frac{810}{36}$ yd. (5) $\frac{1064}{15}$ ft. (6) $\frac{165}{16}$ in. **52**
(7) $\frac{548}{5}$ mi. (8) $\frac{29}{6}$ yr. (9) $\frac{1180}{9}$ bu. (10) $\frac{64}{5}$ lb. (11) $\frac{10872}{135}$ yr. (12) $\frac{3188}{317}$ ft.

98.—(1) $\frac{27}{4}$ mi. (2) $\frac{36}{5}$ rd. (3) $\frac{64}{5}$ tons. (4) $\frac{182}{5}$ acres. (5) $\frac{240}{7}$ pk.
(6) $\frac{378}{5}$ cd. (7) $\frac{878}{11}$ in. (8) $\frac{1085}{8}$ melons. (9) $\frac{12077}{17}$ ft. (10) $\frac{13488}{15}$ bbl.
(11) $\frac{2442}{11}$ lb. (12) $\frac{274412}{45}$ bu. (13) $\frac{2888825}{357}$ yr. (14) $\frac{214888}{9}$ yd. (15)
$\frac{1442851}{785}$ gal. (16) $\frac{28014}{177}$ yd.

99.—(1) 6 ft. (2) 6 yr. (3) 38 bbl. (4) 14 qt. (5) 4$\frac{1}{3}$ tons. (6) 143$\frac{1}{3}$ **53**
yd. (7) 45$\frac{1}{3}$ gal. (8) 453$\frac{1}{7}$ days. (9) 24$\frac{11}{113}$ yr. (10) 7$\frac{23}{371}$ mi. (11) 12$\frac{143}{333}$
bbl. (12) 85$\frac{1}{33}$ gal. (13) 4$\frac{177}{377}$ days. (14) 36$\frac{144}{517}$ yd. (15) 1957 rd. (16)
131181$\frac{1}{7}$ oz.

100.—(1) $\frac{11}{13}$ lb. (2) $\frac{10}{13}$ yd. (3) $\frac{11}{13}$ gal. (4) $\frac{4}{15}$ qt. (5) $\frac{11}{13}$ doz. (6)
$\frac{11}{13}$ gal. (7) $\frac{10}{17}$ hr. (8) $\frac{11}{13}$ bbl. (9) $\frac{17}{135}$ bu. (10) $\frac{40}{135}$ yr. (11) $\frac{11}{13}$ yd.
(12) $\frac{113}{133}$ ft. (13) $\frac{113}{133}$ yr. (14) $\frac{102}{1175}$ gal.

101.—(1) $\frac{2}{3}$ ft. (2) $\frac{5}{13}$ hr. (3) $\frac{1}{3}$ gal. (4) $\frac{1}{3}$ oz. (5) $\frac{5}{33}$ bbl. (6) $\frac{5}{11}$ yr. **54**
(7) $\frac{1}{3}$ yd. (8) $\frac{2}{3}$ yd. (9) $\frac{2}{3}$ hr. (10) $\frac{2}{3}$ bu. (11) $\frac{4}{45}$ tons. (12) $\frac{1}{3}$ gal.

102.—(1) $\frac{2}{3}$ ft. (2) $\frac{1}{3}$ yd. (3) $\frac{1}{3}$ bu. (4) $\frac{2}{3}$ yr. (5) $\frac{1}{3}$ ton. (6) $\frac{1}{3}$ gal. **55**
(7) $\frac{4}{7}$ bbl. (8) $\frac{11}{13}$ ft. (9) $\frac{7}{13}$ hr. (10) $\frac{4}{11}$ lb. (11) $\frac{2}{3}$ oz. (12) $\frac{4}{7}$ yd. (13)
$\frac{4}{11}$ qt. (14) $\frac{4}{7}$ bbl. (15) $\frac{2}{3}$ yr. (16) $\frac{2}{3}$ yd. (17) $\frac{4}{7}$ bbl. (18) $\frac{4}{11}$ yd. (19)
$\frac{2}{3}$ bu. (20) $\frac{2}{3}$ lb. (21) $\frac{4}{7}$ ton. (22) $\frac{4}{11}$ bbl. (23) $\frac{4}{13}$ gal. (24) $\frac{1}{7}$ yd.

56 ***105.***—(1) [illegible] lb.; [illegible] lb.; [illegible] lb. (2) [illegible] oz.; [illegible] oz.; [illegible] oz. (3) [illegible] bu.;
[illegible] bu.; [illegible] bu. (4) [illegible] ft.; [illegible] ft.; [illegible] ft. (5) [illegible] yr.; [illegible] yr.; [illegible] yr. (6) [illegible]
yd.; [illegible] yd.; [illegible] yd. (7) [illegible] gal.; [illegible] gal.; [illegible] gal. (8) [illegible] oz.; [illegible] oz.;
[illegible] oz. (9) [illegible] yr.; [illegible] yr.; [illegible] yr.; [illegible] yr. (10) [illegible] hr.; [illegible] hr.; [illegible]
hr.; [illegible] hr. (11) [illegible] bbl.; [illegible] bbl.; [illegible] bbl.; [illegible] bbl. (12) [illegible] ft.; [illegible] ft.;
[illegible] ft.; [illegible] ft. (13) [illegible] bu.; [illegible] bu.; [illegible] bu.; [illegible] bu. (14) [illegible] rd.;
[illegible] rd.; [illegible] rd.; [illegible] rd. (15) [illegible] yr.; [illegible] yr.; [illegible] yr.; [illegible] yr.
(16) [illegible] bbl.; [illegible] bbl.; [illegible] bbl.
57 ***108.***—(1) 2[illegible] bu. (2) 1[illegible] yr. (3) 1[illegible] hr. (4) 1[illegible] ft. (5) 1[illegible] gal.
(6) 1[illegible] yd. (7) 1[illegible] bbl. (8) 1[illegible] oz. (9) 1[illegible] yd. (10) 2[illegible] ft. (11)
1[illegible] bu. (12) 2[illegible] qt. (13) 146[illegible] lb. (14) 732[illegible] yd. (15) 598[illegible] yr.
(16) 585[illegible] lb. (17) 1183[illegible] rd. (18) 47[illegible] bu. (19) 169^{2} yd. (20) 263^{2}
58 yd. (21) 63 yd. (22) 80[illegible] lb. (23) 1122[illegible] lb.—(24) [illegible] est. (25) $16[illegible].
59 ***111.***—(1) [illegible] ft. (2) [illegible] in. (3) [illegible] bbl. (4) [illegible] lb. (5) [illegible] yr. (6) [illegible]
yd. (7) [illegible] hr. (8) [illegible] gal. (9) [illegible] oz. (10) [illegible] bbl. (11) 1[illegible] yr. (12) [illegible]
bu. (13) 4[illegible] lb. (14) 1[illegible] qt. (15) 1[illegible] yr.

112.—(1) $475[illegible]. (2) 169[illegible]. (3) $234[illegible]. (4) $298[illegible]. (5) $257[illegible]. (6)
$842[illegible]. (7) $21[illegible]. (8) $52[illegible]. (9) $285[illegible]. (10) $263[illegible]. (11) $405[illegible].
(12) $153[illegible]. (13) $130[illegible]. (14) $327[illegible]. (15) $107[illegible]. (16) $195[illegible].
(17) 12^{2} yd. (18) 50^{2} yd.
61 ***115.***—(1) $[illegible]. (2) $[illegible]. (3) $[illegible]. (4) $[illegible]. (5) $[illegible]. (6) $[illegible]. (7) $[illegible]. (8)
$[illegible]. (9) $[illegible]. (10) $[illegible]. (11) $1[illegible]. (12) $2[illegible]. (13) $3. (14) $18[illegible],
(15) $11[illegible]. (16) $[illegible]. (17) $[illegible]. (18) $[illegible]. (19) $[illegible]. (20) $4[illegible].

116.—(1) $588[illegible]. (2) $189[illegible]. (3) $327[illegible]. (4) $494[illegible]. (5) $164[illegible]. (6) $4[illegible].
(7) $1[illegible]. (8) $16. (9) $24. (10) $21. (11) $2[illegible]. (12) $3[illegible]. (13) $396[illegible].
(14) $1[illegible]. (15) $36. (16) $3[illegible]. (17) $2[illegible]. (18) $12[illegible]. (19) $181[illegible]
(20) $27[illegible]. (21) $3[illegible]. (22) $52. (23) $2[illegible]. (24) $20.
62 ***117.***—(1) $558. (2) $424. (3) $3010. (4) $19200. (5) $6954. (6)
$4335[illegible]. (7) $1970[illegible]. (8) $18274[illegible]. (9) $4925[illegible]. (10) $32749[illegible]. (11) $27389[illegible].
(12) $2532[illegible]. (13) $16278[illegible]. (14) $35780[illegible]. (15) $118746[illegible].
63 ***118.***—(1) 858[illegible] oz. (2) 259[illegible] ft. (3) 1054[illegible] yr. (4) 28445[illegible] yd. (5)
1220[illegible] bu. (6) 9354[illegible] lb. (7) 2571[illegible] hr. (8) $141993[illegible]. (9) $32145[illegible]. (10)
8341[illegible] yr. (11) $164539[illegible]. (12) 4855[illegible] bbl. (13) $46784[illegible]. (14) $9146[illegible].
(15) $45817[illegible]. (16) $4.26[illegible].
64 ***123.***—(1) [illegible] times. (2) $33. (3) $5[illegible]. (4) 1[illegible] times. (5) $1[illegible]. (6) $[illegible].
(7) $64[illegible]. (8) 13 times. (9) [illegible] times. (10) $24. (11) $2[illegible]. (12) 2[illegible] times.
(13) 228 times. (14) $1944. (15) 8[illegible] times. (16) $10[illegible]. (17) 605[illegible] times.
(18) $[illegible]. (19) [illegible] times. (20) 9[illegible] times. (21) $[illegible]. (22) $2[illegible]. (23) 4
times. (24) 2[illegible] times.
65 ***124.***—(1) $[illegible]. (2) [illegible] times. (3) $[illegible]. (4) [illegible] times. (5) $[illegible]. (6) $[illegible].
(7) $[illegible]. (8) [illegible] times. (9) $[illegible]. (10) [illegible] times. (11) $[illegible]. (12) $[illegible]. (13)
$[illegible]. (14) $[illegible]. (15) [illegible] times. (16) $[illegible]. (17) [illegible] times. (18) $[illegible]. (19)
$[illegible]. (20) [illegible] times. (21) $[illegible]. (22) $[illegible]. (23) $[illegible]. (24) [illegible] times. (25) $[illegible].
(26) $[illegible]. (27) $1[illegible]. (28) $[illegible]. (29) [illegible] est. (30) $1[illegible].
66 ***125.***—(1) 123455[illegible] times. (2) 783[illegible] yd. (3) 925[illegible] bu. (4) 887[illegible]
times. (5) 31973[illegible] ft. (6) 172773[illegible] times. (7) $91758[illegible]. (8) 63444[illegible] times.

(9) 3591[illegible] times. (10) $85637[illegible]. (11) $8426[illegible]. (12) 694[illegible] times. (13) 66
$1930[illegible]. (14) 1661[illegible] times. (15) $472[illegible]. (16) $3[illegible]. (17) 122[illegible] hr.
126.—(1) $7283[illegible]. (2) 11015[illegible] times. (3) 10794[illegible] times. (4) $4552[illegible].
(5) $14291[illegible]. (6) 4191[illegible] times. (7) $2339[illegible]. (8) $653[illegible]. (9) 28[illegible] times.
(10) $66[illegible]. (11) $49[illegible]. (12) $8[illegible]. (13) 816 casks.
127.—(1) $5[illegible]. (2) $53[illegible]. (3) 17[illegible] times. (4) $94[illegible]. (5) 274[illegible] 67
times. (6) $144[illegible]. (7) 470[illegible] times. (8) $58[illegible]. (9) $5[illegible]. (10)
204[illegible] times. (11) $4[illegible]. (12) $157[illegible].
131.—(1) 6[illegible] yd. (2) $114[illegible]. (3) 6[illegible] lb. (4) $[illegible]. (5) 4[illegible] mo. (6) 69
86[illegible] lb. (7)8[illegible] da. (8) 5[illegible] lb. (9) [illegible] lb. (10) 974[illegible] mi. (11) 8[illegible] hr.
(12) 8 beds. (13) 5¢. (14) 112 hr. (15) 86 bu. (16) 2[illegible] tons. (17) $[illegible]. 70
(18) [illegible] hr. (19) 1[illegible] yd. (20) 15 da. (21) 2[illegible] da. (22) 25[illegible] mi.
133.—(1) [illegible] bbl. (2) $4000. (3) [illegible]. (4) 562[illegible] acres. (5) $6400. (6) 71
81 acres. (7) $7800.——(8) $3825. (9) [illegible]. (10) 14[illegible] yd. (11) 1200 men. 72
(12) [illegible]. (13) $5180. (14) $46942. (15) A [illegible]; B [illegible]; C [illegible]. (16) $5760.
(17) 60[illegible] lb. (18) [illegible].
134.—(1) 767[illegible] sq. rd. (2) 75[illegible] lb. (3) [illegible].——(4) 342[illegible] cu. yd. (5) 656[illegible] 73
bu. (6) $40.92[illegible]. (7) [illegible] orange. (8) 9¢. (9) 389[illegible] bbl. (10) 20[illegible] mi.
(11) [illegible]. (12) $3885. (13) [illegible]. (14) B $18[illegible]; C $26[illegible]. (15) 6 lots. (16)
650[illegible] lb. (17) $46.02[illegible].——(18) $1.19. (19) 226 lb. (20) $5.91[illegible]. (21) 74
8[illegible] da. (22) 7[illegible] da. (23) 1[illegible] da. (24) 26[illegible] gal. (25) $24[illegible]. (26)
$148.57[illegible]. (27) 256 acres. (28) 60 lemons. (29) 3[illegible] hr. (30) A [illegible]; B [illegible];
C [illegible].——(31) 13[illegible] min. (32) 25[illegible] mi. (33) $30720. (34) house, $4500; 75
lot, $2700. (35) $11405; $6843. (36) carriage, $735; horse, $525. (37) $20.
(38) [illegible]. (39) 3 da. (40) $5100. (41) [illegible] bbl. (42) [illegible]. (43) 270 lb. (44)
232 lb.——(45) [illegible] da. (46) 8[illegible] min. (47) 6 da. (48) [illegible]. (49) 70 yd. (50) 76
[illegible]. (51) $42.
146.—(1) .375 gal. (2) .875 yd. (3) .3125 bu. (4) .5625 yr. (5) .46875 80
bbl. (6) .265625 oz. (7) .1625 hr. (8) .35 bu. (9) .96 yd. (10) .056 ft.
(11) .0625 lb. (12) .078125 yr. (13) .008 bu. (14) .0234375 gal. (15)
8.15625 da. (16) 51.04 yd. (17) 47.046875 bu. (18) 6.56 rd. (19) 28.03-
125 mi. (20) 76.006 gal.——(21) .1875 lb. (22) .171875 gal. (23) .15625 81
bu. (24) .1484375 yd. (25) .056 ft. (26) .01125 yr. (27) 3.01953125 tons.
(28) 6.006 rd. (29) $18.67–. (30) $75.88–. (31) $16.417–. (32)
$84.219–, (33) $215.0156+. (34) $761.7778–. (35) $185.81818+. (36)
$940.00781+. (37) 88.27863–lb. (38) 56.13469–bu. (39) 8.48333+yr.
(40) 97.514286–rd. (41) 75.458333+tons. (42) 1.000544–yr. (43) 9.0086-
667–mi. (44) 17.5001273–acres. (45) .0000143– yr.
147.—(1) [illegible] bbl. (2) [illegible] gal. (3) [illegible] yr. (4) [illegible] yd. (5) [illegible] bu. (6) [illegible] hr. 82
(7) 8[illegible] rd. (8) 5[illegible] acres. (9) [illegible] hr. (10) [illegible] yr. (11) 8[illegible] da. (12)
6[illegible] yr. (13) 9[illegible] gal. (14) 5[illegible] yr. (15) [illegible] bu. (16) [illegible] hr. (17) [illegible]
bbl. (18) [illegible] yd. (19) 8[illegible] da. (20) 7[illegible] mi. (21) 4[illegible] gal. (22) [illegible] yd.
(23) 2[illegible] rd. (24) [illegible] bu.
148.—(1) 293.60986 lb. (2) 983.4812 bu. (3) 7694.61217 yr. (4) 979.7106 83
gal. (5) 55.545 yd. (6) 477.309875 rd. (7) 768.7440675 mi. (8) 984.0165075
acres. (9) 838.526875 cd. (10) 79.504575 tons. (11) 819.886+ yr. (12)

83 1.4039 + bu. (13) $271.024–. (14) $1084.955 +. (15) 7814.51018– tons.
(16) 4063.034636 acres. (17) 5407.06444 mi.

84 ***149.***—(1) 52.7491 oz. (2) 1.97791 rd. (3) 5.0687 acres. (4) 38.87137 yr.
(5) 364.91654 yd. (6) 58.712 mi. (7) 286.79142 bu. (8) 15.57065 ft. (9)
3337.3298 hr. (10) 17.415 tons. (11) 42.1838 bu. (12) .4927 yr. (13) 7.1475
bbl. (14) 7.074625 mi. (15) 49.2801375 acres. (16) 57.8271875 cd. (17)
85 235.18163 + mo. (18) 4.821714 + rd.——(19) 57.2369 + wk. (20) 250.50095– ft.
(21) 207.835 yr. (22) 947.9625 yr. (23) 492.000721 fath. (24) 7738.02497
tons. (25) 334.01441 cd.

86 ***150.***—(1) 18539.20 ft. (2) 281.43225 yd. (3) .3261635 acres. (4) 72.34707
rd. (5) 22.486035 mi. (6) 52051.125 gal. (7) .0004524 mo. (8) .0000045752
bu. (9) 42.970875 yd. (10) 3.06787525 bbl. (11) $28.4284312375. (12)
$.0052570943359375. (13) $1.22201⅜. (14) $.2235761⅓. (15) $44.3053[illegible].
(16) $.25675[illegible]. (17) $1834.5. (18) $627800. (19) $22.55. (20) $93060.

87 ***151.***—(1) $8930. (2) 77500 cd. (3) 27000 acres. (4) .0005 times. (5)
.3 bu. (6) 46.8 yd. (7) 10000 mi. (8) .0002 times. (9) 12.38 mo. (10)
2350 fath. (11) .024 times. (12) 97.65625 tons. (13) 1000 times. (14) 1.73
times. (15) $620. (16) 21.6 times. (17) 7.5 times. (18) 716200 yd. (19)
.0087 times. (20) 4.7578125 times. (21) 297.6825 times. (22) 43540 mo.
(23) 7830 rd. (24) 81.4 mi. (25) 316.667– fath. (26) 10.9117– times.
(27) 19.780–mi. (28) 216.708 + yr. (29) 4.89 + times. (30) 2.059– times.
(31) $3214.865–. (32) .00016 times. (33) .001 + times. (34) 18209.3023 +
times. (35) 30.62– pt. (36) .010 + ft. (37) .004– yd. (38) 826.667– cd.
(39) 5.935– rd. (40) .1317 + fath. (41) .0009– bbl. (42) .86 + times.
(43) 2.143– yr. (44) .08– bu. (45) .153 + times. (46) 186.67– gal. (47)
88 1.854– times. (48) 6.37– yr.——(49) 1.8475 times. (50) 3.451298 acres.
(51) 41.52385 mi. (52) .0075 bu. (53) $213.519–. (54) .75 + times. (55)
$.576 +. (56) .2668 + acres. (57) $.01141–. (58) .016 + yr.

152.—(1) 10756.79206 acres. (2) 9.045 bu. (3) $343.4140625. (4)
11999.55 bu. (5) $.0021. (6) .18. (7) 640 bu. (8) $130.62 +. (9) 12654.9
89 loads.——(10) $47.70. (11) .35. (12) 134.37 lb. (13) 41.3635 mi. (14)
264.06232 acres. (15) $394.06–. (16) $114.31. (17) $414.61–. (18) .317.
(19) $.46875; $.53125. (20) 133 sheep. (21) 35 da. (22) $150.08. (23)
90 263.2702 yr. (24) 7.18 yd. (25) 393.3675 mi. (26) $29.40. (27) $72. (28)
$18374. (29) 768 times. (30) .289. (31) .184. (32) $163.11. (33) Mon.
208 lb.; Tues. 159.25 lb.; Wed. 178.75 lb.; Thurs. 172.25 lb.; Fri. 325 lb.;
Sat. 406.25 lb. (34) $5445. (35) $42880. (36) 345 acres. (37) .48. (38)
91 $400. (39) coffee $80; sugar $34.80.

92 ***158.***—(1) $2757. (2) $939. (3) $153. (4) $243. (5) $81. (6) 4890. (7)
$255. (8) $700. (9) $357.75. (10) $845.50. (11) $3541.67–. (12) $7797.

93 ***159.***—(1) 2200 yd. (2) 1892 yd. (3) 1602 yd. (4) 5724 yd. (5) 8224
yd. (6) 837.52 yd. (7) 2008 gal. (8) 1664 gal. (9) 216 gal. (10) 300.4 gal.
(11) 552.48 gal. (12) 351.6 gal. (13) 822 lb. (14) 3290 yd.

94 ***160.***—(1) $197.53 +. (2) $70.15–. (3) $109.38–. (4) $102.18. (5)
$465.63–. (6) $924.19–. (7) $76.89 +. (8) $93.67. (9) $565.37. (10)
$15.47–. (11) $1806.21. (12) $294.

161.—(1) $9.94—. (2) $1918.61+. (3) $56.18+. (4) $143.97—. (5) **94**
$6726. (6) $3.15—. (7) $5.90+. (8) $10.33+. (9) $33.96+. (10)
$1.05+. (11) $78.44—. (12) $7.83—.

163.—(1) $47.43+. (2) $45.57. (3) $11.53—. (4) $13.75. (5) $14.22. **95**
(6) $68.54+. (7) $137.50—. (8) $68.96+. (9) $20.70. (10) $65.64—.
(11) $7.08.

171.—(1) $11.13+. (2) $189.80. (3) $456.87. (4) $631.92. (5) $84. **98**
(6) $146.17. (7) $77.52.——(8) $152.35. (9) $358.44. (10) $43.05. (11) **102**
$19.38 (12) $292.92.——(13) Mar. 3, $35.97; Mar. 18, $27; Mar. 23, $84.15; **103**
Mar. 27, $17.50; monthly statement, $164.62.

208.—(1) 5030 m. (2) 1815¢. (3) 72756 far. (4) 1909 pennies. (5). **115**
183070 m. (6) 13180 m. (7) 2070¢. (8) 120012 far. (9) 25009 m. (10)
11424 d. (11) $183.929. (12) $2661.9755. (13) $9.38. (14) $2126.6605.
(15) $383.2369—. (16) $217.77. (17) $698.3428—. (18) $77.410. (19)
$1908.175. (20) $508.68. (21) 343938 in. (22) 1544510¼ sq. ft. (23) 976
cu. ft. (24) 2464 cu. ft. (25) 1223776188 sq. in. (26) 327⅓ ft. (27)
32150 sq. ch. (28) 6189696 cu. in. (29) 692 cu. ft. (30) 60 in. (31)
100534¼ ft. (32) 467874 in.——(33) 16307 lb. (34) 18160 pwt. (35) ℈1005. **116**
(36) 6240288 oz. (37) ℈23. (38) 29208 gr. (39) 503728 oz. (40) 20928
gr. (41) 728000 lb. (42) 14700 lb. (43) 51968 lb. (44) 10080 lb. (45)
f℥688. (46) 2014 pt. (47) 1508 gi. (48) 2481 pt. (49) 415 gi. (50)
♏1800. (51) 4801 pt. (52) 79380 gal. (53) 64393234 sec. (54) 55967″.
(55) 527040 min. (56) 31536000 sec. (57) 2400 sh. (58) 2678400 sec.
(59) 400 qu. (60) 54953100 sec. (61) 145920 pp. (62) 23040 pp.

209.—(1) $5.678. (2) £18 1s. 9d. 1 far. (3) 38.375 fr. (4) £73 1s. **117**
(5) 61.38 m. (6) £19. (7) 472 m. (8) 768 fr. (9) 861.48+ roubles. (10)
9852.94+ m. (11) 4 T. 16 cwt. 13 lb. 4 oz. (12) 10 lb. 1 oz. 12 pwt. 8 gr.
(13) ℔. 74 ℥9 ʒ7. (14) 21 lb. 11 oz. 13 pwt. (15) 35 cwt. 74 lb. 2 oz.
(16) ℔. 4 ℥11 ℈1 gr. 11. (17) 2450 lb. (18) ℔. 3. (19) 1296 lb. (20)
8 bbl. (21) 80 bu. 17 lb. (22) 166 bu. 6 lb.——(23) 272 cd. 7 cd. ft. (24) **118**
2 mi. 287 rd. 4 yd. 1 ft. 10 in. (25) 8 A. 31 P. 29 sq. yd. 1 sq. ft. 36 sq. in.
(26) 38 cd. 54 cu. ft. (27) 240 ch. (28) 7 A. 130 P. 8 sq. yd. 4 sq. ft. 72
sq. in. (29) 74 cu. yd. 3 cu. ft. 464 cu. in. (30) 1738 A. 1 sq. ch. 9 P. 89
sq. l. (31) 56891 rd. 3 yd. 2 ft. 6 in. (32) 3 Tp. 9 sq. mi. 441 A. 8 sq. ch.
(33) 111 mi. 56 ch. 21 l. (34) 2 cd. 43 cu. ft. 1193 cu. in. (35) 1 gal. 1 pt.
5 f℥ 5fʒ 15♏. (36) 80 bu. 3 pk. 5 qt. (37) 983 gal. 2 qt. 1 gi. (38) 22
bu. 1 pk. 4 qt. (39) 37 gal. 2 pt. 3 f℥ 5 fʒ. (40) 1084 gal. 2 qt. (41)
7 yr. 92 da. 17 hr. 18 min. (42) 9 S. 17° 16′ 2″. (43) 15 rm. 4 qu. 22 sh.
(44) 9 wk. 2 da. 3 hr. 4 min. (45) 3 sc. 18 yr. (46) 427 doz. 3 bottles.
(47) 565 gr. 1 doz. (48) 103° 59.52 com. mi. (49) 15 da. 8 hr. 6 min. 13
sec. (50) 45 bales 1 bundle 1 rm. 5 qr. 22 sh.

210.—(1) 1 pk. 6 qt. (2) 133 rd. 1 yd. 2 ft. 6 in. (3) 6 pt. (4) 4 oz. **119**
16 pwt. (5) ℥9 ʒ4 ℈2 gr. 8. (6) 21 hr. 20 min. (7) 13s. 4d. (8) 43 P.
19 sq. yd. 2 sq. ft. 36 sq. in. (9) 1 qt. 1 pt. (10) 4 cd. ft. 10 cu. ft. 1152
cu. in. (11) 6 cwt. 25 lb. (12) 243 da. 8 hr. (13) 6 qt. 1¼ pt. (14) 2 ft.
1⅞ in. (15) 4 yd. 1 ft. 9 in. (16) 6s. 3d.

119 ***211.***—(1) 1 qt. 1 pt. 3 gi. (2) 8 pwt. 18 gr. (3) 10 oz. 10 pwt. (4) 4.4
qt. (5) 51 da. 22 hr. 6 min. 36 sec. (6) 7s. 6d. (7) 1 pt. 2 gi. (8) 3 cwt.
18 lb. 5 oz. (9) 3 cd. ft. 11 cu. ft. (10) 60 P. 15 sq. yd. 1 sq. ft. 18 sq. in.
(11) 164 da. 6 hr. (12) 1d. 2 far.

120 ***212.***—(1) $\frac{4}{5}$ gr. (2) $\frac{1331}{4608}$ da. (3) $\frac{3}{5}$ pt. (4) $\frac{3}{8}$ cu. ft. (5) .366 d. (6)
.917 m. (7) .4 oz, (8) .25 pk.

213.—(1) $\frac{1}{256}$ bu. (2) $\frac{1}{240}$ lb. (3) $\frac{1}{1440}$ da. (4) $\frac{1}{20}$ T. (5) .0078125
gal. (6) .00075 yd. (7) $.000235. (8) £.000075.

121 ***214.***—(1) £$\frac{1}{8}$. (2) $\frac{55}{144}$ lb. (3) $\frac{5}{8}$ gal. (4) $\frac{3}{4}$ yd. (5) $\frac{11}{12}$ hr. (6) $\frac{3}{160}$ mi.

215.—(1) .921875 bu. (2) .4375 gal. (3) £.0625. (4) .06 T. (5) .75 yd.
(6) .075 hr. (7) .9125 da. (8) £.6875. (9) .0152— yr. (10) .24489— mi.

122 ***216.***—(1) £279 6s. (2) 262 lb. 1 oz. 4 pwt. 17 gr. (3) 189 mi. 307 rd.
4 yd. 1 ft. 4 in. (4) 22 gal. 1 pt. (5) £719 14s. (6) 21 T. 17 cwt. 31 lb. 11
oz. (7) 3 pk. 7 qt. 1 pt. (8) £1 1s. 11.3d. (9) 3 gal. 1.75 pt.

123 ***217.***—(1) 32 lb. 8 oz. 14 pwt. 13 gr. (2) 3 mi. 76 rd. 4 yd. 2 ft. 6 in.
(3) 1 bu. 2 pk. 1 qt. (4) 20 gal. 2 qt. 1 pt. 3 gi. (5) 5 T. 18 cwt. 41 lb.
(6) £2955 17s. 3 far. (7) 7 rd. 2 yd. 9 in. (8) 18 sq. yd. 1 sq. ft. 58½ sq. in.

218.—(1) 8 yr. 5 mo. 28 da. (2) 27 yr. 9 mo. 4 da. (3) 21 yr. 7 mo. 8
da. (4) 12 yr. 11 mo. 29 da. (5) 60 yr. 5 mo. 20 da. (6) 30 yr. 9 mo. 12 da.
(7) 7 yr. 9 mo. 17 da. (8) 3 yr. 1 mo. 14 da. (9) 8 yr. 11 mo. 20 da. (10)
124 79 yr. 8 mo. 24 da.——(11) 12 yr. 8 mo. 22 da. (13) 45 yr. 10 mo. 26 da.
11 hr. 50 mi. (14) 6 yr. 7 mo. 25 da. 8 hr.

219.—(1) 183 da. (2) 162 da. (3) 288 da. (4) 147 da. (5) 120 da.
(6) 208 da. (7) 166 da. (8) 164 da. (9) 541 da. (10) 510 da. (11) 383
da. (12) 530 da. (13) 127 da. (14) 212 da. (15) 238 da. (16) 219 da.

125 ***220.***—(1) 28 da. 2 hr. 5 min. 4 sec. (2) 139 bu. 1 pk. 6 qt. 1 pt. (3) 39
lb. 18 pwt. 18 gr. (4) 65 gal. 1 qt. 2 gi. (5) 159 A. 53 P. 10¾ sq. yd. (6)
202 mi. 109 rd. 2 yd. 1 ft. 6 in. (7) ℔. 15 ℥10 ʒ7 ℈1 gr. 10. (8) £10008
8s. (9) 977° 40′ 16″. (10) 1 mi. 197 rd. 5 yd. 11 in. (11) £68 12s. 8d.
(12) $90.65+. (13) $37. (14) 8183 mi. 40 rd. 4 yd. (15) 92 lb. 8 oz.
(16) 41 A. 63 P. 19 sq. yd. 8 sq. ft. 36 sq. in. (17) £833 8s. 4d. (18) 4 mi.
213 rd. 1 yd. 2 ft. 6 in.; 9 bu. 2 pk. 6 qt. (19) 1 bu. 1 pk. 7 qt. 1 pt.; 9
cwt. 16 lb.

126 ***221.***—(1) £2 1s. 10½d. (2) 24 bu. 6 qt. 1 pt. (3) 14 lb. 7 oz. 17 pwt.
(4) ℥7 ʒ2 ℈1 gr. 8⅓. (5) 9° 53′ 6″. (6) 1 T. 8 cwt. 58⅔ lb. (7) 26 yr. 73
da. 2 hr. 36 min. (8) 8 mi. 106 rd. 3 yd. 2 ft. (9) 15 gal. 3 qt. 1 pt. 2 gi.
127 (10) 1 gal. 2 qt. 0$\frac{8}{25}$ pt.——(12) 3 da. 23 hr. 33 min. 30$\frac{5}{13}$ sec. (13) 1 T. 16
cwt. 72 lb. 13$\frac{5}{13}$ oz. (14) 3 A. 117 sq. rd. 23 sq. yd. 32$\frac{16}{17}$ sq. in. (15) £2
4s. 2d. 3$\frac{111}{343}$ far. (16) 81$\frac{19}{173}$ cu. ft. (17) 54 A. 45 sq. rd. 25 sq. yd. 1 sq. ft.
126 sq. in. (18) £1 5s. 7d. (19) 20 mi. 266 rd. 3 yd. 2 ft. (20) 1 oz. 15
pwt. (21) 113 da. 13 hr. 20 min.; 3 yd. 1 ft. 6 in.; 10 T. 13 cwt. 33 lb. 5⅓
oz. (22) 20 gal. 2 qt. 1 pt.; 9 cwt. 60 lb.; 2 rd.

222.—(1) 13 times. (2) 5 times. (3) 28 times. (4) 383940 times. (5)
128 909$\frac{19}{205}$ times. (6) 8 times.——(7) 14 yd. (8) 15 da. (9) 19 hr. (10) 145
bottles. (11) 20 times; 36⅓ times; 3 times. (12) 640 times; 37$\frac{11}{12}$ times;
.5 times.

223.—(1) \$578.75. (2) \$13.72. (3) \$150. (4) \$105.31+. (5) \$76.25. **128**
(6) 2s. 6d. (7) \$544. (8) 53 A. 3 sq. ch. 5$\frac{1}{5}$ P. (9) \$105.60. (10) \$124.46.
(11) 3 yr. 11 mo. 28 da. (12) 235 da.——(13) 24 da. (14) \$200.64. (15) **129**
17350 bu. 5 qt. (16) \$20.40. (17) 84$\frac{3}{8}$ gal. (18) \$8.82. (19) $\frac{3}{8}$ gold. (20)
20 carats. (21) \$8.13. (22) $\frac{11}{16}$. (23) 9 in. (24) \$75.87+. (25) \$64.20.
(26) 84 bbl. and 1 bu. 2 pk. 4 qt. remainder. (27) \$74.70. (28) 5376
revolutions. (29) 58 min. 19$\frac{1}{2}$ sec. (30) £710 13s. $\frac{1}{2}$d. (31) \$.49. (32)
2 hr. 11 min. 24$\frac{13}{17}$ sec. (33) 1875 mi. (34) \$727.23+ (35) 15 hr. 29 min. **130**
30 sec. (36) 1508$\frac{4}{7}$ times. (37) \$400.73—. (38) 183 bu. 3 pk. 6 qt. (39)
\$1.28. (40) 25 forks. (41) 6 mi. 80 rd. (42) \$15849.58+. (43) \$84.35+.
(44) 415 m. (45) \$68.25. (46) \$172.80. (47) \$3.63—. (48) $\frac{64}{175}$ in.
(49) .2544375 T. (50) ℔ ℥ 6 ℳ24. (51) 186 rd. 3 yd. 2 ft. (52) $\frac{1}{1152}$ da.
(53) \$36.26.

226.—(1) 6° 41′ 21″. (2) 13° 26′ 1″. (3) 95° 55′ 9″. (4) 103° 30′. **132**
(5) 149° 14′ 13″.

230.—(1) 2 hr. 9 min. 37 sec. (2) 44 min. 37 sec. past 10, A. M. (3) 5 **133**
hr. 44 min. 48$\frac{4}{13}$ sec. (4) 45 min. 11$\frac{11}{13}$ sec. past 8, A. M. (5) 2 min. 48$\frac{4}{13}$ sec.
past 5, P. M. (6) 35 min. 18 sec. past 10, A. M. (7) 18 min. 44 sec. past
7, P. M. of preceding day. (8) 16 min. 56 sec. past 6 o'clock, A. M.

231.—(1) 70° 15′ 15″ west. (2) 71° 8′ 30″ west. (3) 88° 19′ 2″ east. **134**
(4) 100° 29′ 15″ east. (5) 16° 19′ east.

248.—(16) 500.15 A. (17) 5848.703 M. (18) 1005.06 G. (19) 5.018- **139**
123009 Cu. M. (20) 150.9 S. (21) 930.074 L. (22) 509.84 sq. dm.

249.—(1) 132718.2 dg. (2) 3728146 cm. (3) 15.718326 Hl. (4) **140**
1782.421 Dg. (5) 8.17513152 Cu. M. (6) 518.541531 Sq. M. (7) 3871341
sq. dm. (8) 51718340 cu. cm. (9) 1.358 Ha. (10) 98716.25 ds. (11) 31.76
S. (12) 18.753128 Ha. (13) 718.6325 Hm. (14) 8.41867321 Kg.

250.—(1) 9.5195275 A. (2) 1064.0625 bu. (3) 95.06961 mi. (4) 1275.- **141**
9411 gal. (5) 166.4473 lb. (6) 389.80025 A. (7) 10.82675 in. (8) 79.5528
M. (9) 677.688 S. (10) 6367.9545 A. (11) 65.4805 Dl. (12) 93.312 G.
(13) 65.8988 Hl. (14) 646.0025 Cu. M.

251.—(1) 280.28873 M. (2) 175.561 L. (3) \$10.90—. (4) \$4.83. (5)
\$69.94—. (6) \$591.13+. (7) \$5820.75. (8) \$63.05—. (9) \$124.68+.
(10) \$400.34+. (11) \$2.95+.

265.—(1) 42 A. (2) 375 sq. ft. 77$\frac{3}{5}$ sq. in. (3) 1050 A. (4) 112 sq. yd. **143**
(5) 61 sq. ft. (6) \$23760. (7) \$24.84. (8) \$150.06—. (9) \$4290. (10) 1
A. 36 sq. rd. 22 sq. yd. 8 sq. ft. 72 sq. in.

266.—(1) 46 ch. (2) 90 rd. (3) 2 mi. (4) 6 yd. 1 ft. (5) 18 ft. (6) **144**
53 rd. (7) 7 yd. 2 ft.

267.—(1) 29 yd. 2 ft. 7$\frac{1}{5}$ in. (2) 41 yd. 1 ft. 10$\frac{1}{8}$ in. (3) 107 yd. 6$\frac{4}{7}$ in. **145**
(4) 305 yd. 11$\frac{1}{25}$ in. (5) 39 yd. 1 ft. 1$\frac{1}{8}$ in. (6) \$80.83+. (7) 2201$\frac{3}{5}$ bricks.
(8) 278 flag-stones. (9) \$62.61+. (10) 10368 shingles. (11) 1139 sods.
(12) \$31.13—.——(14) 37 yd. (15) 117 yd. 1 ft. 4 in. (16) 28-in. carpet. **146**
(17) \$12.

271.—(1) 3024 cu. ft. (2) 60 cu. yd. (3) \$76.80. (4) \$32.50. (5) **148**
1968$\frac{3}{4}$ lb. (6) \$30.94. (7) \$22.95.

149 ***272.***—(1) 6 ft. (2) 7 ft. (3) 1 ft. 6 in. (4) 6 ft. 2 in. (5) 8 ft. (6) 8 ft. (7) 9 ft. 6 in. (8) $138.

150 ***276.***—(1) $10\frac{1}{3}$ bd. ft. (2) $13\frac{1}{3}$ bd. ft. (3) 20 bd. ft. (4) $22\frac{1}{2}$ bd. ft. (5) 33 bd. ft. (6) $41\frac{2}{3}$ bd. ft. (7) 63 bd. ft. (8) 40 bd. ft. (9) $67\frac{1}{2}$ bd. ft. (10) $37\frac{1}{2}$ bd ft. (11) $18.56+. (12) $4.20. (13) $22.00—. (14) 24 bd. ft.
151 (15) $14.69.——(17) 1 ft. 2 in. (18) 15 ft. (19) 1 ft. 8 in. (21) 8 ft. (22) 9 in. (24) 9 in. (25) $145.99+.

152 ***278.***—(1) $353.94—. (2) $2\frac{5}{6}$ ft. (3) $1923.64—. (4) 9 ft.

154 ***280.***—(1) $562785\frac{105}{111}$ bricks. (2) $200220\frac{34}{35}$ bricks. (3) $268311\frac{8}{11}$ bricks.

155 ***282.***—(1) 133.5925+ bu. (2) 110.055+ bu. (3) 18.8666+ bu. (4) 48.2138 bu. (5) $3783\frac{8}{11}$ gal. (6) 34.9117+ bu. (7) 894.9907+ hhd. (8) 167.565+ gal. (10) 3 ft. 7.83+ in. (11) 10 ft. 4.44+ in. (12) $.73+.
156 (13) 6037.312 lb. (14) 4 ft. 5.42+ in. (15) $151.20. (16) $163.20. (17) 5 ft. (18) 128 bu. (20) 1028.16 gal. (21) 1175.04 gal. (22) 713836.8 gal. (23) $74.09—.

159 ***290.***—(1) $.19—. (2) 353.7 lb. (3) $1063.47—. (4) 316 men. (5) 2388 ft. (6) .91 mi. (7) £130 7s. (8) 604.62 m. (9) $8918.20. (10) 3924 ft. (11) 5828 bu. (12) $690. (13) $5377.50. (14) 535 fr. (15) 779 T. (16) $26250. (17) $29500. (18) $822.29. (19) 493 sheep. (20) $22660.
160 (21) $12743.50. (22) 231 girls. (23) $17225. (24) $138.75. (25) $14062.50. (26) $1070.60. (27) $4082\frac{2}{3}$ lb. (28) 468 yd. (29) $225.

161 ***291.***—(1) $33\frac{1}{3}$%. (2) 52%. (3) 20%. (4) 8%. (5) 160%. (6) $6\frac{2}{3}$%. (7) 25%. (8) 5%. (9) $33\frac{1}{3}$%. (10) 18%. (11) 34%. (12) 60%. (13) $16\frac{2}{3}$%. (14) 8%. (15) $33\frac{1}{3}$%. (16) A 48%; B 52%. (17) 20%. (18)
162 14%. (19) 60% more; $37\frac{1}{2}$% less.——(20) 96%. (21) 66%. (22) 40%. (23) 8%. (24) 18%. (25) 35%. (26) $22\frac{2}{3}$%. (27) $11\frac{13}{17}$%. (28) 30%.

163 ***292.***—(1) 800 lb. (2) 630 gal. (3) 1250 mi. (4) 500 T. (5) $3865. (6) $7132. (7) 15 fr. (8) $823. (9) 300 T. (10) $8000. (11) 3425 bu. (12) 9000 yd. (13) 225 da. (14) 36 sheep. (15) $5292. (16) 875 trees.
164 (17) $8077.50. (18) 300 A.——(19) $4322.50. (20) 2825 bu. (21) $38.34. (22) 2150 lb. (23) 500 A. (24) $4836. (25) $2982. (26) $115. (27) $16088.63—. (28) 1850 lb.

293.—(1) 75%. (2) Ger. 13085; Ir. 15702; Fr. 2617. (3) $63869. (4)
165 $1300. (5) Saltpetre 75%; sulphur 10%; charcoal 15%. (6) $817.50. (7) $857\frac{1}{7}$% of yield; $757\frac{1}{7}$ of increase. (8) $4280. (9) 7%. (10) Oct. 30, $366.70; Nov. 18, $374.42; Dec. 24, $386; Jan. 4, $405.30. (11) $699.96. (12) 900
166 lb. (13) $16\frac{2}{3}$%. (14) 1700 bbl. (15) $7716. (16) $351.——(17) $3500. (18) horse $320; buggy $250. (19) dress goods $30; jewelry $34.50. (20) wool $863\frac{1}{13}$ lb.; cotton $258\frac{12}{13}$ lb. (21) A $720. (22) 1st, 2740 bbl.; 2d, 3425 bbl.; 3d, 2329 bbl. (23) oysters $92; fish $41.40; clams $33.35. (24) $34427.43. (25) 30%, $24.51; 45%, $36.765; 25%, $20.425. (26)
167 wife $8750; son $9750; daughter $6500.——(27) 40%. (28) 665 sheep. (29) $3724; $5586. (30) 32500 inhab. (31) 1680 lb. (32) native $76\frac{12}{13}$%; foreign $23\frac{1}{13}$%. (33) $17\frac{11}{17}$%. (34) 80%. (35) tea $5.20; sugar $2.34. (36) 288 bbl. (37) 9%. (38) $19.69—; $14.06+. (39) $7488. (40) $10.24—.

295.—(1) $881.46. (2) $30.06. (3) $191.10. (4) $6.90. (5) $37.50. 169
(6) $1965.42. (7) $35. (8) $89.42—. (9) $3651.64+. (10) $192.30.
(11) $1009.91. (12) $1668.60. (13) $38.28. (14) $252. (15) $1129.50.
(16) 20%. (17) $33\frac{1}{3}$%. (18) 25%. (19) $16\frac{2}{3}$%. (20) $\frac{1}{16}$%. (21) $\frac{5}{8}$%.
(22) $2\frac{1}{2}$% gain. (23) $16\frac{2}{3}$% loss. (24) 10% loss. (25) $33\frac{1}{3}$% gain. (26)
$\frac{1}{8}$% loss. (27) $8976.——(28) $375. (29) $825. (30) $387. (31) $20700. 170
(32) $5900. (33) $325. (34) $63.30. (35) $135.70. (36) $6894. (37)
$47.80. (38) $10. (39) $720. (40) 20%. (41) $260. (42) $1377. (43)
13%. (44) $5076. (45) $53.13. (46) 100%. (47) 135 bu. (48) $101.04.
(49) $6030. (50) $16\frac{2}{3}$%. (51) 74¢. (52) 9%. (53) 15%. (54) 12%. 171
(55) $63. (56) $204.70. (57) $2123.33+. (58) 20%. (59) $45. (60)
$1010.60. (61) 40%. (62) $259.75—. (63) 25%.——(64) $26.25. (65) 30%. 172
(66) A's gain $2600; B's loss $1950. (67) lost $240. (68) 25%. (69)
$11160. (70) $5.70. (71) $162.40. (72) $416. (73) $4000. (74) 33¢.
(75) 60%. (76) $9\frac{3}{8}$%. (77) 4%. (78) 30%. (79) 4%. (80) $7\frac{1}{2}$%. (81)
$311.85. (82) $800. (83) 20%. (84) $155.61. (85) 5% gain. (86) $66\frac{2}{3}$%. 173
(87) $12.83— loss. (88) $2.53. (89) 40% gain. (90) $6\frac{1}{4}$%. (91) $1365.
(92) $1237.48—. (93) $5600; $6720.——(94) $5.46. (95) $1118.91—. (96) 174
$53760. (97) $230. (98) 4% loss. (99) 20%. (100) Rio. 304 lb.; Java.
190 lb. (101) $75. (102) $1111.50. (103) $5096.——(104) $675. (105) 175
21¢. (106) $225. (107) 3%. (108) $7.35. (109) $24. (110) $9.95—.

298.—(1) $243. (2) $413.61—. (3) $792.70—. (4) $2219.64+. (5) 177
$1281.30+. (6) $269.72—. (7) $557.46. (8) $577.39+. (9) $1123.02—.
(10) Of B; $.93—. (11) $6.41+.——(12) $207.63+. 178

299.—(1) $14\frac{1}{3}$%. (2) 28%. (3) $23\frac{1}{3}$%. (4) $40\frac{3}{16}$%. (5) $81\frac{1}{4}$%. (6)
$17\frac{13}{100}$%. (7) $\frac{73}{100}$%. (8) 3%. (9) $14\frac{1}{16}$%.

300.—(2) $3.20. (3) $14.81+. (4) $6.67—. (5) $41.15+. (6) $30. (8) 179
$500. (9) $138.90+. (10) $272.96—. (11) $16.11—. (12) $19.65—.
(14) $16\frac{2}{3}$%. (15) 20%. (16) $33\frac{1}{3}$%. (17) 25%. (18) 20%. (19) $70.
(20) 50%. (21) $12.06+.——(22) $8.04—. (23) $66\frac{2}{3}$¢. (24) $20. (25) 180
$87\frac{1}{2}$%. (26) 8%. (27) $18\frac{3}{4}$%. (28) 20%. (29) $432. (30) $1406.42—;
$352.80; $648. (32) $147.50. (33) $52.66. 181

304.—(2) $1.07. (3) $34.76+. (4) $7.75. (5) $2.95. (6) $56.25. (7) 184
$722.25. (8) $1205.70. (9) $17757.20.——(10) $669.75. (11) $17.42. (12) 185
$7634.95. (13) $2512.75. (14) $80.46+. (16) $1\frac{5}{8}$%. (17) $\frac{3}{13}$%. (18) $\frac{3}{16}$%.
(19) 5%. (20) 4%. (21) 2%. (22) $\frac{1}{4}$%. (23) $3\frac{1}{3}$%. (25) $2515. (26)
$1506. (27) $2440. (28) $3900. (30) $82. (31) $350. (32) $512. (34)
$180. (35) $560. (36) $1600. (38) $151.02+. (39) $720. (40) $800. 186
(42) $32. (43) $136. (44) $248. (45) $11.04. (46) $\frac{1}{13}$%. (47) $536.
(48) $530. (49) 3%. (50) $20.25.——(51) $7761.88—. (52) $2\frac{1}{2}$%. (53) 187
$173.20. (54) $322.26—. (55) 2%. (56) 7360 bu. (57) $3370. (58)
$1\frac{1}{3}$%. (59) $9930.27. (60) $1\frac{1}{4}$%. (61) 184 bbl. (62) 5%.——(63) $361. 188
(64) 3600 bu. (65) $\frac{1}{2}$%. (66) $8.75. (67) 26300 lb. (68) $591.07. (69)
$205.17—. (70) 3%. (71) $11.34.——(72) 77 bbl.; $2.91—. (73) 56 bbl.; 189
42¢. (74) 116 bbl.; $2.51. (75) $1823.25. (76) $1840; $46.90. (77) $320.
(78) $680.40. (79) 75¢.——(80) flour $2250; bacon $2100. (81) 2%. (82) 190

190 14000 lb. coffee; $142.87. (83) 78 bbl. (84) $1.50. (85) $15.90. (86)
191 $3008. (87) $1033.50.——(88) 2%. (89) $128.19–. (90) $43000. (91)
192 $1167.25. (92) $3255.69–.——(93) $431.09.
196 ***317.***—(1) $2.20+. (2) $30.88–. (3) $2.29+. (4) $9.49–. (5)
$1.37–. (6) $44.08–. (7) $121.90+. (8) $70.33–. (9) $1.13+. (10)
$32.05+. (11) $50.20. (12) $1.74+. (13) $5.66–. (14) $3.71+. (15)
$51.00–. (16) $11.80+. (17) $10.20–. (18) $37.35–. (19) $10.70+.
(20) $410.81+. (21) $70.27–. (22) $5.37+. (23) $24.42–. (24)
$61.47–. (25) $12.15+. (26) $3.17+. (27) $28.83–. (28) $.35–.
(29) $7.23. (30) $2.76–. (31) $205.04. (32) $16.92–. (33) $237.51+.
(34) $36.08–. (35) $12.28+. (36) $30.18–. (37) $9.72+.
198 ***318.***—(1) $6.25+. (2) $10.84–. (3) $8.07–. (4) $5.46+. (5) $1.16+.
(6) $1.69–. (7) $5.36–. (8) $90.29+. (9) $17.47–. (10) $4.31–.
(11) $3.42–. (12) $1.11+. (13) $.17–. (14) $.06–. (15) $.04+.
199 ***319.***—(1) $17.13–. (2) $5.06+. (3) $106.52+. (4) $19.40+. (5)
$53.24–. (6) $42.13+. (7) $297.34+. (8) $146.26–. (9) $13.51+.
(10) $5.92+. (11) $59.23+. (12) $12.03+. (13) $144. (14) $19.81–.
(15) $15.92–. (16) $37.35–. (17) $27.59–. (18) $81.73–. (19) $9.19–.
(20) $1.51+. (21) $2.91–. (22) $283.55+. (23) $34.70+. (24) $62.63+.
(25) $226.72+. (26) $67.80+. (27) $62.90+. (28) $189.95+. (29)
$734.95+. (30) $37.37+. (31) $43.57–. (32) $123.31–. (33) $90.13.
(34) $363.98+.
201 ***320.***—(1) $131.25. (2) $390.02+. (3) $54.33+. (4) $169.87+. (5)
$20.81+. (6) $33.71–. (7) $222.09+. (8) $219.97–. (9) $14.47+.
(10) $68.96+. (11) $30.27–. (12) $121.90–. (13) $694.72–. (14)
$6.23+. (15) $111.55–. (16) $24.28–. (17) $1781.76+. (18) $98.37–.
(19) $274.78+. (20) $123.74+. (21) $62.42+. (22) $438.39–. (23)
$7.40+. (24) $188.27–. (25) $2.09–. (26) $8.24+. (27) $1965.68+.
202 (28) $44.40–. (29) $2.37+. (30) $100.16+.——(31) $224.73–. (32)
$44.42–. (33) $60.17+. (34) $1401.01–. (35) $305.00–.
203 ***321.***—(1) $3.66+. (2) $3.95. (3) $4.08–. (4) $2.34–. (5) $202.71–.
(6) $16.55–. (7) $7.82–. (8) $295.58+. (9) $1524.75. (10) $9.48+.
(11) $4.92. (12) $24.63+. (13) $20.99+. (14) $25.81+. (15) $166.65–.
(16) $9.65+. (17) $392.35+. (18) $196.46–. (19) $337.94+. (20)
$14.90+.
205 ***322.***—(1) $.68+. (2) $10.20. (3) $11.85. (4) $56.02–. (5) $7.72+.
(6) $19.18–. (7) $108.61–. (8) $33.48+. (9) $5.83–. (10) $24.35+.
(11) $10.38–. (12) $560.58–. (13) $93.16+. (14) $34.37–. (15)
$105.27+. (16) $633.32+. (17) $31.25. (18) $5.10–. (19) $128.31–.
(20) $35.16+. (21) $131.71–. (22) $325.87–. (23) $21.41–. (24)
$18.12+. (25) $675.04+. (26) $299.10+. (27) $64.73+. (28) $137.09+.
206 ***323.***—(1) $893.99–. (2) $317.82+. (3) $3774.36+. (4) $863.52.
(5) $25.51+. (6) $418.35–. (7) $521.81+. (8) $773.24+. (9) $621.25.
(10) $327.83–. (11) $862.07–. (12) $6027.30–.
207 ***324.***—(1) $.38+. (2) $23.16+. (3) $2.52+. (4) $1.54–. (5) $5.26–.
(6) $9.09–. (7) $62.18+. (8) $12.11+. (9) $19.90–. (10) 59.99–.

(11) $864.54+. (12) $75.07—. (13) $20.94—. (14) $66.84—. (15) 207
$12.33—. (16) $284.91—. (17) $337.01—. (18) $56.32+.——(20) £2 19s. 208
5$\frac{1}{4}$+d. (21) £2 13s. 10$\frac{1}{4}$+d. (22) £5 0s. $\frac{1}{4}$+d. (23) 208.16— m.

325.—(1) 5%. (2) 8%. (3) 4%. (4) 10%. (5) 6%. (6) 9%. (7) 4%.
(8) 8%. (9) 9%.

326.—(1) 3 yr. 5 mo. 18 da. (2) 2 yr. 1 mo. 24 da. (3) 11 mo. 5 da. 209
(4) 6 da. (5) 18 da. (6) 28 da. (7) 33 yr. 4 mo.; 25 yr.; 20 yr.; 16 yr. 8
mo. (8) 25 yr.; 42 yr. 10 mo. 9 da.

327.—(1) $2280. (2) $630. (3) $1500. (4) $595. (5) $350. (6) $600. 210
(7) $320. (8) $865. (9) $3576.42—.

328.—(1) $171. (2) $400. (3) $666. (4) $7331.94+. (5) $8960. 211
(6) $1200. (7) $4383.55+. (8) $479.80+. (9) $12.75.

329.—(1) 8%. (2) $32.49—. (3) $590.57—. (4) $606.06. (5) May 212
16, 1884. (6) $963.19+. (7) $5293.60. (8) $787.79—. (9) Aug. 10, 1884.
(10) $375.75. (11) $518.80. (12) $2060.43+. (13) Nov. 4, 1884. (14) 213
4%. (15) $9001.48+. (16) $326.71—. (17) $413.43—. (18) 7 yr. 6 mo.
(19) 4%.——(20) accurate interest $4.14 less. (21) 1 mo. 4 da. (22) 9%. 214
(23) 5%. (24) 31$\frac{11}{18}$%. (25) 1$\frac{23}{27}$% disct.

330.—(1) $12692. (2) $440.72. (3) $5467.50. (4) $12509. (5) $6687.50. 216
(6) $8719.86. (7) $11039.88—. (8) $10027.28. (9) $437.05—. (10)
$4343.78. (11) $185.45—. (12) $980.95. (13) $3222. (14) $9369.60.
(15) $3960.47—. (16) $1848.71—. (17) $3805.64+.

331.—(1) $1339.81+. (2) $1053.12+. (3) $594.50+. (4) $1397.18—. 218
(5) $974.42+. (6) $1339.94—. (7) $1157.66—. (8) $71.71+. (9)
$434.58—. (10) $89.83+. (11) $165.67+. (12) $468.92+. (13)
$3161.69+. (14) $3972.11—.

333.—(2) $20724.63—. (3) $90305.77+. (4) $23262.27+. (5) $709.94+. 221
(6) $2581.80—. (7) $6108.95—. (8) $3486.95—. (9) $3065.22—. (10)
$13384.60—. (11) $358.24+. (12) $10782.96—.——(14) 5%. (15) 5%. 222
(16) 6%. (17) 9%. (18) 6%. (19) 4%. (21) 11 yr. (22) 9 yr. 6 mo.
(23) 4 yr. (24) 52 yr.——(26) $500. (27) $5000. (28) $40. (29) $2000. 223
(30) $4370.77—. (31) $531.50+. (32) $5000.

337.—(1) $832; $2.08. (2) $8220; $100.01. (3) $300; $6.20. (4) $291; 224
$14.55. (5) $270; $6.30. (6) $259.88—. (7) $580.——(8) $2782.50 at 2 mo. 225
(9) $3.60. (10) $242.24+. (11) $6342.54 [at simple int.] (12) $26.37—.
(13) $3231.37. (14) $3611.94.

346.—(1) Apr. 22. (2) May 3. (3) Nov. 6. (4) Feb. 22, 1886. (5) 228
Nov. 12. (6) Apr. 21. (7) June 6. (8) Oct. 28. (9) Aug. 16. (10) Jan.
3, 1886.

347.—(1) 125 da. (2) 64 da. (3) 94 da. (4) 125 da. (5) 94 da. (6) 229
156 da. (7) 63 da. (8) 155 da. (9) 95 da. (10) 126 da.

348.—(1) 138 da. (2) 111 da. (3) 99 da. (4) 97 da. (5) 128 da. (6) 230
101 da. (7) 66 da. (8) 149 da. (9) 98 da. (10) 124 da.

354.—(1) Mar. 9, 1885; 36 da.; $9; $1491. (2) Jan. 1, 1886; 54 da.; 232
$9.98; $940.02. (3) Apr. 11, 1886; 30 da.; $1.63; $323.37. (4) Feb. 11,
1886; 41 da.; $1.37; $148.63. (5) Apr. 8, 1884; 40 da.; $3.33; $496.67.

232 (6) Mar. 3, 1884; 55 da.; $9.17; $1190.83. (7) June 6, 1885; 52 da.; $2.38;
$272.62. (8) Nov. 16, 1883; 71 da.; $38.66; $2761.34. (9) Nov. 9, 1886;
52 da.; $45.07; $5154.93. (10) $218.98–. (11) 53 da.; $35.33; $2964.67.
233 (12) $7.31; $795.24. (13) Jan. 12, 1887; 60 da.; $5.38; $532.62. (14)
234 Jan. 11, 1886; 30 da.; $3.26; $486.16.——(15) $5.11; $494.89. (16) $593.25.
(17) $34.29; $1802.61. (18) $1.08. (19) discounts, $12.47; $2.22; $8.45;
235 total proceeds, $1728.41. (20) $573.97.——(21) Feb. 1, 1885; 56 da.; $12.83;
$1487.17. (22) Sept. 17, 1884; 45 da.; $7.02; $1116.38. (23) Oct. 12,
1884; 102 da.; $21.25; $1228.75. (24) Oct. 8, 1885; 50 da.; $4.27; $435.23;
(25) No. 1—Mar. 3, 1886; 118 da.; $14.75; $985.25. No. 2—Mar. 3, 1886;
118 da.; $10.33; $689.67. No. 3—Mar. 3, 1886; 118 da.; $8.26; $551.74.
No. 4—Mar. 3, 1886; 118 da.; $.93; $62.27.
236 ***355.***—(1) $680. (2) $740. (3) $372. (4) $400. (5) $120. (6) $240.
(7) $1400. (8) $540. (9) $1181.73–. (10) $400. (11) $722.63–. (12)
$438.33+. (13) $761.81–. (14) $1400. (15) $1020.93–. (16) $503.69+.
(17) $2549.70–.
237 ***356.***—(1) 8%. (2) 6%. (3) Aug. 7, 1883. (4) May 18, 1888.
238 ***357.***—(1) $806.23. (2) $1814.79. (3) $6501.37.
241 ***360.***—(1) $206.62. (2) $358.58. (3) $512.68. (4) $594.78.
244 ***361.***—(1) $230.57. (2) $263.44. (3) $265.38. (4) $344.05.
246 ***364.***—(1) $493.19. (2) $1317.07. (3) $719.38. (4) $2543. (5) $2790.03.
249 ***365.***—(1) $315.13. (2) $520.84. (3) $46.86. (4) $1017.16. (5) $810.25.
250 (6) $864.89. (7) $2906.55. (8) $1572.26. (9) $365.15.——(10) $2445.21.
253 ***383.***—(2) $112. (3) $1087.50. (4) $656.25. (5) $876.38–. (6) $324.
254 (7) $629.63–.——(9) $19216.13–. (10) $3258.75. (11) $2681. (12)
$15413.06+. (13) $1553.38–. (14) $2925.38–. (16) $9849.38–. (17)
$2107.38–. (18) $3692.50. (19) $5859.75. (20) $6754.50. (21) $6205.
(23) 3%. (24) $6\frac{5}{8}$%. (25) $14\frac{1}{4}$%. (26) $12\frac{3}{8}$%. (27) $117\frac{7}{8}$. (28) $8\frac{1}{4}$%.
255 (29) $32\frac{7}{8}$%. (31) $103\frac{1}{8}$.——(32) $23\frac{5}{8}$%. (33) $15\frac{7}{8}$%. (34) $83\frac{3}{4}$. (35) $19\frac{5}{8}$%.
(36) $13\frac{1}{8}$%. (38) $3100. (39) $12500. (40) $17000. (41) $11700.
(42) $6900. (43) $8400. (44) $7000. (46) $5000. (47) $12000. (48)
$15000. (49) $13800. (50) $3800. (51) $32000. (52) $16014.38–.
256 (53) $562.50. (54) $85\frac{7}{8}$.——(55) 150 sh. (56) $3365. (57) $15\frac{3}{8}$%. (58) 74
sh. (59) $157.50. (60) $196\frac{5}{8}$. (61) $421.88–. (62) $12800. (63) $34\frac{3}{8}$%.
(64) $10145.
257 ***384.***—(2) $75. (3) $70. (4) $150. (6) $87\frac{3}{8}$. (7) $59\frac{7}{8}$. (8) $79\frac{7}{8}$. (10)
258 5%. (11) 8%. (12) $6\frac{5}{8}$%. (14) $12\frac{1}{2}$%. (15) $5\frac{1}{3}$%. (16) $3\frac{1}{3}$%.——(18) 7%.
(19) 5%. (20) 3%. (22) 6%. (23) 12%. (24) $7000; $87\frac{3}{8}$. (25) 4%.
(26) 5% bonds are $\frac{15}{16}$% on inv. better. (27) $32000. (28) $7\frac{1}{2}$%. (29)
259 $4820. (30) $13200. (31) $10\frac{5}{8}$%. (32) 4%. (33) $36000.——(34) $149\frac{7}{8}$.
(35) $10000. (36) $1125. (37) $9\frac{1}{11}$%. (38) $1300. (39) $8460. (40)
$109\frac{7}{8}$. (41) $67230. (42) 18 sh. (43) $2026. (44) $258. (45) $72\frac{8}{11}$.
260 (46) $166\frac{3}{8}$. (47) 4%; $50. (48) $288.75. (49) 2%. (50) $5205. (51) $370.
(52) 3%. (53) $85020. (54) $3200. (55) $2000. (56) $200 gain. (57).
261 4%.——(58) $300. (60) $89\frac{7}{17}$. (61) $92\frac{4}{11}$. (62) 107[illegible]. (64) 4[illegible]%. (65).
6% bonds are $1\frac{11}{128}$% better. (66) $5\frac{1}{3}$%.

391.—(2) \$7236. (3) \$8436.25. (4) \$3180. (5) \$1505.63—. (8) 265
\$3489.50. (9) \$1795.80. (10) \$2466.67—. (11) \$752.81+. (12) \$4928.75. 266
(14) \$824. (15) \$352. (16) \$7500. (17) \$136. (18) \$1200.——(21) \$120. 267
(22) \$1600. (23) \$720. (24) \$1200. (25) \$1824.53—. (26) \$3530.70. (27)
\$1200. (28) \$12013.96+. (29) \$2396.80—. (30) \$5549.80. (31) \$21128.85+.
(32) \$15387.98—.

397.—(1) \$1649.85. (2) \$3889. (3) \$1699.42—. (4) \$3549.92+. (5) 271
\$611.96—. (6) \$80.66+. (7) \$1000.77+. (8) \$300.94—. (9) \$1523.23—.
(10) \$736.58—. (11) \$2301.38—. (12) \$354.84+. (13) \$428.63—. (14)
\$759.08+. (15) \$4908.09+. (16) \$669.54+. (17) \$1103.05—. (18)
\$2211.77+. (19) \$3646.08—. (20) \$194.08—.

398.—(1) £1260. (2) £560. (3) 4161 fr. (4) 1682.37 fr. (5) 3600 gu. 272
(6) 448 gu. (7) 1280 m. (8) 3600 m. (9) £1195 17s. $6\frac{1}{4}$+ d. (10)
9505.73— fr. (11) 18600.20+ gu. (12) 35697.82+ m.——(13) £800. (14) 273
£740. (15) 4888 fr. (16) 800.93— gu. (17) 3200 m. (19) 112.43—%.
(20) 105.15+%. (21) 89.51—%. (22) 91.98+%. (24) 126.23%. (25)
114.70—%.——(26) 102.93—%. (27) 114.40+%. 274

407.—(1) \$447.60. (2) \$521.55. (3) \$1872.50. (4) \$406.70. (5) 276
\$363.75. (6) 134.25. (7) \$365.04. (8) \$177.20.——(9) \$1958.75. (10) \$6720. 277
(11) \$900. (12) \$67.50. (13) \$955.40. (14) \$182.40. (15) \$466.35.
(16) \$1409. (17) \$3465.40. (18) \$6.44—. (19) \$385.80. (20) \$87.15.
(21) \$15.60. (22) \$399. (23) duty \$469; exchange \$955.46+.

415.—(1) \$35. (2) \$27. (3) \$122.50. (4) \$227.70. (5) \$52.29. (6) 279
\$131.25. (7) \$103.34+. (8) \$27.93. (9) 112.50. (10) \$228.92. (11) \$210.
(12) \$3600.35.

416.—(1) 12 m. (2) 9 m. (3) 11 m. (4) \$1.38. (5) 98¢. (6) \$1.13. 280
(7) \$1.52. (8) rate 6 m.; A's taxes \$50.70. (9) rate 32¢ on \$100; B's tax
\$26.56.——(10) rate 30¢ on \$100; C's tax \$47.40. 281

429.—(1) \$40.50. (2) \$531.25. (3) Eagle \$5294.12—; Manhattan 284
\$7941.18—; Globe \$4764.70+. (4) Home \$2500; Essex \$1500; Howe
\$2000. (5) \$30; \$37.50. (6) \$109.375. (7) \$800; \$1400.——(8) \$1333.33+; 285
\$5000. (9) $\frac{7}{8}$%; $87\frac{1}{2}$¢ on \$100. (10) \$300. (11) $1\frac{1}{2}$%. (12) 8%; \$400.
(13) 2%. (14) $\frac{5}{14}$%. (15) 96¢. (16) \$5400. (17) \$12000. (18) \$105.
(19) \$3200. (20) \$12000; \$1200. (21) \$8000. (22) \$8501.77. (23) \$720. 286
(24) \$52006.08—.

433.—(2) \$7220.85. (3) \$383.80. (4) \$28.34. (5) \$80.88. (6) \$147.06. 289
(7) \$3010.14. (8) \$78.30. (9) \$9.20—.——(11) \$2361.75. (12) \$6979.12. 290

437.—(1) 7. (2) $2\frac{1}{3}$. (3) 4. (4) $3\frac{1}{15}$. (5) $\frac{5}{6}$. (6) $\frac{1}{3}$. (7) $2\frac{1}{4}$. (8) $\frac{4}{5}$. 291
(9) $\frac{1}{3}$. (10) $\frac{3}{7}$. (11) $2\frac{2}{15}$. (12) $5\frac{1}{3}$. (13) 23. (14) $7\frac{1}{3}$. (15) $\frac{2}{15}$. (16) $5\frac{7}{8}$.

442.—(1) $11\frac{1}{3}$¢. (2) \$4. (3) 7 mi. (4) \$27. (5) \$$\frac{11}{16}$. (6) \$5.125. 293
(7) 2 doz. (8) \$13. (9) \$6. (10) 198 men. (11) 7 mi. (12) 8 hr.

443.—(1) \$1.17. (2) \$1.35. (3) 2175 bu. (4) \$120. (5) 7 lb. tea; 15 295
lb. coffee. (6) \$16.45. (7) 135 oranges. (8) 2190 bbl. (9) \$9.60. (10)
15 sheep. (11) \$7.20. (12) $3\frac{1}{4}$ lb. (13) \$$\frac{11}{16}$. (14) \$246. (15) \$465.
(16) $24\frac{1}{3}$ gi.——(17) $7\frac{1}{2}$ ft. (18) 39 ft. (19) $22\frac{1}{2}$ ft. (20) 36 men. (21) 9 296
men. (22) A \$900; B \$600. (23) \$9000. (24) \$78.91. (25) 21 da. (26)

297 25%. (27) 4 min. (28) 3⅓ min. (29) $15. (30) $552.—(31) 1 yr. (32)
10 da. (33) 8 da. (34) 6/11. (35) ⅛. (36) 21⅜ da. (37) 2990 stones.
(38) 6 men. (39) 48 da. (40) 12¢. (41) 144 da.
299 ***444.***—(1) $7.68. (2) 32 pages.—(3) 54 cd. (4) $129.60. (5) 9 hr.
(6) $1368.50. (7) 14 half-barrels. (8) $59.80. (9) $306.60. (10) $63.56 +.
300 (11) 9%. (12) 23 da. (13) 1430 reams. (14) 15 ft.—(15) 31 min. 15 sec.
(16) 77⅓ da. (17) 18⅓ da.
301 ***446.***—(1) 18⅓¢. (2) 90¢. (3) 71%. (4) 20 carats.
303 ***447.***—(1) 1 lb. @ 60¢; 2 lb. @ 70¢; 1 lb. @ 85¢; 1 lb. @ 90¢. (2) 6 lb.
@ 8¢; 3 lb. @ 9¢; 3 lb. @ 10¢. (3) 1 gal. of 60%; 1 gal. of 70°; 2 gal. of
75°; 4 gal. of 90°. (4) 5 bbl. @ $6; 1 bbl. @ $6.50; 1 bbl. @ $7; 3 bbl.
@ $8. (5) 1 gal. @ 20¢; 2 gal. @ 25¢; 2 gal. @ 40¢. (6) 1 gal. each of 50°,
70°, and 80°; 17 gal. of 100°. (7) 1 gal. water; 2 gal. whiskey @ $1.10;
1 gal. @ $1.40; 4 gal. @ $1.50.
304 ***448.***—(1) 1 lb. each @ 50¢, 60¢, 80¢, and 4 lb. @ 75¢. (2) 3 lb. @ 20¢;
8 lb. @ 25¢; 4 lb. @ 30¢; 12 lb. @ 35¢; 8 lb. @ 40¢. (3) 3 parts ⅞ pure;
10 parts ¼ pure; 2 parts ⅝ pure. (4) 3 gal. of 50%; 1 gal. of 55%; 2 gal.
of 60%; 9 gal. of 70%; 2 gal. of 75%. (5) 3 chickens; 6 ducks; 3 geese;
2 turkeys.
305 ***449.***—(1) 24 prs. of each. (2) 30 yd. @ 9¢; 1 yd. each @ 10¢ and 14¢;
15 yd. @ 18¢. (3) 3 lb. each @ 14¢ and 17¢; 8 lb. @ 18¢; 4 lb. @ 21¢; 7
lb. @ 22¢. (4) 30 shares. (5) 10 each at $8 and $10.
306 ***450.***—(1) 80 bu. each @ 90¢ and 95¢; 40 bu. each @ $1.10 and $1.20.
(2) 6⅓ qt. (3) 100 bbl. @ $1.90; 20 bbl. @ $2.25; 10 bbl. @ $2.50. (4)
4 gal. water; 16 gal. rum.
309 ***458.***—(1) 60¢ on $1; $10388.40. (2) $5299.14. (3) Smith $1800;
Owens $1260. (4) Manning & Co. $2931; Whiting & Co. $1490; French
and Sons $950. (5) Miles $15579; Thomas $6855.30; Lewis $3937.50.
(6) Devon $4588.92; Heitmueller $4020.66; Patrick $3650.22; Lackland
310 $3003.21; Hopper $2497.32.—(7) Jordan & Co. $1499.20; Henson $540;
Smith & Son $308; Sinclair $227.20. (8) 82¢ on $1; Whitney $5096.30.
313 ***467.***—(1) Perry & Co. receive $912.86−; Steamer pays $2262.32+. (2)
314 A pays $369.93; B $513.79; C $452.14; D $308.28; Ship receives $1469.14.
(3) $308.27. (4) A receives $2759.69; B $465.36; Ship pays $2810.71; C
$87.30; D $477.04. (5) A $715.56; D $477.04. (6) Ship receives $115.16;
315 B receives $1326.01; A pays $231.98; C pays $1339.19.—(7) A $1785.58;
B $1339.19.
316 ***471.***—(1) $65.80. (2) $60.78. (3) $85.44.
318 ***472.***—(1) $7.60. (2) $21. (3) $115.70.
320 ***473.***—(1) $78.90. (2) $43.80. (3) $220.
322 ***480.***—(1) 6 mo. (2) 3 mo. (3) 4 mo. (4) 4 mo. 3 da. (5) 5 mo. 2 da.
323 (6) 3 mo. 9 da.—(7) July 5. (8) Aug. 20. (9) 7 mo. 8 da. (10) July 19,
1886. (11) 4 mo. (12) 10 mo. (13) Sept. 30, 1885. (14) Sept. 2, 1884.
324 (15) July 11, 1885. (16) Oct. 8, 1886.—(17) Oct. 31, 1885. (18) July 6,
1886.
327 ***482.***—(1) July 3, 1886. (2) Mar. 1, 1886. (3) Apr. 4, 1884. (4) Sept.

30, 1885. (5) $2065; Feb. 13, 1885. (6) Apr. 23, 1886. (7) Apr. 8, 1885. **327**
(8) July 6. (9) Aug. 3. **328**

483.—(1) May 16, 1884. (2) Jan. 19, 1886. (3) May 16, 1886. (4) **329**
Sept. 27, 1886. (5) Dec. 20, 1885. **330**

484.—(1) May 20, 1886. (2) Oct. 29, 1885. (3) Dec. 2, 1885. (4) Apr. **331**
17, 1885.—(5) June 21, 1886. (6) Oct. 9, 1886. (7) Aug. 9, 1884. (8) **332**
July 13, 1885. (9) May 18, 1885. (10) July 1, 1886. (11) Mar. 28, 1885.
(12) June 3, 1884.

487.—(1) July 18. (2) July 2, 1883. (3) Dec. 10, 1884. (4) Jan. 5, **334**
1885. (5) Sept. 1, 1883.—(6) Oct. 26, 1884. (7) Dec. 12, 1886. (8) Sept. **335**
30, 1884. (9) Aug. 30, 1883. (10) Jan. 18, 1885.

489.—(1) May 2, 1886. (2) Oct. 7, 1885. (3) Feb. 6, 1886. (4) Dec. **339**
28, 1885. (5) May 20, 1886. (6) July 30, 1884. (7) Apr. 12, 1886. (8) **340**
May 25, 1885. (9) Apr. 6, 1885. (10) Aug. 20, 1886.—(11) Apr. 28, 1884. **341**
(12) July 16, 1885. (13) Apr. 7, 1886. (14) Aug. 4, 1884. (15) June 22,
1882.—(16) Aug. 3, 1884. (17) Oct. 21, 1884. (18) Jan. 11, 1885. (19) **342**
Jan. 5, 1884. (20) Oct. 12, 1882.—(21) June 20, 1884. (22) Oct. 12, 1885. **343**
(23) Nov. 25, 1884.

491.—(1) Net proceeds, $2917.10; equated date, Mar. 16, 1884. (2) **345**
Net proceeds, $1881.65; equated date, Apr. 8, 1884. (3) Net proceeds, **346**
$10364.11; equated date, Oct. 15, 1883. (4) Net proceeds, $7365.48;
equated date, Apr. 24, 1886. (5) Net proceeds, $2798.93; equated date,
June 8, 1886.

493.—(1) $299.43–. (2) $428.86. (3) $2260.47. (4) $1440.14. (5) **349**
$604.42. **350**

501.—(1) $4024.34. (2) $5222.10. (3) $7013.13. (4) $3998.91. (5) **352**
$6641.10. (6) $1881.25.—(7) $2935. (8) $8430.15. (9) $6129.20. (10) **353**
$989. (11) $11870.70. (12) $3109.50. (13) $13548.31. (14) $15505.92.
(15) $2434.24. (16) $2158.76. (17) $3741.62. (18) $255.80. (19) $25000.
(20) $8980.50. (21) insolvent $1500. (22) capital Jan. 1, 1887, $2406.95;
gain in 1886 $3906.95. (23) net gain $2825.—(24) $3380. (25) $499. **354**
(26) Will $10000; Meyers $6000. (27) Wells $7195.20; McComas $4796.80.
(28) Devon $10000; Green $6000. (29) Insolvency at commencing—Waite
$2000; Foley $2000; Wiley $1000; capital at closing—Waite $1593; Foley
$1593; Wiley $2593.

502.—(1) Langworthy $2100; Shimer $2400; Dickinson $2700. (2) **355**
$870; $1020; $1260. (3) Hodges $1280; Creel $994.—(4) Herrel $2325.55; **356**
Bell $2099.45. (5) Russell $21592.94; Florence $18657.06. (6) Barnette
$4500; Brown $8600. (7) Reid $26647.06; Haines $22352.94. (8) Joyce
$1200; Arnold $900.

503.—(1) A $4998.09; B $4266.67; C $3535.24. (2) A $3566.71; B **358**
$3713.29. (3) Wilson $1758.93; Holmes $1687.50; Farr $1553.57. (4) **359**
Harvey $1900; Roberts $2225; Williams $875. (5) A $4233.05; B $5016.95.
(6) Merritt $2029.75; Reeves $3970.25.

504.—(1) Cross $15708.43; Davies $11091.57. (2) Gould's interest in **362**
firm $17355.13; Hunter's indebtedness $989.60; Nichols receives from **363**

363 Hunter $515.20. (3) Herndon $13291.10; Newman $5597.73; Dulin
$9211.16. (4) Horn $19920.36; McLane $13479.64; Horn $1860.36; Mc-
Lane $13539.64. (5) Hixon's insolvency $1534.01; Merritt's capital $666.19;
Pickett's insolvency $2032.19. (6) Smith $912.78; Harrison $793.69; Yeat-
364 man $633.53.——(7) Whitney $920; Norman $1120. (8) White $21010;
365 Moore $34580.——(9) $5130. (10) net loss $2358.62; A $4685.69; B $6314.69;
sales $1695.55. (11) at commencing, each $331.69½; Hale's cap. at closing
$476.66; Greer's $1111.67. (12) Scott $11703; Gordon $11363; Hall $575.
366 (13) $7620.56. (14) Mackenzie to pay Sonn $262. (15) net gain $3013;
agent owes firm $22826. (16) Coe $5655.81; May $3817.67; Lewis $4072.19;
Hill $3393.49; to reserve profit account $1019.67.

374 ***517.***—(9) $8934.99; gain $934.99. (10) $1741.33. (11) balance
375 $6243.52; loss $2256.48. (12) $1162.50. (13) $752.33. (14) $17319.93.
376 (15) $26989.75.——(16) $875.— (17) $6037.50. (18) $118.75. (19)
377 $12126.67.——(20) $150.

378 ***518.***—(1) 57. (2) 57. (3) 52. (4) 638. (5) 574. (6) 664. (7) 6949.
(8) 6739. (9) 8664781780. (10) 103685041. (11) 1147421109. (12)
11599992904.

379 ***519.***—(1) $14542.93. (2) $3639.58. (3) $8024.49.

380 ***520.***—(1) $572. (2) $792. (3) $396. (4) $583. (5) 275 rd. (6) 198
mi. (7) $176. (8) $891. (9) $297. (10) $495. (11) $374. (12) $682.

381 ***521.***—(1) $56496. (2) $799018. (3) $5663152. (4) $410091.55. (5)
$453952.95. (6) $36033.25. (7) $530895.75. (8) 1043606.30. (9) $167-
625.15.

522.—(1) $139510.50. (2) $147804.75. (3) $158233.30. (4) $131011.65.
(5) $452339.40. (6) $754503.75. (7) $151524.65. (8) $238939.80. (9)
$28434.23.

382 ***523.***—(1) $114007.20. (2) $10598.90. (3) $10617.88. (4) $16656.30.
(5) $46457.10. (6) $22464. (7) $14677.76. (8) $10111.04. (9) $155129.25.
(10) $155322.15. (11) $7473.24. (12) $3216.90.

524.—(1) $525. (2) $756. (3) $384. (4) $810. (5) $5400. (6) $900.
(7) $13000. (8) $14700. (9) $7200. (10) $1600. (11) $630. (12) $12600.
(13) $1200. (14) $1200. (15) $320. (16) $1200.

383 ***525.***—(1) $247715.70. (2) $243540. (3) $60226335. (4) $1087638.75.
(5) $5209451.52. (6) $131602.24. (7) $401026686.72. (8) $167186.25.
(9) $10149180.80.

384 ***526.***—(1) $424575. (2) $84770. (3) $733779.50. (4) $26863.20. (5)
$830062.74. (6) $526.32. (7) $981088. (8) $9603. (9) $1007010.

527.—(1) $1296. (2) $1764. (3) $3136. (4) $7225. (5) $5476. (6)
$8649. (7) $6724. (8) $4096. (9) $1444. (10) $361. (11) $12769.
(12) $14884.

385 ***528.***—(1) $3024. (2) $2204. (3) $1281. (4) $2176. (5) $3999. (6)
$1776. (7) $2975. (8) $3744. (9) $1581. (10) $4784. (11) $12144.
(12) $10125. (13) $11169. (14) $12325. (15) $11264. (16) $11856.

529.—(1) $3074. (2) $5925. (3) $1088. (4) $6885. (5) $4288. (6) $575.
(7) $342. (8) $1968. (9) $8556. (10) $4030. (11) $6972. (12) $2860.

530.—(1) \$625. (2) \$4225. (3) \$2025. (4) \$7225. (5) \$1225. (6) \$5625. (7) \$3025. (8) \$13225. **386**

531.—(1) \$7216. (2) \$5621. (3) \$3024. (4) \$9024. (5) \$621. (6) \$2009. (7) \$625. (8) \$1224. (9) \$11021. (10) \$221. (11) \$9016. (12) \$13216.

532.—(1) \$2436. (2) \$4875. (3) \$1599. (4) \$4896. (5) \$4851. (6) \$4875. (7) \$8064. (8) \$2491. (9) \$8019. (10) \$14384. (11) \$12036. (12) \$9964. **387**

533.—(1) \$35$\frac{1}{16}$. (2) \$55$\frac{1}{4}$. (3) \$16$\frac{1}{64}$. (4) \$38$\frac{1}{16}$. (5) \$33$\frac{1}{4}$. (6) \$39$\frac{1}{3}$. (7) \$30$\frac{1}{4}$. (8) \$56$\frac{1}{4}$. (9) \$72$\frac{1}{4}$.

534.—(1) \$90$\frac{5}{8}$. (2) \$112$\frac{2}{35}$. (3) \$160$\frac{5}{13}$. (4) \$339$\frac{1}{3}$. (5) \$.12$\frac{4}{9}$. (6) \$3681$\frac{2}{35}$. (7) \$16.25$\frac{3}{25}$. (8) \$650$\frac{6}{25}$. (9) \$.28$\frac{4}{7}$. (10) \$.72$\frac{11}{13}$. (11) \$.42$\frac{14}{17}$. (12) \$152$\frac{5}{81}$. **388**

535.—(1) \$276. (2) \$800. (3) \$929$\frac{1}{3}$. (4) \$950. (5) \$2552. (6) \$5952. (7) \$1422. (8) \$2100. (9) \$3363. (10) \$3040. (11) \$1692. (12) \$6989. **389**

536.—(1) \$.232. (2) \$.45. (3) \$36. (4) 3.5 times. (5) \$5.12. (6) \$.1305. (7) \$10.18. (8) \$61.20. (9) \$77.60.

537.—(1) $\frac{aed}{moe}$. (2) $\frac{br}{it}$. (3) $\frac{lh}{me}$. (4) $\frac{ah}{ma}$. (5) $\frac{cs}{om}$. (6) $\frac{led}{mre}$. (7) $\frac{lo}{or}$. (8) $\frac{cbr}{ode}$. (9) $\frac{br}{me}$. (10) $\frac{la}{or}$. **390**

539.—(1) \$4.25. (2) \$1. (3) \$12.50. (4) \$1.25. (5) \$17.50. (6) \$1.80. (7) \$5.52. **391**

540.—(1) \$8.50. (2) 30¢. (3) 91¢. (4) 50¢. (5) 70¢. (6) \$9. **392**